

GENERAL MECHANICS

A MEASURING SYSTEM WITH AN ADDITIONAL CHANNEL FOR ELIMINATING THE DYNAMIC ERROR*

DIMITAR DICHEV, HRISTOFOR KOEV

*Department of Machine and Precision Engineering, Technical University of Gabrovo,
4, Hadji Dimitar St., 5300 Gabrovo, Bulgaria,
e-mails: dichevd@abv.bg, koevh@abv.bg*

PETR LOUDA

*Department of Material Science, Technical University of Liberec,
2, Studentska St., 46117 Liberec, Czech Republic,
e-mail: petr.louda@tul.cz*

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ABSTRACT. The present article views a measuring system for determining the parameters of vessels. The system has high measurement accuracy when operating in both static and dynamic mode. It is designed on a gyro-free principle for plotting a vertical. High accuracy of measurement is achieved by using a simplified design of the mechanical module as well by minimizing the instrumental error. A new solution for improving the measurement accuracy in dynamic mode is offered. The approach presented is based on a method where the dynamic error is eliminated in real time, unlike the existing measurement methods and tools where stabilization of the vertical in the inertial space is used. The results obtained from the theoretical experiments, which have been performed on the basis of the developed mathematical model, demonstrate the effectiveness of the suggested measurement approach.

KEY WORDS: Dynamic error, measuring system, mathematical model, mechano electro-mechanical systems (MEMS).

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Corresponding author e-mail: dichevd@abv.bg

1. Introduction

One of the main features distinguishing the development of measuring equipment today refers to the broader scope of application of the instruments and systems intended for measuring time-varying physical quantities. This is, to a great extent, due to the rapid development of the microprocessor and computer equipment, as well as to their successful application in improving the measuring systems used in the area of dynamic measurements [1, 2]. However, to improve the accuracy characteristics of such systems, we need to work out new and/or to improve the existing measurement methods so as to minimize or eliminate the dynamic error in the measurement result.

The development and improvement of the measuring equipment for determining the parameters of moving objects can be viewed from this perspective. For example, some of these parameters are the ones determining the position of the ship against the sea surface, such as heel, trim, etc. They are dynamic quantities. Therefore the efficiency of the ship steering depends on the accurate and the time data obtained from the ship's measuring systems in relation to the above parameters.

Dynamic measurements are distinguished for the presence of a dynamic error whose basic component depends on the instrument inertia and the dynamic characteristics of the measured and the interfering quantities. This component is referred to as inertial. It usually obtains values which significantly exceed the values of the other components of the measurement error [3, 4]. Within the heel and trim measuring instruments, the inertial component is mainly determined by the deviations of the vertical reproduced in the instrument from the ideal astronomical vertical. The deviations have dynamic characteristics and in most cases they are random time functions resulting from the dynamic actions and the inertia of the elements and the units reproducing the vertical in the specific measuring instrument [5].

The instruments for determining the heel and the trim are designed on the basis of sensors where physical pendulums, gyroscopes and accelerometers are used [6]. The gyroscope verticals have the widest application for reproducing the vertical in this type of measuring instruments and systems [7, 8]. This is mainly due to their physical nature defining their stability in relation to inertial actions caused by the ship motion and the existing at the place, where the instrument is set up. Under the conditions of dynamic actions, these instruments provide relatively high accuracy, which reaches dynamic error values up to several tenths of a degree for the best samples [9]. On the other hand, the measuring instruments built on the basis of gyro-verticals are distinguished for

a number of disadvantages such as a sophisticated design, less reliability under extreme conditions, requirement of special systems ensuring the gyro-vertical operation; large sizes, high prices, etc. [10], thus limiting, to a great extent, their application.

In terms of metrology, the disadvantage related to the sophisticated design of these instruments has the greatest impact on their quality. This leads to an increase of the value of the instrumental error which is of vital importance for the measurement accuracy in static mode [11]. Taking into consideration, that the operating values of the heel and trim are set in this mode linked to the ship loading, it is obvious that instrument accuracy characteristics both in dynamic and static mode play a key role. Hence, in a number of cases, this limits the practical applications of gyro-vertical-based instruments for heel and trim measurement.

The disadvantages presented above can be overcome by designing a measuring system based, on one hand, on a very simplified mechanical module, and on the other hand, on the possibilities of modern microprocessor and computer equipment, as well as by successfully integrated processing algorithms [12] intended for minimizing the dynamic error.

On the basis of the analysis performed so far, it can be summarized that within the existing methods and tools for measuring the parameters of moving objects the dynamic accuracy is ensured by stabilizing the vertical in the inertial space. Unlike the above mentioned approaches, the measuring system considered in this article is based on a different concept targeting the elimination of the dynamic error caused by its deviation from the inertial space in real time rather than the stabilization of the vertical.

Therefore the purpose of this article is to develop and present the structural and mathematical model of a heel and a trim measuring system with an additional channel for elimination the inertial component of the dynamic error, and based on the above principles.

2. A structural model of the measuring system

The basic concept of the present measuring system is focused on the simplified design of the vertical in the form of a physical pendulum. On one hand, the comparatively simple design of the mechanical module, consisting of a small number of elements, results in limiting the magnitude of the instrumental error. On the other hand, the error caused by the instability of this type of a vertical in the inertial space, due to the dynamic actions of the ship motion, can be eliminated by obtaining real-time additional information on the

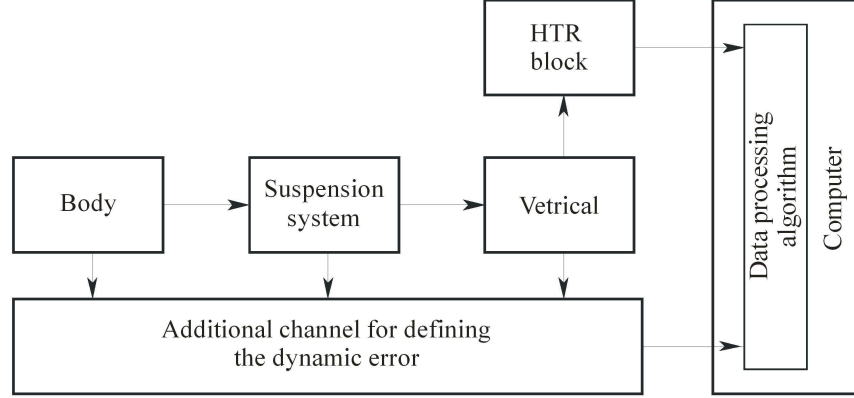


Fig. 1. Block diagram of the measuring system for defining the ship's heel and trim

pendulum deviations from the electro-mechanical systems (MEMS) positioned in the measuring circuit.

The concept of developing the data processing algorithm is based on a model minimizing the dynamic error due to the deviations of the physical pendulum from the ideal astronomical vertical. These principles can be used to design a measuring system of high metrological characteristics achieved both in static and dynamic mode of measurement.

The block diagram of such a system used for measuring the ship's heel and trim is shown in Fig. 1. The body of the mechanical module is fixed to the ship. By means of a suspension system a physical pendulum of two degrees of freedom, reproducing the instrument's astronomical vertical, is mounted on the body. The suspension system consists of two cylindrical joints connected in series. The joints have inter perpendicular axes of rotation, which intersect at one point. The data about the heel and the trim angles is obtained from a block registering the heel and trim angles (HTR block), which consists of two absolute encoders connected to the respective measuring axis.

The measurement accuracy in dynamic mode is ensured by an additional channel for determining the dynamic error. It consists of two pairs of identical MEMS accelerometers used for measuring the linear acceleration. The accelerometers are mounted on the body of the mechanical module, on the first cylindrical joint (two accelerometers), and on the physical pendulum, respectively. The first two accelerometers are mounted in such a way that their measuring axes are sensitive to the accelerations generated by the roll whereas the measuring axes of the other two accelerometers are sensitive to the accel-

ations generated by the pitch. This circuit of mounting of the MEMS sensors ensures the sensitivity of the first accelerometer of each sensor pair to all accelerations generated by the roll and the pitch. Every second accelerometer is sensitive not only to the accelerations of the first sensor but also to those generated by the pendulum motion according to its degree of freedom. This makes possible the development of a procedure involving the subtraction of the signals from the first and the second accelerometer of each sensor pair where the output signals are proportional to the accelerations generated by the pendulum motion according to its degree of freedom. A double integration of the accelerations is performed by means of a data processing algorithm, where signals defining the pendulum deviations from the vertical according to its two degrees of freedom are obtained.

As it was mentioned earlier, within instruments measuring the ship's heel and trim the inertial component of the dynamic error is mainly determined by the deviations of the vertical reproduced in the instrument from the ideal astronomical vertical. Therefore, the signals obtained represent the values of the dynamic error in the form of a measuring procedure.

Due to the simplified design of the mechanical module, it can be assumed, with accuracy enough for practical solutions, that the measuring systems under consideration are linear. They have the significant property of superposition, upon which it can be claimed that the output signals obtained from the block registering the heel and the trim angles (HTR block) are the sum of the desired signal and the dynamic error defined by the procedure described above. Then, the dynamic error can be eliminated from the measurement result by subtracting the signals from the output of the HTR block and those obtained

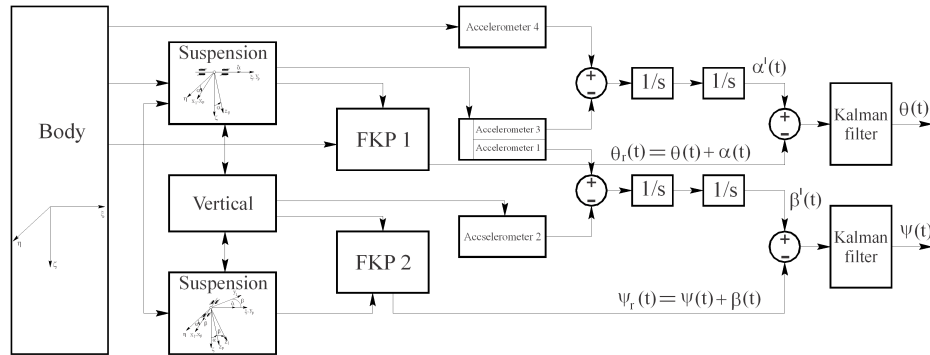


Fig. 2. Operating diagram of the measuring system

as a result of determining the dynamic error, which is actually underpinned in the data processing algorithm.

The operation of the measuring system is illustrated in details in Fig. 2. Within the metrological circuit of the diagram, the absolute encoders (FKP1 and FKP2) are mounted along the basic measuring axes and register the angles of rotation of the instrument's body in relation to the vertical in two inter-perpendicular directions. Some measurement information on the change of these quantities is obtained at the outputs of FKP1 and FKP2, since the instrument's body is firmly fixed to the ship and its measuring axes are set so as to be sensitive to the roll θ and pitch ψ . The information is registered with regard to the time coordinate in the form of the functions $\theta_r(t)$ and $\psi_r(t)$. The instability of the physical pendulum in the inertial space generates a dynamic error as a result whose characteristics are defined by the functions $\alpha(t)$ and $\beta(t)$. The latter are formed by the pendulum deviations from the ideal astronomical vertical, generated along the two measuring coordinates of the instrument. Then the values obtained from the two measuring channels equal the sums of the desired signals and the respective dynamic errors, i.e. $\theta_r(t) = \theta(t) + \alpha(t)$; $\psi_r(t) = \psi(t) + \beta(t)$, where $\theta(t)$ and $\psi(t)$ are the signals defining the actual values of the heel and the trim in relation to the time coordinate, respectively.

To sum up, within the selected measurement approach the high dynamic accuracy is achieved by determining and eliminating the dynamic error in real time. The dynamic errors $\alpha(t)$ and $\beta(t)$ included in the two measuring channels are determined by means of a HTR block and a data processing algorithm. The position of the four identical accelerometers of the HTR block has been clarified in the operating diagram in Fig. 2 and their operation has been described in details above. To clear the signals $\alpha(t)$ and $\beta(t)$ from the additional interference effects generated in the accelerometer measuring channels, a Kalman filter is envisaged to be used. Its parameters and characteristics will be a subject of another article.

3. A mathematical model of the measuring system

The dynamic characteristics of the instruments under consideration refer to their metrological characteristics as they affect the formation of the error obtained as a result of the measurement. The mathematical model presents the main ratios of the measuring system, the measured and the interfering quantities, in a form which is suitable for analytical study of the dynamic characteristics. The model is developed on the basis of the block and the operating diagrams described above. The differential equations related to the motion of

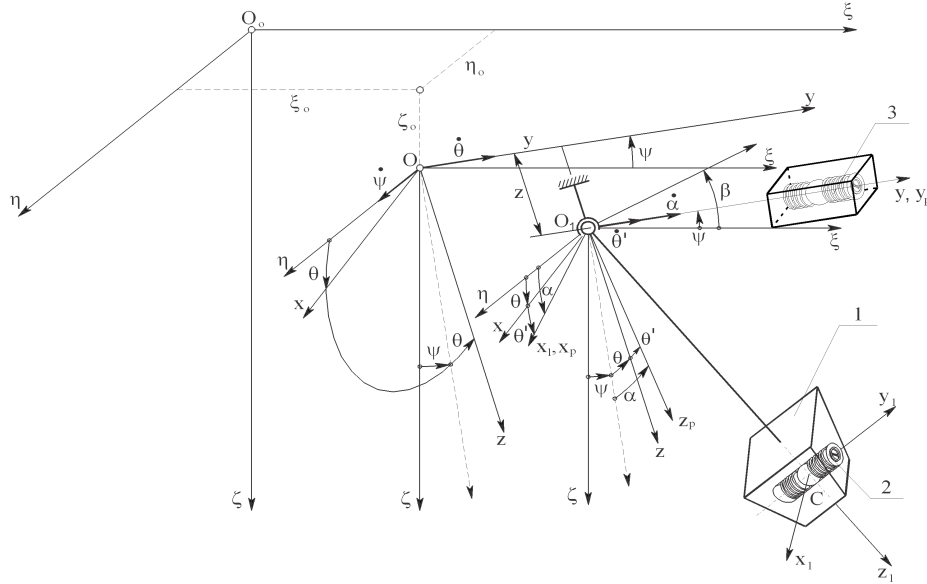


Fig. 3. Diagram of the dynamic system *moving object – measuring instrument*. 1 – physical pendulum; 2 – first accelerometer; 3 – second accelerometer; O_0, η, ξ, ζ – supporting coordinate system; O, x, y, z – coordinate system connected to the ship; C, x_1, y_1, z_1 – coordinate system connected to the physical pendulum; O, x_p, y_p, z_p – intermediate coordinate system; η_0, ξ_0, ζ_0 – coordinates of the position of the ship's centre of gravity in relation to the supporting coordinate system

the instruments' inertial components present the most complete description of the dynamic characteristics of the instruments. The equations are worked out for operating conditions close to the real ones. The latter are mainly defined by the ship's motion in real wind-generated rough seas. They represent a sophisticated dynamic process which can be considered as a set of deviations according to each degree of freedom.

In order to make the mathematical operations easier, only one of the two measuring channels will be viewed – that related to the trim. In addition, the mathematical model of the accelerometer mounted on the cylindrical joint can be easily deduced from the equations of the second accelerometer positioned on the instrument pendulum, due to which it will not be taken into account when working out the differential equations.

The differential equations are worked out on the basis of a dynamic system shown in Fig. 3. The motions of a ship are defined as angular and lin-

ear fluctuations of a rigid body around or along with its centre of gravity. The moving object (the ship), to which the coordinate system O, x, y, z is connected, changes its position randomly in relation to the supporting system O_0, ξ, η, ζ . The measuring instrument is mounted on the ship and its sensor (the physical pendulum) is connected to the coordinate system C, x_1, y_1, z_1 . The suspension point O_1 of the instrument's sensor coincides with the diametral plane of the ship and its position with regard to the centre of gravity of the moving object O is defined by the z and y coordinates. The position of the moving object in relation to the supporting system O_0, ξ, η, ζ is set by the three coordinates of its centre of gravity $O - \xi_0, \eta_0, \zeta_0$ and the matrix $A = \|a_{ij}\|$ ($i, j = 1, 2, 3$) of the given angle cosines of the trim ψ and the heel θ , defining the angular displacement between the axes of the systems O_0, ξ, η, ζ and O, x, y, z . The mechanical system consists of two bodies – a physical pendulum which is free to rotate with regard to the coordinate axes O_1x_1 and O_1y_1 , and an accelerometer mounted in the centre of gravity C of the physical pendulum. The accelerometer's inertial body of a mass m_2 stays at an equilibrium position in relation to the y_1 coordinate by means of two horizontal springs of an elastic constant c .

Therefore, the mechanical system has three degrees of freedom and the generalized coordinates are, α, β and y_1 , respectively. The α and β coordinates define the angular displacement of the physical pendulum from the vertical in relation to O_1y_1 and O_1x_1 axes, respectively, whereas y_1 determines the relative motion of the inertial mass m_2 . By means of the β coordinate the inertial component of the dynamic error for the measuring channel under consideration is defined. The latter determines the trim values of a ship.

The Lagrangian method is used upon working out the differential equations. The kinetic energy of the system is the sum of the kinetic energies of two bodies. The first one is a physical pendulum of two degrees of freedom and the second is the moving mass of an accelerometer of one degree of freedom.

$$(1) \quad E_k = E_{k1} + E_{k2}.$$

The kinetic energy of the first body is defined according to König's theorem related to rigid bodies, i.e.:

$$(2) \quad E_{k1} = \frac{1}{2} \cdot m_1 \cdot V_C^2 + \frac{1}{2} \cdot J_{C\omega} \cdot \omega^2,$$

where m_1 – the mass of the first body; $V_C = \sqrt{[\dot{\eta}_C(t)]^2 + [\dot{\xi}_C(t)]^2 + [\dot{\zeta}_C(t)]^2}$ – the absolute velocity of the point C ; $J_{C\omega}$ – the body's moment of inertia

in relation to the current axis C_ω through the body centre of mass; ω – the absolute velocity of the body.

Since the C, x_1, y_1, z_1 coordinate system is constantly connected to the physical pendulum, its inertial characteristics remain constant in time. In this case, the sensor's mass moments of inertia with regard to the coordinate axes of C, x_1, y_1, z_1 remain constant. Then, for the second addend in (2), which defines the rotary motion of the sensor, the following is obtained:

$$(3) \quad T_r = \frac{1}{2} \cdot J_{C\omega} \cdot \omega^2 = \frac{1}{2} \cdot (J_{x_1} \cdot \omega_{x_1}^2 + J_{y_1} \cdot \omega_{y_1}^2 + J_{z_1} \cdot \omega_{z_1}^2 - 2 \cdot J_{x_1 y_1} \cdot \omega_{x_1} \cdot \omega_{y_1} - 2 \cdot J_{x_1 z_1} \cdot \omega_{x_1} \cdot \omega_{z_1} - 2 \cdot J_{y_1 z_1} \cdot \omega_{y_1} \cdot \omega_{z_1}),$$

where $J_{x_1}, J_{y_1}, J_{z_1}$ are the sensor's mass moments of inertia in relation to the respective axes of the C, x_1, y_1, z_1 system; $J_{x_1 y_1}, J_{x_1 z_1}, J_{y_1 z_1}$ are the centrifugal mass moments of inertia of the body with regard to the respective axes of the C, x_1, y_1, z_1 system.

The axes of C, x_1, y_1, z_1 are assumed to be the principal axes of inertia of the body, i.e. $J_{x_1 y_1} = J_{x_1 z_1} = J_{y_1 z_1} = 0$, by means of which the above expression is simplified to:

$$(4) \quad T_r = \frac{1}{2} \cdot (J_{x_1} \cdot \omega_{x_1}^2 + J_{y_1} \cdot \omega_{y_1}^2 + J_{z_1} \cdot \omega_{z_1}^2).$$

The projections of the angular velocity ω on the axes of the C, x_1, y_1, z_1 system can be derived from Fig. 3, and their final expression is:

$$(5) \quad \begin{aligned} \omega_{x_1} &= \dot{\beta} + \dot{\psi} \cdot \cos \alpha; \\ \omega_{y_1} &= \dot{\theta} \cdot \cos(\beta - \psi) + \dot{\alpha} \cdot \cos(\beta - \psi) + \dot{\psi} \cdot \sin \alpha \cdot \sin(\beta - \psi); \\ \omega_{z_1} &= \dot{\theta} \cdot \sin(\beta - \psi) + \dot{\alpha} \cdot \sin(\beta - \psi) - \dot{\psi} \cdot \sin \alpha \cdot \cos(\beta - \psi). \end{aligned}$$

The kinetic energy of the second body is equal to the sum of the kinetic energies of the two components of the absolute motion, i.e.:

$$(6) \quad E_{k2} = \frac{1}{2} \cdot m_2 \cdot V_m^2 + \frac{1}{2} \cdot m_2 \cdot \dot{y}_1^2,$$

where m_2 – the mass of the inertial component of the accelerometer; $V_m = \sqrt{[\dot{\eta}_m(t)]^2 + [\dot{\xi}_m(t)]^2 + [\dot{\zeta}_m(t)]^2}$ – the absolute velocity of the mass m_2 ; $\dot{y}_1$ – the relative velocity of the mass m_2 .

After doing all necessary mathematical operations for the final definition of (2) and (6), and after substituting in (1), the following final formula for the kinetic energy of the dynamic system under study is obtained:

$$\begin{aligned}
 E_k = & q_1 \cdot \dot{\eta}_0^2 + q_2 \cdot \dot{\xi}_0^2 + q_3 \cdot \dot{\zeta}_0^2 + q_4 \cdot \dot{\theta}^2 + q_5 \cdot \dot{\psi}^2 + q_6 \cdot \dot{\eta}_0 \cdot \dot{\theta} + q_7 \cdot \dot{\xi}_0 \cdot \dot{\theta} + \\
 & + q_8 \cdot \dot{\zeta}_0 \cdot \dot{\theta} + q_9 \cdot \dot{\eta}_0 \cdot \dot{\psi} + q_{10} \cdot \dot{\xi}_0 \cdot \dot{\psi} + q_{11} \cdot \dot{\zeta}_0 \cdot \dot{\psi} + q_{12} \cdot \dot{\theta} \cdot \dot{\psi} + q_{13} \cdot \dot{\beta}^2 + \\
 & + q_{14} \cdot \dot{\alpha}^2 + q_{15} \cdot \dot{\eta}_0 \cdot \dot{\beta} + q_{16} \cdot \dot{\xi}_0 \cdot \dot{\beta} + q_{17} \cdot \dot{\zeta}_0 \cdot \dot{\beta} + q_{18} \cdot \dot{\eta}_0 \cdot \dot{\alpha} + q_{19} \cdot \dot{\xi}_0 \cdot \dot{\alpha} + \\
 & + q_{20} \cdot \dot{\zeta}_0 \cdot \dot{\alpha} + q_{21} \cdot \dot{\theta} \cdot \dot{\beta} + q_{22} \cdot \dot{\theta} \cdot \dot{\alpha} + q_{23} \cdot \dot{\psi} \cdot \dot{\beta} + q_{24} \cdot \dot{\psi} \cdot \dot{\alpha} + q_{25} \cdot \dot{\alpha} \cdot \dot{\beta} + \\
 (7) \quad & + q_{26} \cdot \dot{y}_1^2 + q_{27} \cdot \dot{y}_1 \cdot \dot{\beta} + q_{28} \cdot y_1 \cdot \dot{y}_1 \cdot \dot{\beta} + q_{29} \cdot y_1 \cdot \dot{y}_1 \cdot \dot{\alpha} + q_{30} \cdot \dot{y}_1 \cdot \dot{\alpha} + \\
 & + q_{31} \cdot \dot{y}_1 \cdot \dot{\zeta}_0 + q_{32} \cdot \dot{y}_1 \cdot \dot{\xi}_0 + q_{33} \cdot \dot{y}_1 \cdot \dot{\theta} + q_{34} \cdot \dot{y}_1 \cdot \dot{\psi} + q_{35} \cdot y_1^2 \cdot \dot{\beta}^2 + \\
 & + q_{36} \cdot y_1^2 \cdot \dot{\alpha}^2 + q_{37} \cdot y_1^2 \cdot \dot{\beta} \cdot \dot{\alpha} + q_{38} \cdot y_1 \cdot \dot{\beta}^2 + q_{39} \cdot y_1 \cdot \dot{\beta} \cdot \dot{\alpha} + \\
 & + q_{40} \cdot y_1 \cdot \dot{\zeta}_0 \cdot \dot{\beta} + q_{41} \cdot y_1 \cdot \dot{\xi}_0 \cdot \dot{\alpha} + q_{42} \cdot y_1 \cdot \dot{\xi}_0 \cdot \dot{\beta} + q_{43} \cdot y_1 \cdot \dot{\theta} \cdot \dot{\beta} + \\
 & + q_{44} \cdot y_1 \cdot \dot{\psi} \cdot \dot{\beta} + q_{45} \cdot y_1 \cdot \dot{\psi} \cdot \dot{\alpha} + q_{46} \cdot y_1 \cdot \dot{\theta} \cdot \dot{\alpha} + q_{47} \cdot y_1 \cdot \dot{\alpha}^2,
 \end{aligned}$$

where $q_1, q_2, \dots, q_{47}$ are the coefficients depending on the design parameters of the instrument's inertial components, on the angular quantities representing time functions and determining the position of the ship in relation to the water surface and the position of the pendulum with regard to the ideal vertical, as well as on the geometric parameters defining the position of the measuring instrument in relation to the ship's centre of gravity.

The number of Lagrange's equations used for writing the differential equations are three as well, since the generalized co-ordinates α, β and y_1 , in relation to which some solutions of the model are sought, are three. After defining the generalized forces and substituting them in Lagrange's equations along with the derivatives of (7) in relation to the generalized coordinates and time, we obtain the differential equations of the dynamic system under study, whose matrix form could be written as follows:

$$\begin{aligned}
& \left\| \begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{y}_1 \end{matrix} \right\| + \left\| \begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \\ \dot{y}_1^2 \end{matrix} \right\| + \\
& + \left\| \begin{matrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{y}_1 \end{matrix} \right\| + \left\| \begin{matrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} \alpha \\ \beta \\ y_1 \end{matrix} \right\| + \left\| \begin{matrix} d_1 \\ d_2 \\ d_3 \end{matrix} \right\| \cdot \dot{\alpha} \cdot \dot{\beta} + \\
& + \left\| \begin{matrix} h_1 \\ h_2 \\ h_3 \end{matrix} \right\| \cdot \dot{y}_1 \cdot \dot{\alpha} + \left\| \begin{matrix} g_1 \\ g_2 \\ g_3 \end{matrix} \right\| \cdot \dot{y}_1 \cdot \dot{\beta} + \left\| \begin{matrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} y_1 \cdot \ddot{\alpha} \\ y_1 \cdot \ddot{\beta} \\ y_1 \cdot \ddot{y}_1 \end{matrix} \right\| + \\
(8) \quad & + \left\| \begin{matrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} y_1 \cdot \dot{\alpha}^2 \\ y_1 \cdot \dot{\beta}^2 \\ y_1 \cdot \dot{y}_1^2 \end{matrix} \right\| + \left\| \begin{matrix} u_1 \\ u_2 \\ u_3 \end{matrix} \right\| \cdot y_1 \cdot \dot{\alpha} \cdot \dot{\beta} = \left\| \begin{matrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{matrix} \right\| \cdot \left\| \begin{matrix} \ddot{\eta}_0 \\ \ddot{\xi}_0 \\ \ddot{\zeta}_0 \end{matrix} \right\| + \\
& + \left\| \begin{matrix} e_{11} & e_{12} \\ e_{21} & e_{22} \\ e_{31} & e_{32} \end{matrix} \right\| \cdot \left\| \begin{matrix} \ddot{\theta} \\ \ddot{\psi} \end{matrix} \right\| + \left\| \begin{matrix} f_{11} & f_{12} \\ f_{21} & f_{22} \\ f_{31} & f_{32} \end{matrix} \right\| \cdot \left\| \begin{matrix} \dot{\theta}^2 \\ \dot{\psi}^2 \end{matrix} \right\| + \left\| \begin{matrix} n_1 \\ n_2 \\ n_3 \end{matrix} \right\| \cdot \dot{\theta} \cdot \dot{\psi} + \\
& + \left\| \begin{matrix} p_{11} & p_{12} \\ p_{21} & p_{22} \\ p_{31} & p_{32} \end{matrix} \right\| \cdot \left\| \begin{matrix} \dot{\theta} \cdot \dot{\alpha} \\ \dot{\theta} \cdot \dot{\beta} \end{matrix} \right\| + \left\| \begin{matrix} r_{11} & r_{12} \\ r_{21} & r_{22} \\ r_{31} & r_{32} \end{matrix} \right\| \cdot \left\| \begin{matrix} \dot{\psi} \cdot \dot{\alpha} \\ \dot{\psi} \cdot \dot{\beta} \end{matrix} \right\|,
\end{aligned}$$

where the elements of the separate matrixes are functions of the parameters defining the geometric and the mass inertial characteristics of the measuring instrument, as well as the quantities determining the position of the ship and the instrument's sensors with regard to their equilibrium position.

The differential equations are formed under the influence of a great number of parameters which can be divided in the following groups, depending on the defining factors: parameters defining the characteristics of the sea roughness; parameters defining the shape, the dimensions, the geometric and the mass characteristics of the ship; parameters defining the position of the vessel in relation to the wave direction and kinematics parameters defining its motion; hydrodynamic parameters defining the interaction between the water

and the ship; geometric and mass parameters defining the design of the instrument; as well as parameters defining its position with regard to the ship's centre of gravity.

The mathematical model determined by equations (8) describes best the properties and characteristics of the measuring system because it expresses the interrelation between the system's readings and the values of the measured quantity, the design parameters and the influencing quantities. This model allows us to predict the measurement results when the system operates under different running conditions, as well as to optimize its design and parameters, thus subjecting their selection to the conditions that best realize a minimum error measurement.

Equations (8) are reduced to simplified form by linearizing the quantities defining the movement of the system's points and by including only small first-order quantities. As a result, the following system of differential equations is obtained:

$$(9) \quad \begin{cases} (J_{y_1} + m_1.l^2) \cdot \ddot{\alpha} + k_\alpha \cdot \dot{\alpha} + m_1.g.l.\alpha = m_1.l.\ddot{\eta}_0 - (J_{y_1} + m_1.l.z) \cdot \ddot{\theta}; \\ (J_{x_1} + m_1.l^2) \cdot \ddot{\beta} + k_\beta \cdot \dot{\beta} + m_1.g.l.\beta = -m_1.l.\ddot{\xi}_0 - (J_{x_1} + m_1.z.l) \cdot \ddot{\psi}; \\ m_2.\ddot{y}_1 + k_{y_1} \cdot \dot{y}_1 + c.y_1 = -\frac{1}{2}m_2 \cdot (l.\ddot{\beta} + \ddot{\xi}_0 + z.\ddot{\psi}), \end{cases}$$

where k_α , k_β and k_{y_1} are damping coefficients according to the three generalized coordinates α , β and y_1 ; l – the length of the physical pendulum.

The third equation in (9) defines the link between the readings of the accelerometer mounted on the physical pendulum and the quantities entering the input of the instrument. A differential equation representing the motion of the sensor of the second accelerometer can be easily worked out from this equation, taking into account the identical design characteristics of the two sensors and the fact that this accelerometer is not sensitive to the motion of the physical pendulum. Then the differential equation will be:

$$(10) \quad m_2.\ddot{y}_p + k_{y_p} \cdot \dot{y}_p + c.y_p = -\frac{1}{2}m_2 \cdot (\ddot{\xi}_0 + z.\ddot{\psi}),$$

where y_p is the coordinate of the motion of the sensor of the second accelerometer.

In this case, the difference between the readings of the two accelerometers will be proportional to the function $\ddot{\beta}(t)$, by means of which we can easily determine the quantity $\beta(t)$ defining the dynamic error.

Actually, additional interferences are superimposed on the signal. They can be easily identified if equations (8) are set in a form where first- and second-order quantities are used, i.e.:

$$\begin{aligned}
 (11) \quad & \left(J_{y_1} + m_1.l^2 \right) .\ddot{\alpha} + b_1.\dot{\alpha} + m_1.g.l.\alpha = \\
 & = m_1.l. \left(\ddot{\eta}_0 + \alpha.\ddot{\zeta}_0 \right) - (J_{y_1} + m_1.l.z) .\ddot{\theta} + \\
 & + m_1.l.y. \left(\theta.\ddot{\psi} - \alpha.\ddot{\psi} - 2.\dot{\alpha}.\dot{\psi} + 2.\dot{\theta}.\dot{\psi} + \psi.\ddot{\theta} \right) \\
 & (J_{x_1} + m_1.l^2) .\ddot{\beta} + 2.m_2.l.\beta.\ddot{y}_1 + b_2.\dot{\beta} + \\
 & + m_1.g.l.\beta = m_1.l. \left(\beta.\ddot{\zeta}_0 - \ddot{\xi}_0 \right) + \\
 & + m_2. \left(\ddot{\zeta}_0 - y.\ddot{\psi} \right) .y_1 - (J_{x_1} + m_1.z.l) .\ddot{\psi} + \\
 & + m_1.y.l. \left(\psi.\ddot{\psi} - \beta.\ddot{\psi} + \dot{\psi}^2 \right) \\
 & m_2.\ddot{y}_1 + k_{y_1}.\dot{y}_1 + c.y_1 = \frac{1}{2}.m_2. \left(\ddot{\zeta}_0.\beta - l.\ddot{\beta} - y.\ddot{\psi}.\beta \right) + \\
 & + \frac{1}{2}.m_2. \left[y. \left(\ddot{\psi}.\psi + \dot{\psi}^2 \right) - z.\ddot{\psi} - \ddot{\xi}_0 \right].
 \end{aligned}$$

It can be seen in the third equation in (11), that the functions $\ddot{\zeta}_0.\beta$ and $y.\ddot{\psi}.\beta$ appear on the right hand side. They are superimposed on the signal $\ddot{\beta}(t)$. Although the values of these functions are formed as small quantities of second order in relation to $\ddot{\beta}(t)$, they introduce an additional error when determining the quantity $\beta(t)$. Therefore, they have to be eliminated from the signal before the final formation of the function $\beta(t)$. In this case, the use of Kalman filter is very suitable. Its position in the measuring procedure is given in the operating diagram shown in Fig. 2.

4. Results from the theoretical study

A simulation model of the dynamic system *moving object - measuring instrument* has been developed (Fig. 4) to study the logic of operation of the measuring system and to ensure the theoretical experiments. The latter has been built by using Simulink, which is part of the Matlab software package. The model is designed on the basis of the differential equations (9) and follows the sequence of logic of the operating diagram presented in Fig. 2.

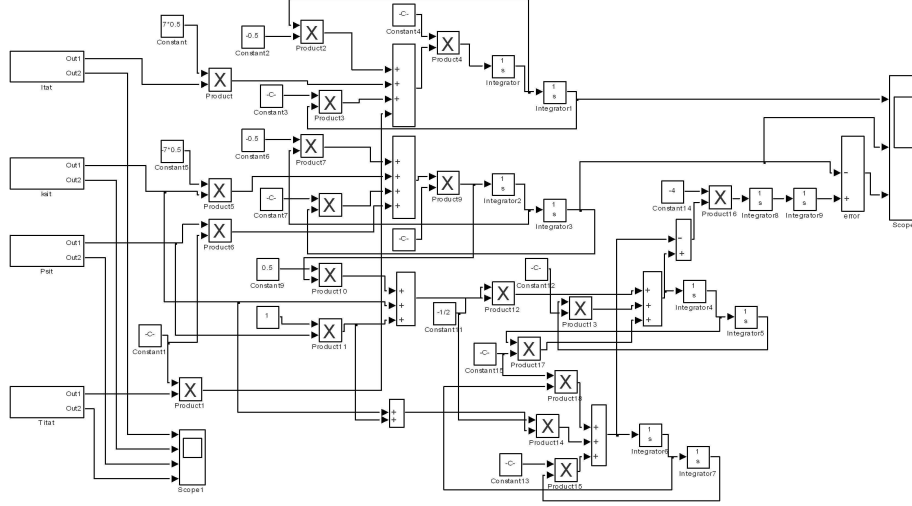


Fig. 4. Simulation model of the dynamic system *moving object – measuring instrument*

The disturbances and the measured quantity are modelled by means of four subsystem blocks. The third one is involved in the formation of the quantity measured $\psi(t)$. The block diagram allows the signals in these blocks to be set in the form of both harmonic time functions and stochastic processes.

The results from the operation of the simulation model are presented in a graphic form by two Scope blocks. Within the first Scope block, the metrological accuracy of the measuring system operating without and with the additional channel for defining the dynamic error, respectively is illustrated. Figure 5 presents concrete results, where the first graph defines the deviations $\alpha(t)$ of the physical pendulum in relation to the astronomical vertical along the α co-ordinate. The second graph shows the values of the dynamic error $\beta(t)$ obtained upon measuring without using the additional channel. The third graph in Fig. 5 illustrates the result from the operation of the additional channel by means of the dynamic error obtained in this case.

It can be seen from the graphs, that due to the proposed solution for improving measurement accuracy the dynamic error decreases ten times and is reduced to a permissible range. The theoretical studies show, that when using this measuring approach, the maximal value of the dynamic error does not exceed $0.3 [^\circ]$. It could be seen, that in the transient period, where the influence of the auto-fluctuations is significant (graphs 1 and 2 in Fig. 5), the

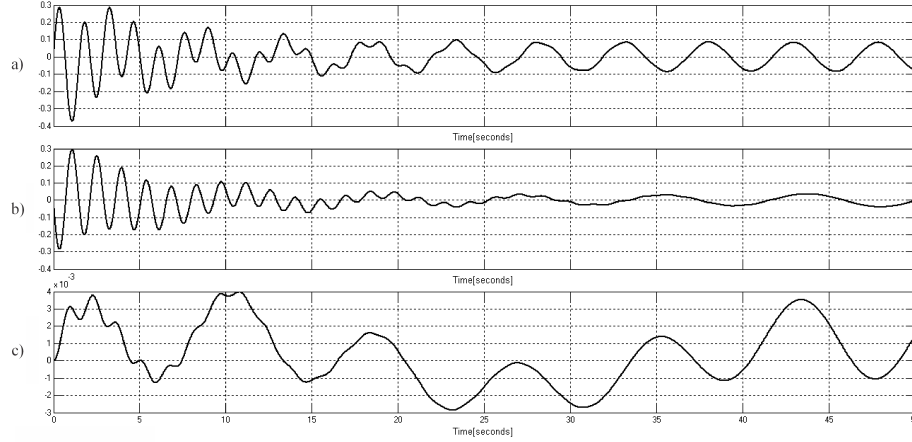


Fig. 5. Values of the dynamic error

dynamic error (graph 3 in Fig. 5) does not exceed its value by more than 8%. Actually, this proves the stability of the measuring procedure in relation to the characteristics of the functions $\alpha(t)$ and $\beta(t)$. In this case, the solutions are developed by the fourth-order Runge-Kutta method. The values of the external actions under which these results have been obtained are presented in a graphic form in Fig. 6. The first two graphs define the values of the sway and the surge of the ship, respectively whereas the other two are the pitch and the roll. Taking into consideration that pitch is the quantity measured, the relative dynamic error of its measurement, in this particular case, will be 3%.

a) Values of the dynamic error when the heel is measured without using the additional channel [rad]; b) Values of the dynamic error when the trim is measured without using the additional channel [rad]; c) Values of the dynamic error when the trim is measured with the involvement of the additional channel [rad]

The simulation model makes possible the determination of the optimal values of the design parameters influencing the dynamic characteristics of the measuring system, by means of which maximum dynamic accuracy is achieved. The design of the mechanical module of the measuring system has been developed on this basis. Illustrations of the prototype are presented in Fig. 7. It consists of a physical pendulum (1) with two degrees of freedom. Two identical absolute encoders (2) of FKP 13.A type and 213-bit resolution are mounted on its axes of rotation. Two MEMS accelerometers (3 and 4) of SCA2100-D01 type are used for the realization of the additional channel in compliance with the operating diagram shown in Fig. 2.

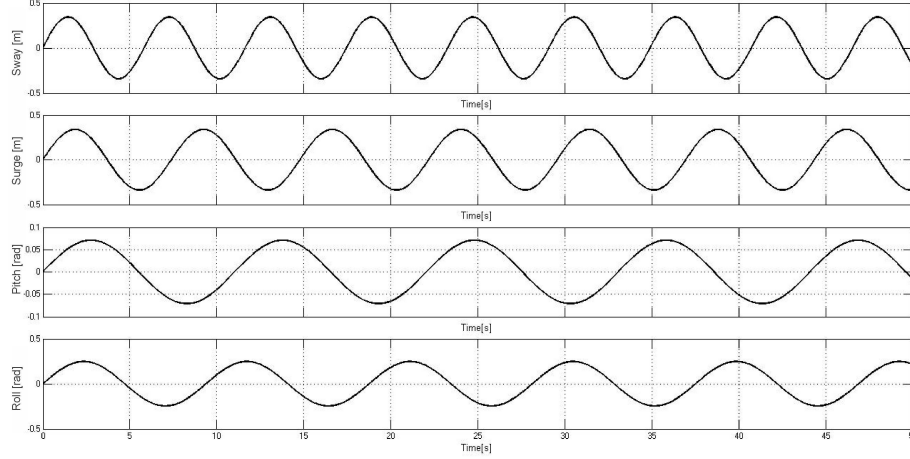


Fig. 6. Values of the external actions and the measured quantity

5. Conclusions

The measuring system, proposed in this article, ensures high accuracy of measurement of the ship's heel and trim both in static and dynamic operating mode. The metrological circuit of the system is based on a gyro-free design of the vertical unlike most systems built on this principle. The simplified design of the system's mechanical module guarantees a minimal instrumental error of the measuring system, which appears of great importance for ensuring the accuracy of measurement, especially in static mode. The measurement error in this mode does not exceed 2 ['] for the whole measuring interval. The sensors and measuring transducers used in the system are the comparatively cheap MEMS accelerometers and absolute encoders, which actually ensures the low cost of the system.

The high measurement accuracy of the system in dynamic mode is provided by an additional measuring channel. Its principle of operation is based on eliminating the dynamic error due to the deviation of the vertical in the inertial space in real time. When a Kalman filter is connected to the system, the measurement accuracy in dynamic mode is increased because the filter reduces not only the secondary fluctuations caused by external actions, defined by (9), but also the influence of the main interference presented in this article. In this case, the dynamic error is within the range of 0.1–0.2 [°], thus, outlining the positive perspectives for the practical application of the proposed

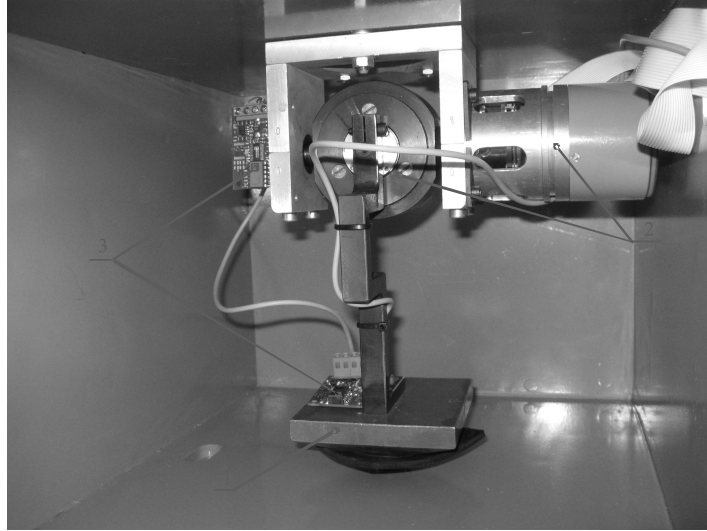


Fig. 7. Prototype of the mechanical module of the measuring system. 1 – physical pendulum; 2 – absolute encoders of FKP 13.A type and 213-bit resolution; 3 and 4 – accelerometers of SCA2100-D01 type

approach. The analysis related to the application of the Kalman filter into this measuring system will be a subject of another article.

The correctness of the suggested approach for eliminating the dynamic error has been proven in the present article by mathematical means defined on the basis of the dynamic system “*moving object – measuring instrument*”. The results from the theoretical experiments performed with the help of the simulation model show the effectiveness of the measuring approach proposed.

REFERENCES

- [1] PRONKIN, N. S. Basics of Metrology for Dynamic Measurements, Moscow, Logos, 2003.
- [2] STEPETOV, A. G. Theory, Calculation and Design of Measuring Instruments, Moscow, Standartinform, 2006.
- [3] NAZAROV, N. G. Metrology. Basic Concepts and Mathematical Models, Moscow, Vishshaya Shkola, 2002.
- [4] GRANOVSKIY, V. A. Dynamic Measurements, St. Petersburg, Energoatomizdat, 1984.

- [5] RIVKIN, S. S. Definition of Dynamic Errors of Gyro-Instruments on a Moving Base, Moscow, Azimut, 1991.
- [6] IVANOV, Y. V. Autonomous Sensors for Heal, Trim and Vertical Displacements of Underwater and Above-Water Objects. *Sensors and Systems Magazine*, **5** (2000), 33–37.
- [7] IVANOV, Y. V. Inertial Measurements in Dynamic Mode by Means of High-frequency Physical Pendulums. *Precision Engineering Magazine*, **46** (2003), No. 9, 56–60.
- [8] PELPOR, D. S. Orientation and Stabilization Gyroscopic Systems, Moscow, Mashinostroene, 1982.
- [9] BELYANIN, L. N., E. V. YAKIMOVA. Vertical Accelerometer Plotter, XVI Scientific Symposium, Electronic and Electromechanical Systems and Devices, Tomsk, 2000, 148–153.
- [10] DANILOV, A. T. A Gyroscopic Measuring System for Parameters of Moving Objects. *Problems of Special Machinebuilding Magazine*, **4** (2001), 178–181.
- [11] DICHEV, D. A. Models of Errors due to Measurement in Static and Dynamic Mode, XIX National Scientific Symposium, Metrology and Metrology Assurance, Sazopol, 2009, 51–57.
- [12] ZHU, R., D. SUN, Z. ZHOU, D. WANG. A Linear Fusion Algorithm for Attitude Determination using Low Cost MEMS-based Sensors. *Measurement*, **40** (2007), 322–328.
- [13] KAZAKOFF, A. Advances in Engineering Software for Lift Transportation System. *Journal of Theoretical and Applied Mechanics*, **42** (2012), No. 1, 3–22.
- [14] VENEVA, I., B. VANDERBORGHT, D. LEFEBER, P. CHERELLE. Propulsion System with Pneumatic Artificial Muscles for Powering Ankle-Foot Orthosis. *Journal of Theoretical and Applied Mechanics*, **43** (2013), No. 4, 3–16.