

SOLID MECHANICS

MODELLING OF ACOUSTIC EMISSION SOURCE AND WAVE RESPONSE IN LAYERED MATERIALS*

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ABSTRACT. This study proposes a model of wave propagation in layered media for the use in acoustic emission (AE) studies. This model aims to find an AE response at a free surface to the propagating waves originating at a dislocation source either in one layer medium or a layer-to-layer interface. Each of the layered media is assumed to be homogenous, linear elastic and isotropic. An integral transformation method has been applied to determine the wave response in frequency-wave number domain, which is then converted to time-space domain.

In the numerical examples, we first select truncated values with the finite integral transformation, so that no wave interference happens in the responses from wave reflection at truncated boundaries. Next, we simulate wave propagation in an elastic half space, and compare results obtained with that from other kind bottom boundary. Next, we introduce a dislocation source in interface and compare a simulated AE wave response obtained with that computed in the layered medium to demonstrate the performance of the model. In each simulation, the results show good agreement with the reference solutions.

KEY WORDS: Acoustic emission, dislocation source, layered medium, wave propagation.

1. Introduction

Acoustic emission (AE) is a naturally occurring phenomenon that occurs when there is a crack, or dislocation source, in a material [1]. A significant

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amount of energy is released due to a loss of cohesion and changes in an internal structure of the crack. Part of the energy released can be transformed into an acoustic wave, also known as an AE signal. This signal can be used in investigation of whether materials have been damaged because the signals emit different amplitudes depending on the type or amount of damage to the material [2–5]. The detection of the changes in the internal structure of the materials is a non-destructive method recognized as AE technique. The AE technique is typically based on the detection and conversion of high frequency acoustic waves to electrical signals using a sensor in real time [1]. Following the conversion, the signals are conditioned (preamplifier, amplifier and filter) prior to being analyzed. Analyzing AE signals can help locate and identify the cracking or dislocation source inside the material [6–8], estimate material damages [9–11], and identify damage mechanisms [12–16]. Since the AE testing is not reproducible due to the nature of the source, exemplified as sudden and sometimes random formation of a crack, modelling of AE source and wave propagation is essential [1, 17]. Indeed, AE modelling and technology has been widely used in various applications, such as nuclear power, aerospace pressure vessel, wood, power stations test and materials processing and testing [18–26].

Many studies suggested that AE be a form of micro-seismicity generated during a failure process as the material is loaded [27]. Consequently, modelling of earthquake wave propagation with a dislocation source in seismology can be and have already been used in that of AE waves [28]. In other words, material crack, seismic rupture source and the like, share the same source mechanism, i.e., time-dependent dislocation over a finite fault area, or simply the finite dislocation source. The finite dislocation source can be modelled as the summation (or integration in the limit) of point sources, each of which accounts for the evolutionary dislocation over a discretized sub-area triggered at different time instant. Therefore, truthfully characterizing the point source is a key in understanding of nature of the finite dislocation source.

The mechanism of the above point source is typically modelled as the product of a source time function characterizing the dislocation growth (e.g., a ramp function), a factor combining nine couples of impulsive forces that is equivalent to the unit dislocation, and a scaling factor or magnitude (= final dislocation $\times$ material rigidity $\times$ fault area). Each couple can be represented mathematically by two impulsive forces acting in opposite directions, with an infinitesimal separation distance either along or perpendicular to the impulse direction or, in the limit, by the derivative of the impulse with respect to the separation-distance parameter. The combination of the nine couples can be theoretically proved to be equivalent to any kind of a dislocation (normal and

shear) in an arbitrary orientation from an elastodynamic approach, which uses the generalized Betti reciprocal relation by introducing a time parameter, as first described in [29]. Alternatively and more generally, Bakus et al. [30–31] use the concept of stress glut to derive the same results. That derivation uses the assumption that the earthquake and indigenous sources are considered to be the result of a localized, transient failure of the linearized elastic constitutive relation, and leads to the stress glut as a function of the dislocation quantity.

While the above point-source characterization based on equivalent forces is useful in solving wave motion equations, which have been well developed and widely used in study of seismology and recently in fracture mechanics, the source description can be directly given with a dislocation form and subsequently used for finding wave responses. In addition, the equivalent forces are valid only for a dislocation within a medium, which is not applicable to generalized dislocations at the layer-to-layer interface, exemplified as de-bonding damage in reality.

Built upon the aforementioned studies as well as pertinent others [32–34], this paper presents a continuum-mechanics model for AE wave propagation in layered materials, which is generated by time-dependent point dislocation over a fault area buried within one of the layers or at the layer-to-layer interface.

2. Wave motion to a point, impulsive dislocation

In this study, the AE source is described as an impulsive dislocation at a point of ruptured fault with given area, while the materials are modelled as vertically non-homogeneous or layered media with each layer being isotropic, homogeneous and linearly elastic, as schematically shown in Fig. 1.

The AE waves can then be obtained by solving force-free wave equations of motion with a dislocation over a fault area. In doing so, it is also required of pertinent conditions that are the continuity conditions at each layer-to-layer interface and boundary conditions at both surfaces. For ease in describing the solution procedure, the following displacement and traction vectors are defined:

$$(1, 2) \quad \vec{u}(x, y, z, t) = u_x \vec{e}_x + u_y \vec{e}_y + u_z \vec{e}_z, \quad \vec{t}(x, y, z, t) = \tau_{xz} \vec{e}_x + \tau_{yz} \vec{e}_y + \tau_{zz} \vec{e}_z,$$

where $\vec{e}_x, \vec{e}_y$ and $\vec{e}_z$ are orthogonal unit vectors, respectively in the x, y and z directions, u the displacement, and τ the stress.

The wave motion can then be found by solving the following three dimensional (3D) wave equations in force-free layer i :

$$(3) \quad \tau_{jx,x} + \tau_{jy,y} + \tau_{jz,z} = \rho_i \ddot{u}_j, \quad j = x, y, z, \quad i = 1, 2, \dots, n;$$

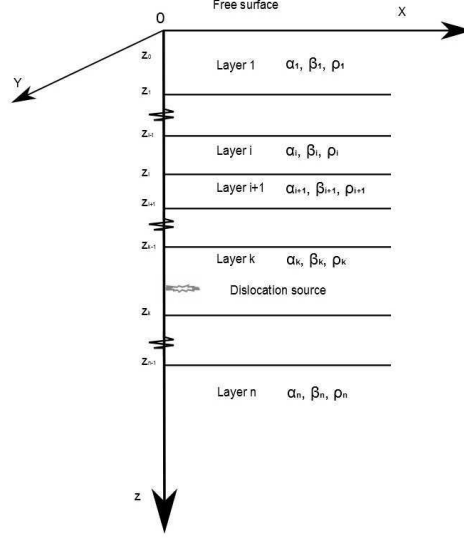


Fig. 1. Profile of layered materials with buried dislocation source

where ρ is the density and prime in subscript denotes partial derivative (e.g., $\tau_{xy,y} = \partial \tau_{xy} / \partial y$).

The constitutive relationship between displacements and stresses is expressed as:

$$(4) \quad \tau_{jk} = \lambda_i \delta_{jk} (u_{x,x} + u_{y,y} + u_{z,z}) + \mu_i (u_{j,k} + u_{k,j}), \quad j, k = x, y, z, \quad i = 1, 2, \dots, n;$$

in which δ_{jk} is Kronecker delta, equal to one if $j = k$ and zero otherwise, and λ and μ are the Lamé's elastic constants and can be found in terms of P and S wave speeds (α and β) and density (ρ) as follows:

$$(5, 6) \quad \lambda = \rho(\alpha^2 - 2\beta^2), \quad \mu = \rho\beta^2.$$

The continuity condition at each layer-to-layer interface requires that displacement and traction vectors be continuous across $z = z_i$:

$$(7, 8) \quad \vec{u}(x, y, z_i^-, t) = \vec{u}(x, y, z_i^+, t), \quad \vec{t}(x, y, z_i^-, t) = \vec{t}(x, y, z_i^+, t),$$

where z_i^- and z_i^+ represent the negative and positive sides of interface $z = z_i$, respectively.

The boundary condition at the free surface and the other with fixed, free or infinite boundary are:

$$(9, 10) \quad \vec{t}(x, y, z = 0, t) = 0 \text{ and } \vec{t}(x, y, z = z_{n-1}^+, t) = 0 \text{ or } \vec{u}(x, y, z_{n-1}^+, t) = 0,$$

or radiation condition with no propagating waves coming from the place where z is infinity.

The point, impulsive dislocation source can be described as normal and shear dislocation components ($\Delta u_n, \Delta u_s$) with shear slip direction (θ_s) over a fault area A at a point ($x_s = 0, y_s = 0, z_s$) as shown in Fig. 2. It can be expressed as:

$$(11) \quad \vec{u}'(x_s, y_s, z_s^+, t) - \vec{u}'(x_s, y_s, z_s^-, t) = \vec{u}'_s \delta(t),$$

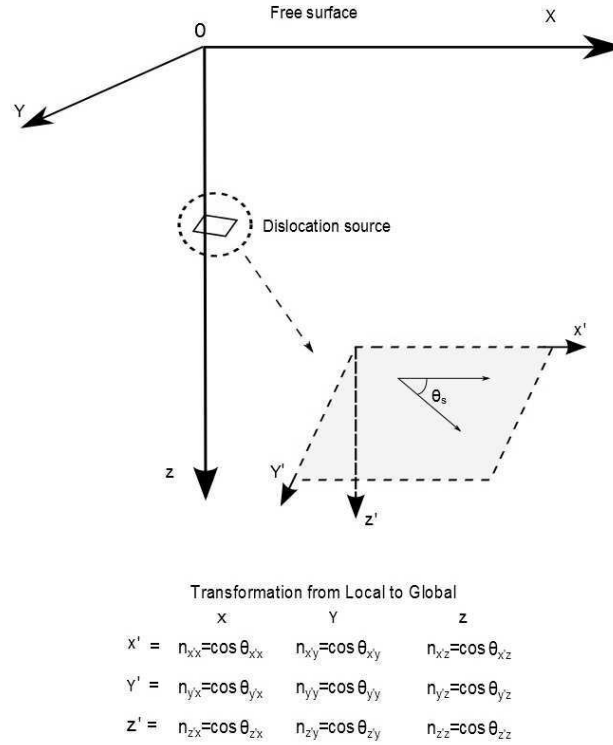


Fig. 2. Description of dislocation in an inclined-to-surface fault

$$(12) \quad \vec{u}'_s = [\Delta u_s (\cos \theta_s \vec{e}_{x'} + \sin \theta_s \vec{e}_{y'}) + \Delta u_n \vec{e}_{z'}] \delta(x) \delta(y) \delta(z - z_s),$$

where quantities with prime indicate that they are described with local $x' - y' - z'$ coordinate system in the fault and can be converted in the global $x - y - z$ coordinates as:

$$(13) \quad \vec{u}(x_s, y_s, z_s^+, t) - \vec{u}(x_s, y_s, z_s^-, t) = \vec{u}_s \delta(t) = [\Delta u_x \vec{e}_x + \Delta u_y \vec{e}_y + \Delta u_z \vec{e}_z] \delta(t),$$

$$(14) \quad \Delta u_i = [\Delta u_s (\cos \theta_s n_{x'i} + \sin \theta_s n_{y'i}) + \Delta u_n n_{z'i}] \delta(x) \delta(y) \delta(z - z_s), \quad i = x, y, z.$$

As an example for $\Delta u_n = 0$, $\theta_s = 0$ and local $x' - y' - z'$ coordinate system consistent with global $x - y - z$ one, the equivalent force vector at source location can be found based on Eq. (14) as:

$$(15) \quad \vec{f}_e = [A\mu_k \Delta u_s] \left[\delta(x) \delta(y) \frac{\partial}{\partial z} \delta(z - z_s) \vec{e}_x + \delta(y) \delta(z - z_s) \frac{\partial}{\partial x} \delta(x) \vec{e}_z \right],$$

which are the double couples of impulsive forces in x - and z -directions with seismic moment as $M_0 = A\mu_k \Delta u_s$. It can be verified that the equivalent forces to the general dislocation in Eq. (13) are consistent with the traditional seismic-moment based nine couples.

3. Solution for wave motion

To solve for wave response, integral transform procedure is applied. Particularly, triple Fourier transforms are used between time-space domain (x, y, z, t) and frequency-wave number domain (k_x, k_y, z, ω) , i.e.,

$$(16) \quad \vec{u}(x, y, z, t) = \int_{-\infty}^{\infty} e^{i\omega t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (W_R \vec{e}_R + W_S \vec{e}_S + W_T \vec{e}_T) dk_x dk_y d\omega,$$

$$(17) \quad \vec{t}(x, y, z, t) = \int_{-\infty}^{\infty} e^{i\omega t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \omega (F_R \vec{e}_R + F_S \vec{e}_S + F_T \vec{e}_T) dk_x dk_y d\omega,$$

where orthogonal vector harmonics $(\vec{e}_R, \vec{e}_S, \vec{e}_T)$ are defined as:

$$(18) \quad \begin{aligned} \vec{e}_R &= e^{i(k_x x + k_y y)} \vec{e}_z, \quad \vec{e}_S = \left(\frac{ik_x}{k_r} \vec{e}_x + \frac{ik_y}{k_r} \vec{e}_y \right) e^{i(k_x x + k_y y)}, \\ \vec{e}_T &= \left(\frac{ik_y}{k_r} \vec{e}_x - \frac{ik_x}{k_r} \vec{e}_y \right) e^{i(k_x x + k_y y)}, \end{aligned}$$

in which k_x and k_y are wave numbers in x and y directions, respectively ω is the frequency, and $k_r = \sqrt{k_x^2 + k_y^2}$ is the wave number in radial direction.

Variables W_X and F_X ($X = R, S$ or T) in Eqs (17) and (18) may also be expressed in terms of displacement vector $\vec{u}$ and traction vector $\vec{t}$ as:

$$(19) \quad W_X(k_x, k_y, z, \omega) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} dt e^{-i\omega t} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \left\{ \vec{u}(x, y, z, t) \cdot [\vec{e}_X]^* \right\},$$

$$(20) \quad F_X(k_x, k_y, z, \omega) = \frac{1}{\omega (2\pi)^3} \int_{-\infty}^{\infty} dt e^{-i\omega t} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \left\{ \vec{u}(x, y, z, t) \cdot [\vec{e}_X]^* \right\},$$

where the asterisk denotes the complex conjugate and the dot between two vectors represents an inner product.

Equations (16) and (17) imply that the 3D wave motion in Cartesian coordinates $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ in the time-space domain can be projected into three new orthogonal harmonic directions $(\vec{e}_R, \vec{e}_S, \vec{e}_T)$ in the transformed domain. The 3D wave motion in the Cartesian coordinates $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ can also be projected in the Cartesian coordinates $(\vec{e}_x, \vec{e}_y, \vec{e}_z)$ themselves in the transformed domain. For example, equation (21) may alternatively be written in the following explicit form:

$$(21) \quad \vec{u}(x, y, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\tilde{u}_x \vec{e}_x + \tilde{u}_y \vec{e}_y + \tilde{u}_z \vec{e}_z) e^{i(k_x x + k_y y + \omega t)} dk_x dk_y d\omega,$$

where the relations between displacement components in the frequency-wave number domain $\tilde{u}_d$ ($d = x, y, z$) and variables W_X ($X = R, S$ or T) can be easily found as:

$$(22) \quad \tilde{u}_x(k_x, k_y, z, \omega) = \frac{ik_x}{k_r} W_S + \frac{ik_y}{k_r} W_T,$$

$$(23) \quad \tilde{u}_y(k_x, k_y, z, \omega) = \frac{ik_y}{k_r} W_S - \frac{ik_x}{k_r} W_T,$$

$$(24) \quad \tilde{u}_z(k_x, k_y, z, \omega) = W_R.$$

The traction vector may also be written in a way similar to Eqs (22)–(24). It can be shown [27] that W_R , W_S , F_R and F_S are associated with P – SV waves, while W_T and F_T are associated with SH waves. Therefore, the displacement responses in the x and y directions are composed of both P – SV and SH wave motions, while the displacement response in the z direction is only contributed by the P – SV wave motion. After performing the transformation, the wave motion in Eqs (3)–(4) in the time-space domain (x, y, z, t) in each layer may be rearranged in a general form in the transformed domain (k_x, k_y, z, ω) , i.e.:

$$(25) \quad \frac{d}{dz} \begin{Bmatrix} \vec{w} \\ \vec{f} \end{Bmatrix} = [A(i)] \begin{Bmatrix} \vec{w} \\ \vec{f} \end{Bmatrix}, \quad i = 1, 2, \dots, n;$$

where $[A(i)]$ is the coefficient matrix, which is a function of the physical parameters of layer i , $\vec{w} = \vec{w}_{P-SV} \equiv \{W_R, W_S\}^T$ and $\vec{f} = \vec{f}_{P-SV} \equiv \{F_R, F_S\}^T$ for the P – SV waves, and $\vec{w}_{SH} \equiv W_T$ and $\vec{f}_{SH} \equiv F_T$ for the SH waves. Here, $\equiv$ denotes “by definition.” Coefficient matrix $[A(i)]$ in Eq. (25) for the P – SV waves is:

$$(26) \quad [A] \equiv [A_{P-SV}] = \begin{bmatrix} 0 & k_r \left(1 - \frac{2\beta^2}{\alpha^2}\right) & \frac{\omega}{\rho\alpha^2} & 0 \\ -k_r & 0 & 0 & \frac{\omega}{\rho\beta^2} \\ -\rho\omega & 0 & 0 & k_r \\ 0 & \rho\omega \left(\frac{\gamma k_r^2}{\omega^2} - 1\right) & -k_r \left(1 - \frac{2\beta^2}{\alpha^2}\right) & 0 \end{bmatrix},$$

and for the SH waves:

$$(27) \quad [A] \equiv [A_{SH}] = \begin{bmatrix} 0 & \frac{\omega}{\rho\beta^2} \\ \rho\omega \left(\frac{\beta^2 k_r^2}{\omega^2} - 1\right) & 0 \end{bmatrix},$$

where $\gamma = 4\beta^2(1 - \beta^2/\alpha^2)$.

Correspondingly, the continuity conditions at each interface become:

$$(28) \quad \begin{Bmatrix} \vec{w} \\ \vec{f} \end{Bmatrix}_{z=z_i^+} = \begin{Bmatrix} \vec{w} \\ \vec{f} \end{Bmatrix}_{z=z_i^-},$$

and the boundary conditions in Eqs (9)–(11) are:

$$(29) \quad \vec{f}|_{z=0} = 0 \quad \text{and} \quad \vec{f}|_{z=z_{n-1}^+} = 0 \quad \text{or} \quad \vec{w}|_{z=z_{n-1}^+} = 0.$$

Introducing the transformation:

$$(30) \quad \begin{Bmatrix} \vec{w} \\ \vec{f} \end{Bmatrix} = [D(i)] \begin{Bmatrix} \vec{\mu}_u \\ \vec{\mu}_d \end{Bmatrix} = \begin{bmatrix} M_u(i) & M_d(i) \\ N_u(i) & N_d(i) \end{bmatrix} \begin{Bmatrix} \vec{\mu}_u \\ \vec{\mu}_d \end{Bmatrix},$$

where columns in matrix $[D(i)]$ are the eigenvectors of matrix $[A]$, $\vec{\mu}_u$ and $\vec{\mu}_d$ denote, respectively, an up- and down-going wave vectors.

For $P - SV$ wave motion:

$$(31) \quad M_{u,d}(i) = \begin{bmatrix} \mp i q_{Pi} \varepsilon_{Pi} & \kappa_r \varepsilon_{Si} / \omega \\ \kappa_r \varepsilon_{Pi} / \omega & \mp i q_{Si} \varepsilon_{Si} \end{bmatrix},$$

$$(32) \quad N_{u,d}(i) = \begin{bmatrix} \rho_i (2\beta_i^2 \kappa_r^2 / \omega^2 - 1) \varepsilon_{Pi} & \mp 2i \rho_i \beta_i^2 \kappa_r q_{Si} \varepsilon_{Si} / \omega \\ \mp 2i \rho_i \beta_i^2 \kappa_r q_{Pi} \varepsilon_{Pi} / \omega & \rho_i (2\beta_i^2 \kappa_r^2 / \omega^2 - 1) \varepsilon_{Si} \end{bmatrix},$$

and for SH wave motion:

$$(33, 34) \quad M_{u,d}(i) = \varepsilon_{Si} / \beta_i, N_{u,d}(i) = \mp i \rho_i \beta_i q_{Si} \varepsilon_{Si},$$

where:

$$(35, 36) \quad q_{Pi} = \sqrt{1/\alpha_i^2 - \kappa_r^2 / \omega^2}, q_{Si} = \sqrt{1/\beta_i^2 - \kappa_r^2 / \omega^2},$$

$$(37, 38) \quad \varepsilon_{Pi} = (2\rho q_{Pi})^{-1/2}, \varepsilon_{Si} = (2\rho q_{Si})^{-1/2}.$$

To account for damping, the real-valued P and S wave speeds can be replaced by a pair of complex ones, i.e., by $\alpha_i[1 + i \operatorname{sgn}(\omega)/(2Q_{Pi})]$ and $\beta_i[1 + i \operatorname{sgn}(\omega)/(2Q_{Si})]$, where $\operatorname{sgn}(\omega)$ denotes the sign of frequency ω , Q_{Pi} and Q_{Si} are the attenuation factors for P and S waves in layer i . The branch cuts for the radicals in the expressions for q_{Pi} and q_{Si} are taken to be:

$$(39, 40) \quad \operatorname{Im}(\omega q_{Pi}) \geq 0, \quad \operatorname{Im}(\omega q_{Si}) \geq 0.$$

Substituting Eq. (30) into Eq. (25), the output wave vectors $\vec{\mu}_u(i)$ and $\vec{\mu}_d(k)$ for medium bounded with (z_i, z_k) or simply (i, k) are related to input wave vectors $\vec{\mu}_d(i)$ and $\vec{\mu}_u(k)$ through reflection and transmission vectors R and T or the scattering matrix as:

$$(41) \quad \begin{Bmatrix} \vec{\mu}_u(i) \\ \vec{\mu}_d(k) \end{Bmatrix} = \begin{bmatrix} R(i, k) & T(k, i) \\ T(i, k) & R(k, i) \end{bmatrix} \begin{Bmatrix} \vec{\mu}_d(i) \\ \vec{\mu}_u(k) \end{Bmatrix},$$

which is schematically shown in Fig. 3.

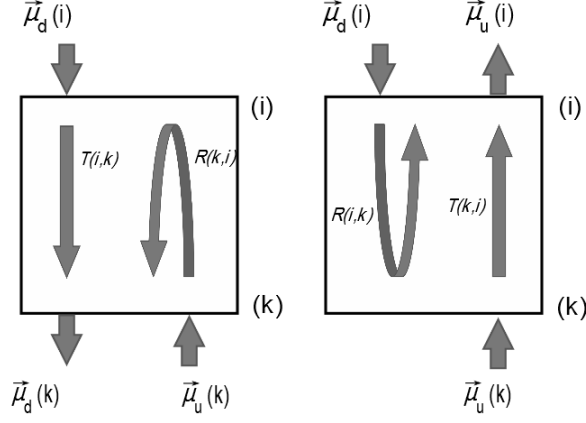


Fig. 3. Schematic of relation between out-going wave vectors $\vec{\mu}_u(i)$ and $\vec{\mu}_d(k)$ and input wave vectors $\vec{\mu}_d(i)$ and $\vec{\mu}_u(k)$ through reflection and transmission vectors R and T

The reflection and transmission matrices in layer i , which is bounded by depths z_{i-1}^+ and z_i^- , are:

$$(42) \quad R(i-1^+, i^-) = R(i^-, i-1^+) = 0,$$

$$(43) \quad T(i-1^+, i^-) = T(i^-, i-1^+) = \begin{cases} \text{diag}[e^{i\omega q_{Pi}\Delta z} & e^{i\omega q_{Si}\Delta z}], & (P-SV) \\ e^{i\omega q_{Si}\Delta z}, & (SH) \end{cases},$$

where diag indicates a diagonal matrix and $\Delta z = |z_i^- - z_{i-1}^+|$. The reflection and transmission matrices at the interface z_i bounded by z_i^- and z_i^+ or (i^-, i^+) are:

$$(44, 45) \quad R(i^-, i^+) = Q_{12}Q_{22}^{-1}, \quad R(i^+, i^-) = -Q_{22}^{-1}Q_{21},$$

$$(46, 47) \quad T(i^-, i^+) = Q_{22}^{-1}, \quad T(z_i^+, z_i^-) = Q_{11} - Q_{12}Q_{22}^{-1}Q_{21},$$

where

$$(48) \quad Q_{11} = -iN_d^T(i)M_u(i+1) + iM_d^T(i)N_u(i+1),$$

$$(49) \quad Q_{12} = -iN_d^T(i)M_d(i+1) + iM_d^T(i)N_d(i+1),$$

$$(50) \quad Q_{21} = iN_u^T(i)M_u(i+1) - iM_u^T(i)N_u(i+1),$$

$$(51) \quad Q_{22} = iN_u^T(i)M_d(i+1) - iM_u^T(i)N_d(i+1).$$

The reflection matrix at one free surface and the other as free or fixed in Eq. (29) are respectively:

$$(52) \quad R(0^+, 0) = -[N_d(1)]^{-1}N_u(1),$$

$$(53, 54) \quad R(n^-, n^+) = -[N_d(n)]^{-1}N_u(n) \quad \text{or} \quad R(n^-, n^+) = -[M_d(n)]^{-1}M_u(n).$$

Based on the fundamental reflection and transmission matrices within each layer, at each interface and surface, the corresponding reflection and transmission matrices between any two depths can be constructed using the following composite rule:

$$(55) \quad R(i, k) = R(i, j) + T(j, i)R(j, k)[I - R(j, i)R(j, k)]^{-1}T(i, j),$$

$$(56) \quad T(i, k) = T(j, k)[I - R(j, i)R(j, k)]^{-1}T(i, j),$$

where (i, k) can be used reversely, i.e., (k, i) .

For discontinuity source, Eq. (13) becomes:

$$(57) \quad \left\{ \begin{array}{c} \vec{w} \\ \vec{f} \end{array} \right\}_{z=z_s^+} - \left\{ \begin{array}{c} \vec{w} \\ \vec{f} \end{array} \right\}_{z=z_s^-} = \left\{ \begin{array}{c} \vec{w}_s \\ 0 \end{array} \right\},$$

Substituting Eq. (30) into Eq. (57) and using wave reflection and matrices at two depths, one can find:

$$(58) \quad \left[\begin{array}{cc} M_u(k) & M_d(k) \\ N_u(k) & N_d(k) \end{array} \right] \left\{ \begin{array}{c} R(s^+, n-1^+) \vec{\mu}_d(s^+) \\ \vec{\mu}_d(s^+) \end{array} \right\} - \left[\begin{array}{cc} M_u(k) & M_d(k) \\ N_u(k) & N_d(k) \end{array} \right] \left\{ \begin{array}{c} \vec{\mu}_u(s^-) \\ R(s^-, 0) \vec{\mu}_u(s^-) \end{array} \right\} = \left\{ \begin{array}{c} \vec{w}_s \\ 0 \end{array} \right\}.$$

Equations (57) can be solved as:

$$(59) \quad \vec{\mu}_u(s^-) = [Q_1(k) - R(z_{sj}^+, \infty)Q_2(k)R(s^-, 0)]^{-1}i[N_d^T(k) + R(s^+, n-1^+)N_u^T(k)]w_s,$$

where for dislocation within the k th layer:

$$(60) \quad Q_1(k) = Q_2(k) = I,$$

and for dislocation at the interface of layer $k - 1$ and k :

$$(61) \quad Q_1(k) = Q_{11}(k, k-1) + Q_{12}(k, k-1), \quad Q_2(k) = Q_{21}(k, k-1) + Q_{22}(k, k-1).$$

The wave responses at $z = 0$ in transformed domain can be found as follows:

$$(62) \quad \vec{w} = [M_u(1) + M_d(1)R(0^+, 0)][I - R(0^+, s^-)R(0^+, 0)]^{-1}T(s^-, 0^+)\mu_u(s^-).$$

Subsequently with the use of Eq. (16), wave response in time-space domain can be found.

4. Numerical examples

In numerical calculation, finite integral transform procedure is applied to solve for wave response. Specifically, triple finite Fourier transforms are used between time-space domain (x, y, z, t) and frequency-wave number domain (k_x, k_y, z, ω) . It can be shown that the displacement and traction vectors can be represented in a concise form with $|t| \leq t_t$, $|x| \leq x_t$ and $|y| \leq y_t$, that is, equation (16) can be represented as:

$$(63) \quad \vec{u}(x, y, z, t) = \frac{\pi}{t_t} \frac{\pi}{x_t} \frac{\pi}{y_t} \sum_{l=-\infty}^{\infty} e^{i\omega t} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (W_R \vec{e}_R + W_S \vec{e}_S + W_T \vec{e}_T),$$

in which $k_x = m\pi/x_t$ and $k_y = n\pi/y_t$ are respectively wave numbers in x and y directions, $\omega = l\pi/t_t$ is the frequency.

As a limiting case, summations in Eq. (63) become integrals when the truncated parameters of t_t , x_t and y_t approach infinity, indicating the conventional integral transform approach.

4.1. Features of wave responses to point impulsive dislocation source

For illustration, displacement wave responses at surface locations (see Table 1) are computed to a point, impulsive shear-dislocation source buried in a uniform half-space.

The material parameters used for an half space of 7000-series aluminum alloys [33] can be found in the second layer of Table 2, while dislocation parameters are selected as $\Delta u_n = 0$, $\Delta u_s = 0.01$ mm, $\theta_s = 0$, and fault area $A = 0.001$

Table 1. Observation positions at the surface

Observation positions	Surface Locations	
	x (mm)	y (mm)
1	15	20
2	22.5	30
3	30	40

mm. The truncated values in Eq. (63) are selected to be $x_t = y_t = 193.8$ mm and $t_t = 125 \mu\text{s}$, so that no wave interference in the responses from wave reflection at truncated boundaries (x_t, y_t) within the time window (t_t), i.e., $2x_t/c > t_t$ and $2y_t/c > t_t$ where c is the fastest or P wave speed in the material. The truncated values correspond to the resolution of frequency and wave-numbers as $\Delta k_x = \Delta k_y = 4.0537 \text{ m}^{-1}$, and $\Delta\omega = 4.0537 \text{ rad/s}$. For comparison, the displacements at the surface locations 1, 2 and 3 are also computed for the three-layer medium given in Table 2.

Table 2. Physical properties of layers

Layer	α (m/s)	β (m/s)	Q_α	Q_β	ρ (kg/m ³)	z (mm)
1	4.320	2.200	25	50	1670	0–0.5
2	6.320	3.180	12.5	25	2700	0.5–4.7
3	7.320	4.150	6.25	12.5	2610	4.7– ∞

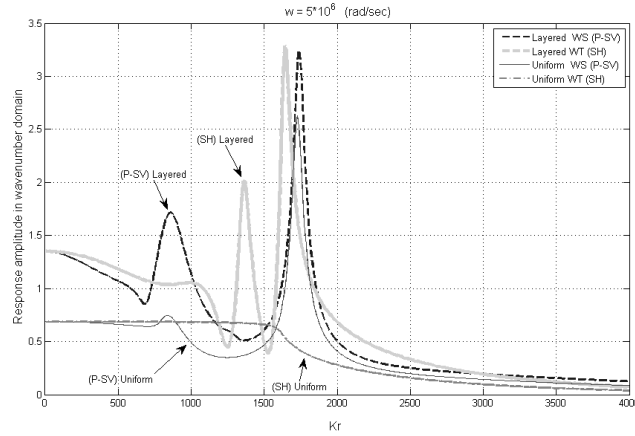


Fig. 4. $P-SV$ and SH wave responses at surface with 5 M rad/s for a uniform and a layered media

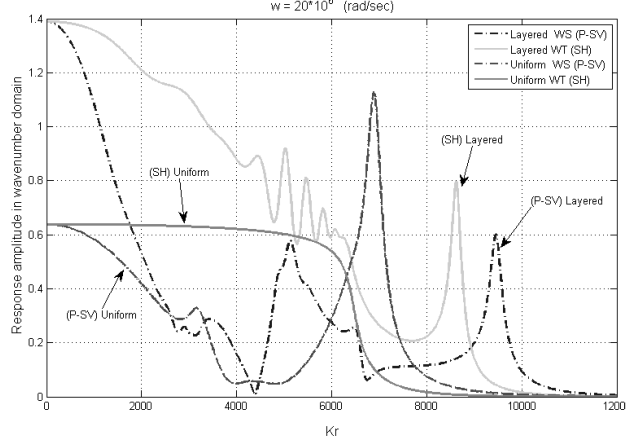


Fig. 5. $P-SV$ and SH wave responses at surface with 20 M rad/s for a uniform and a layered media

Figures 4 and 5 show the $P-SV$ and SH wave response amplitudes at surface (i.e., W_S and W_T) versus the wave number in radial direction (k_r) at selected frequencies 5 and 20 Mega-rad/s using Eq. (62). The dominant peaks of $P-SV$ wave response (W_S) in Fig. 4 for the uniform half space correspond to the P - and S -wave modes at $k_r = \omega/\alpha = 791$ rad/m and $k_r = \omega/\beta = 1572$ rad/m respectively, while similar peaks for the three-layer medium are found, showing the averaged P - and S -wave modes over the three layers. No peaks of SH wave response (W_T) in Fig. 4 are observed for uniform half-space, indicating that no SH -wave mode or surface waves exist for the uniform half-space. In contrast, a couple of peaks show off for the three-layer medium in Fig. 4, corresponding to the different surface wave modes such as Love waves. Figure 5 shows the similar wave response features at different frequency.

Figures 6 and 7 present the x - and z -direction displacement amplitudes in the wave number domain (Eqs. (22) and (24)) at selected frequencies for a uniform and a layered media, indicating respectively the combined $P-SV$ and SH wave responses for the former and only $P-SV$ wave responses for the latter. Figure 8 shows the amplitudes of x -direction displacement in the frequency domain for a uniform and a three-layer media, indicating more complexity of wave responses in a layered medium than the uniform half space in addition to the similar overall frequency features.

Figure 9 shows the x -direction displacements at observation locations 1, 2 and 3. It can be seen from this figure that the displacement at location

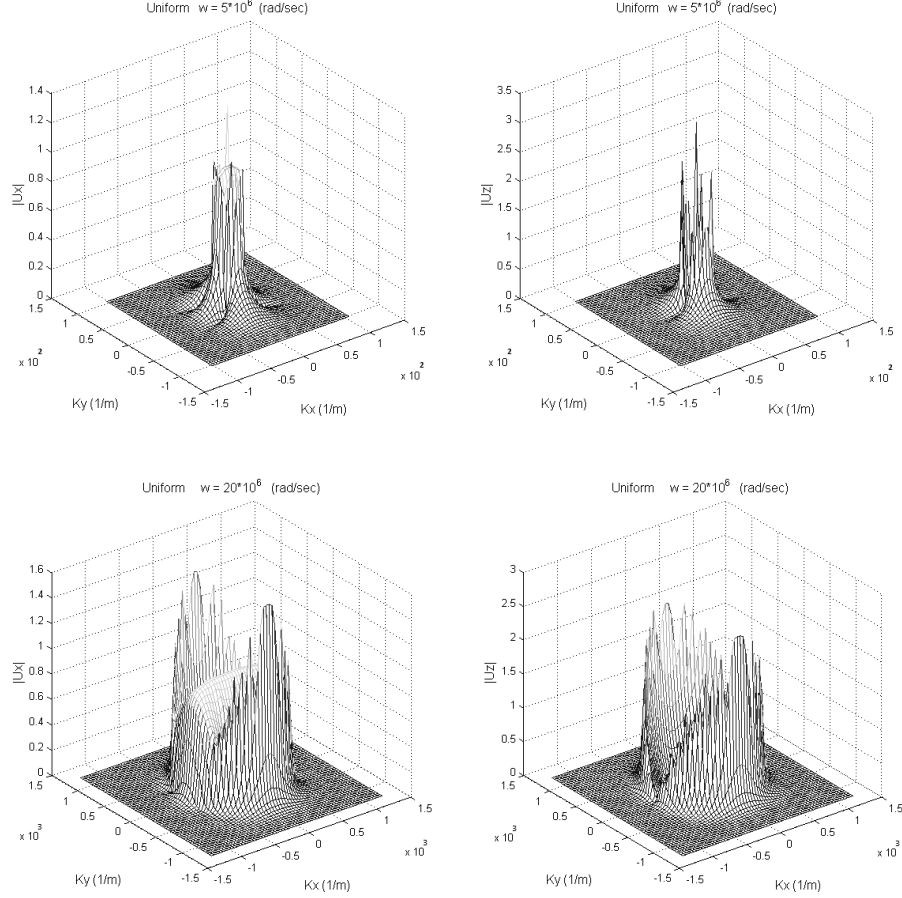


Fig. 6. The x - and z -direction displacement amplitudes in the frequency-wave number domain in a uniform medium

1 is zero until the first P wave signal arrives at $4.0(10^{-6})s$ which is measured in Fig. 9 and also consistent with the theoretical calculation based on the P wave speed and source-to-response distance. The S wave signals arrive later and give rise to larger displacement than that of the P waves at $8.2(10^{-6})s$. The displacements at locations 2 and 3 show similar wave propagation features with later arrival times and smaller, damping-related amplitudes, which can be further verified in Fig.10, the response amplitudes in frequency domain at observation locations 1, 2 and 3.

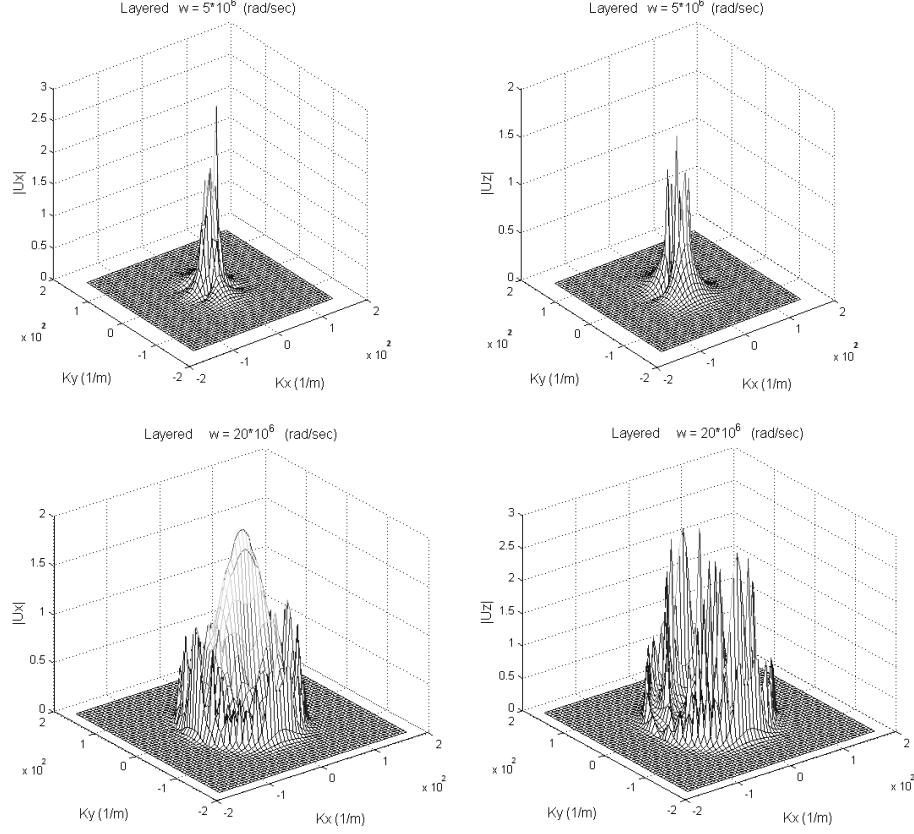


Fig. 7. The x - and z -direction displacement amplitudes in the frequency-wave number domain in a layered medium

For wave displacement response in a three-layer medium, Fig. 11 demonstrates that in addition to the similar, first arrival wave times, the AE signals show more wave reflection from the layers below the source. This figure also exposes that the amplitude in the x -direction is larger than those in the y - and z -directions, attributable to the dislocation source direction.

Figures 12 and 13 show displacement responses in frequency and time domains at free surface ($x = 15$, $y = 20$) mm from the epicentre of the source in a half space to the dislocation in the interface between the first and second layers at a depth of 0.5 mm. The results show, that the peak of displacement

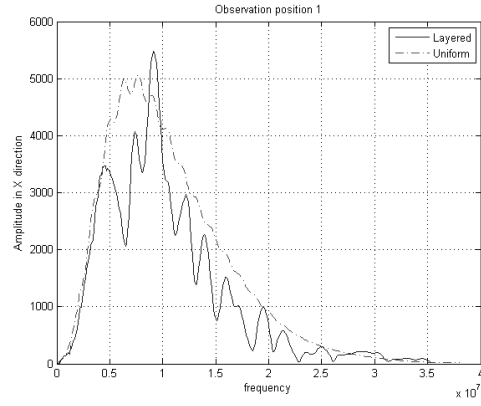


Fig. 8. The x -direction displacement amplitude spectra in a uniform and a three-layer media

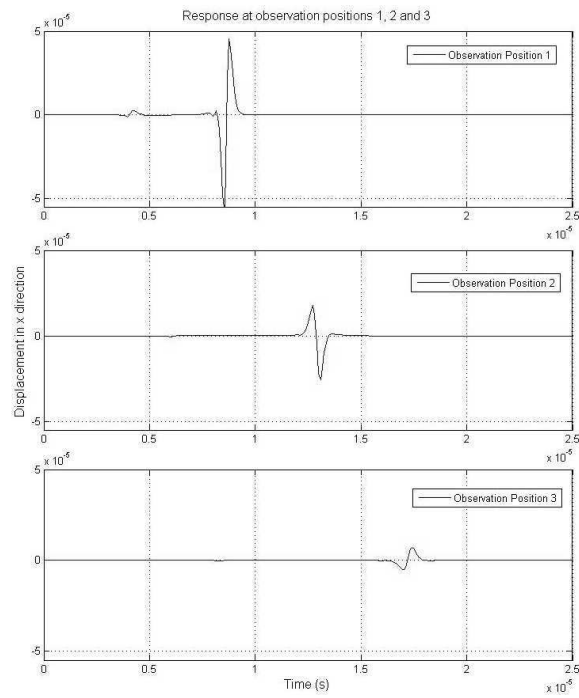


Fig. 9. Displacement wave responses at observation locations 1, 2 and 3

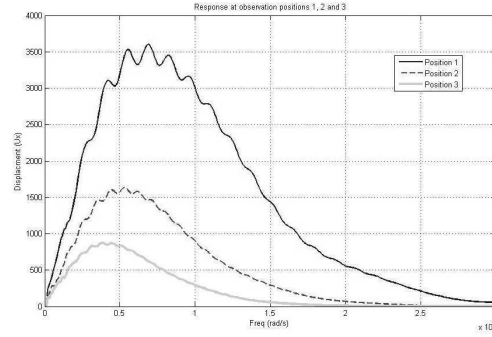


Fig. 10. Displacement wave responses at observation locations 1, 2 and 3 in frequency domain

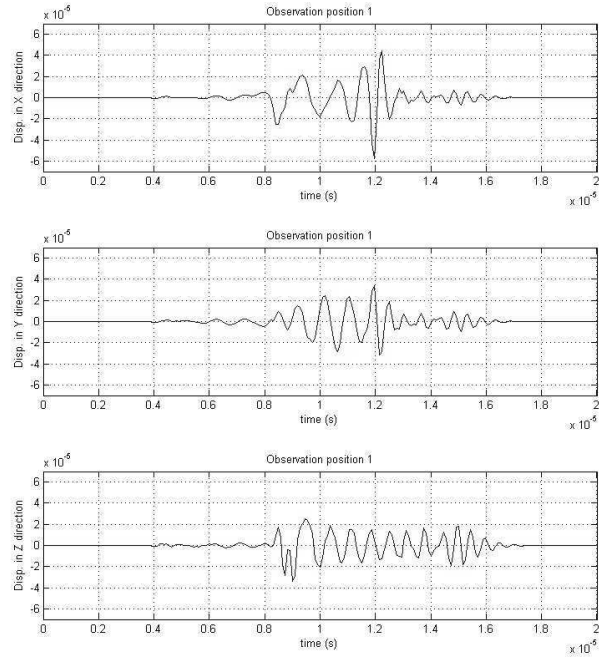


Fig. 11. Displacements computed for a three-layer medium in the x -, y - and z -directions

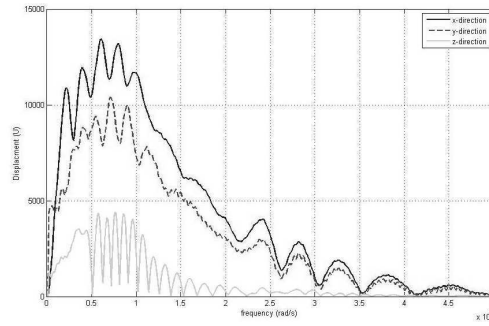


Fig. 12. The displacement wave responses in frequency domain to the dislocation source in the interface between the first and second layers

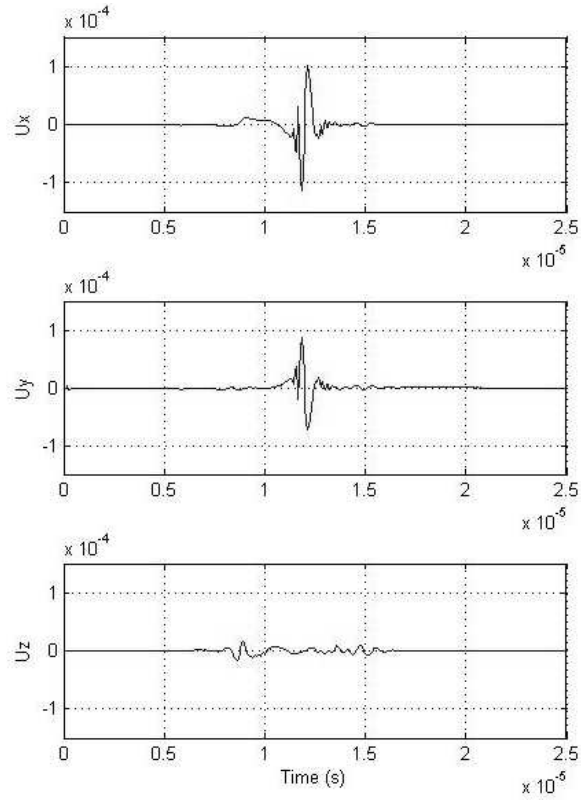


Fig. 13. The displacement wave responses in time domain to the dislocation source in the interface between the first and second layers

in the x -direction is the largest, due to the fact that the dislocation occurs in the x -direction.

4.2. Features of wave responses to general dislocation source

The d -direction displacement wave response g_d to a time-dependent, point dislocation source can be found as:

$$(64) \quad g_d(x, y, z, t) = \frac{M_0}{2\pi} \int_{-\infty}^{\infty} m(\omega) \tilde{u}_d(x, y, z; \omega) e^{i\omega t} d\omega,$$

where $m(\omega)$ is the Fourier transformation of a source time function characterizing the dislocation growth, and $\tilde{u}_d(d = x, y, z)$ is the displacement response to a unit-impulse, point dislocation in frequency domain. For the source time function, selected as a ramp function with the rise time T_r , $m(\omega)$, is given by:

$$(65) \quad m(\omega) = \frac{1}{\omega^2 T_r} (e^{-i\omega T_r} - 1) e^{-i\omega \tau}.$$

The x -direction displacement at location 1 has also been computed for layered medium as shown in Fig. 14, with the same material properties as given in Table 2.

Figures 14 and 15 describe the amplitudes of the computed frequency- and time-domain displacement responses, showing generally negligible response beyond $\omega = 2(10^7)$ rad/s and zero responses at $\omega_n = (n)0.418(10^7)$ rad/s, where n is an integral. This is related to the selected ramp function in which $m(\omega, t) = 0$ at $\omega_n T_r = 2n\pi$, for $T_r = 1.5(10^{-6})$ s.

5. Conclusion

In this study, a numerical modelling of AE source and wave propagation in layered materials has been examined. The general dislocation source is modelled and the corresponding traction discontinuity found, which is also validated with the traditional seismic-moment description. Three-dimensional (3D) wave equations in each homogeneous, isotropic and linearly-elastic layer are solved with the use of an integral transformation approach. With the model, AE wave propagation features have been examined in general, and wave responses at free surface from the perspective of damage detection in particular.

Numerical examples have been carried out to simulate AE wave propagation. Wave responses at surface were found to test the model by comparing with analytical solutions. The results showed good agreement with the analytical ones. Furthermore, the AE response was calculated for a dislocation source

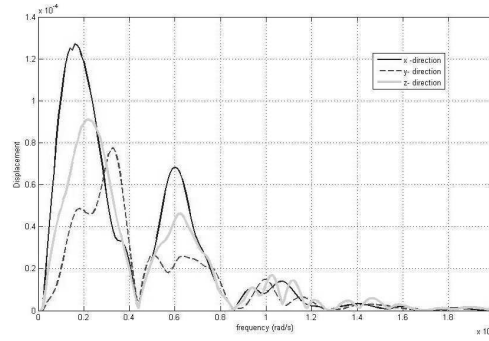


Fig. 14. Displacement responses in frequency domain to general dislocation source at location 1 in the x -, y - and z -directions

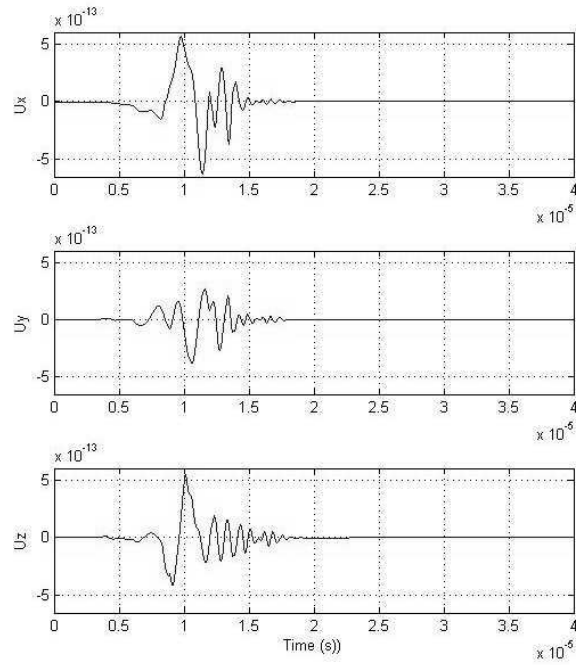


Fig. 15. Displacement responses in time domain to general dislocation source at location 1 in the x -, y - and z -directions

in the interface. Subsequently, the results obtained were compared with those from dislocation source in the layered medium.

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