

## On a Stability Criterion for Visco-Plastic Bodies

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### 1. Statement of the problem

Phenomena of local instability are very often observed in viscoplastic bodies under high-speed thermomechanical processes. Such are the cases of necking in bars subjected to dynamic tension, of forming of thin zones at cutting of metals [1] and dynamic torsion of bars [2] etc.

An energetic stability criterion for such kind of processes in dynamically loaded bodies under temperature field is proposed in this paper. A thermodynamic method of investigation as well as the Liapunov's stability definition are used. The thermodynamic method is often used in the cases of equilibrium stability problems of elastic bodies [3, 4, 5 etc.]. The consideration of the dynamic character of the process involves essential changes in the statement of the stability problem.

Consider the body B occupying the region  $\Omega_0$  of the three-dimensional Euclidean space  $E_3$  at a certain moment  $t_0$ . The volume of the body at that moment is  $V$  and it is bounded by a closed regular surface  $S$ . At the moment  $t_1$  the body occupies the region  $\Omega_t$  of  $E_3$ . We assume  $\Omega_0$  to be the reference configuration with the rectangular Cartesian coordinate system  $\{\mathbf{X}\}$ ;  $\Omega_t$  is the actual configuration with the rectangular Cartesian coordinate system  $\{\mathbf{x}\}$ ; the law of motion is  $\mathbf{x}=\mathbf{x}(\mathbf{X}, t)$ . The body is loaded on the part  $S_p$  of the boundary surface with dynamically applied traction  $\hat{\mathbf{T}}\mathbf{N}$ , varying according to a given programme in the time interval  $(t_0, t_1)$ . On the remaining part  $S_v$  of the surface ( $S_v \cup S_p = S$ ) the velocities  $\mathbf{v}$  are prescribed. The surroundings are under a constant temperature  $\hat{\theta}$ .

The boundary conditions of the process are:

$$\begin{aligned} \mathbf{T}\mathbf{N} &= \hat{\mathbf{T}}(\mathbf{X}, t)\mathbf{N}, \quad \mathbf{X} \in S_p, \quad t \in (t_0, t_1), \\ \mathbf{v} &= \hat{\mathbf{v}}(\mathbf{X}, t), \quad \mathbf{X} \in S_v, \quad t \in (t_0, t_1) \\ \mathbf{Q} \cdot \mathbf{N} &= -(\theta - \hat{\theta})h_t, \quad \mathbf{x} \in S, \quad t \in (t_0, t_1). \end{aligned} \quad (1)$$

$\mathbf{T}$  is the first Piola-Kirchoff stress tensor;  $\mathbf{N}$  is the outward unit normal to the surface  $S$  in  $\Omega_0$ ;  $\mathbf{v}=\dot{\mathbf{x}}(\mathbf{X}, t)$  is the velocity in terms of the material variables  $(\mathbf{X}, t)$ ;  $\mathbf{Q}$  is the heat flux vector in  $\Omega_0$ ;  $\theta=\theta(\mathbf{X}, t)$  is the absolute temperature;  $h_t$  is the heat transfer coefficient.

The body is considered as a closed thermodynamic system, consisting of an infinite number of local thermodynamic systems — the infinitesimal neighbourhoods of the particles  $X$ , having mass  $dm$ . The full thermodynamic state of any local thermodynamic system  $(X, dm)$  is described by the following system of ordered functions  $\{x, \varrho, E, \eta, X^{(\alpha)}, u, \dot{G}, h\}(X, t)$ .  $\varrho$  is the density in  $\Omega_t$ ;  $\eta$  is the entropy per unit mass;  $X^{(\alpha)}$  ( $\alpha = 1, 2, \dots, n$ ) are internal state variables, describing the visco-plastic properties of the material.  $G = \partial_x \theta$  is the temperature gradient;  $h$  is the entropy displacement  $\dot{h} = \frac{1}{\theta} Q$ ;  $E = \frac{1}{2} (F^T F - I)$  is the strain tensor;  $F = \partial_x x$  is the deformation gradient;  $I$  is the unit tensor;  $u$  is the internal energy per unit mass [6]. In the case of visco-plastic bodies, the visco-plastic strain tensor is often used as internal state variable  $X^{(1)} = E^a$  [7].

Taking into account the character of the considered process and the relations between the functions describing the full thermodynamic state of the local system, we can reduce the description of the process to determining the following system of functions

$$M_X(t) = \{v, E, \eta, X^{(\alpha)}\}(X, t), \quad X \in \Omega_0, \quad t \in (t_0, t_1).$$

Knowing  $M_X$  we may obtain the remaining functions of the process by means of the first group constitutive equations, the geometrical relations and the law of mass conservation

$$(2) \quad u = u(E, \eta, X^{(\alpha)}), \quad \tilde{T} = \varrho_0 \partial_E u, \quad \theta = \partial_\eta u, \quad P^{(\alpha)} = -\varrho_0 \partial_{X^{(\alpha)}} u,$$

$$FF^T = 2E + I, \quad \varrho = \varrho_0 / \det \{F\},$$

$\tilde{T}$  is the second Piola-Kirchoff stress tensor;  $P^{(\alpha)}$  are the dissipative forces, corresponding to the internal state variables  $X^{(\alpha)}$ .

The local thermodynamic process is now described by  $M_X(t)$ . The time-change of  $M_X(t)$  is given by the following system differential equations

$$(3) \quad \begin{aligned} \varrho_0 \dot{v} &= \text{Div } T, \\ \dot{E} &= \frac{1}{2} (\dot{F}^T F + F^T \dot{F}), \quad \dot{F} = \partial_x v(X, t), \\ \varrho_0 \dot{\eta} &= \text{Div} \left( \frac{1}{\theta} Q \right) + \frac{1}{\theta} \varrho_0 \delta, \\ \dot{X}^{(\alpha)} &= \Phi^{(\alpha)}(E, \eta, X^{(\alpha)}) \quad (\alpha = 1, 2, \dots, n), \end{aligned}$$

where

$$(4) \quad \begin{aligned} Q &= F(E, \theta, X^{(\alpha)})G, \\ \varrho_0 \delta &= G \cdot \dot{h} + \sum_{\alpha=1}^n \text{tr} \{P^{(\alpha)} \dot{X}^{(\alpha)}\} \geq 0. \end{aligned}$$

Equ. (3)<sub>1</sub> is the equation of motion, neglecting the body forces. Equ. (3)<sub>2</sub> is the geometrical relation between the material time derivatives of the strain tensor and the deformation gradient; equ. (3)<sub>3</sub> is the balance of entropy; equ. (3)<sub>4</sub> are the equations of evolution of the internal state variables; (4)<sub>1</sub> is the heat conduction equation; (4)<sub>2</sub> represents the energy dissipation which must be nonnegative, according to the second thermodynamic law. In the particular case of  $X^{(1)} = E^a$  equ. (3)<sub>4</sub> may take the form [6]

$$(5) \quad \dot{\mathbf{E}}^a = \mathbf{B}(\tilde{\mathbf{T}}, \theta, \mathbf{E}^a) f(\Delta z), \quad \dot{\mathbf{E}}^a = 0, \quad \Delta z = z - z_0 > 0, \\ \dot{\mathbf{E}}^a = 0, \quad \Delta z \leq 0,$$

or [7]

$$(6) \quad \dot{\mathbf{E}}^a = \gamma(\theta) \langle \Phi(F) \rangle \mathbf{M}(\tilde{\mathbf{T}}, \theta, \mathbf{E}^a), \quad \langle \Phi \rangle = \Phi, \quad F > 0, \\ \langle \Phi \rangle = 0, \quad F \leq 0.$$

$\mathbf{B}$  and  $\mathbf{M}$  are tensors characterizing the direction of the anelastic strain rate in the stress space;  $f$  and  $\Phi$  are functions characterizing the viscoplastic properties of the material;  $z(\tilde{\mathbf{T}}, \theta, \mathbf{E}^a)$  is the free enthalpy per unit mass;  $z_0$  is the boundary value of  $z$ ;  $F(\tilde{\mathbf{T}}, \theta, \mathbf{E}^a) = 0$  is the static yield condition;  $\gamma$  is the viscosity coefficient.

The linear Fourier relation may be assumed for the heat conduction equation (4)<sub>1</sub>

$$(7) \quad \mathbf{Q} = \kappa_t \mathbf{G},$$

$\kappa_t = \text{const}$  is the heat conduction coefficient.

Let us consider thermomechanic processes in the body  $B$ , described locally by means of eqs. (3) and (4), boundary conditions (1) and initial conditions  $M_{\mathbf{x}}(t_0)$  leading to sufficiently smooth functions  $M_{\mathbf{x}}(t)$ ,  $\mathbf{x} \in \Omega_0$ ,  $t \in (t_0, t_1)$ . The functions composing  $M_{\mathbf{x}}(t)$  may have discontinuities on certain lines, surfaces or points in  $\Omega_t$ . The total thermodynamic process in  $B$  must obey the first and second thermodynamic principles.

$$(8) \quad \frac{d}{dt} \int_V \left( \rho_0 u + \frac{1}{2} \mathbf{v} \cdot \mathbf{v} \right) dV = \int_S \mathbf{v} \cdot \mathbf{T} \mathbf{N} dS + \int_S \mathbf{Q} \cdot \mathbf{N} dS, \\ \frac{d}{dt} \int_V \rho_0 \eta dV - \int_S \frac{1}{\theta} \mathbf{Q} \cdot \mathbf{N} dS = \int_V \frac{1}{\theta} \rho_0 \delta dV \geq 0.$$

Let us introduce the space  $R$  composed of the elements  $M = \{\mathbf{a}, \mathbf{A}, c, \mathbf{Y}^{(a)}\}(\mathbf{X}, t)$ . The functions composing  $M$  are:  $\mathbf{a}$  — vector;  $\mathbf{A}$  — second order symmetric tensor;  $c$  — scalar;  $\mathbf{Y}^{(a)}$  — tensors of an order corresponding to the order of  $\mathbf{X}^{(a)}$ . All these functions are sufficiently smooth with respect to  $\mathbf{X} \in \Omega_0$  and  $t \in (t_0, t_1)$ . We introduce the metrics in  $R$  by means of the distance between two elements  $M_1, M_2 \in R$

$$(9) \quad R_D^2(M_1, M_2) = R_I^2(\mathbf{a}_1, \mathbf{a}_2) + R_{II}^2(\mathbf{A}_1, \mathbf{A}_2) + R_{III}^2(c_1, c_2) + \sum_{a=1}^n R_a^2(\mathbf{Y}_1^{(a)}, \mathbf{Y}_2^{(a)}),$$

where

$$(10) \quad R_I^2(\mathbf{a}_1, \mathbf{a}_2) = \int_V \mathbf{a}_1 \mathbf{a}_2 dm, \quad R_{II}^2 = \int_V \text{tr} \{ \mathbf{A}_1 \mathbf{A}_2 \} dm, \\ R_{III}^2(c_1, c_2) = \int_V c_1 c_2 dm, \quad R_a^2 = \int_V \text{tr} \{ \mathbf{Y}_1^{(a)} \mathbf{Y}_2^{(a)} \} dm.$$

The appropriate norm of the element  $M_{\mathbf{x}}(t) \in R$  for each  $\mathbf{X} \in \Omega_0$  and  $t \in (t_0, t_1)$  is

$$(11) \quad \|M_{\mathbf{x}}(t)\|_R^2 = R_I^2(\mathbf{v}, \mathbf{v}) + R_{II}^2(\mathbf{E}, \mathbf{E}) + R_{III}^2(\eta, \eta) + \sum_{a=1}^n R_a^2(\mathbf{X}^{(a)}, \mathbf{X}^{(a)}).$$

## 2. Stability criterion

The stability of a thermomechanical process described locally by eqs. (3) and (4) will be investigated with the boundary conditions (1) and initial conditions  $M_X^0(t_0)$ , leading to trajectories  $M_X^0(t)$ ,  $X \in \Omega_0$ ,  $t \in (t_0, t_1)$  in the space  $R$ . In order to obtain a stability criterion for the process at the moment  $t_k \in (t_0, t_1)$ , we consider another thermo-viscoplastic dynamic process, described also by means of eqs. (3) and (4). For each  $t \in (t_0, t_1)$ , the second process obeys boundary conditions (1) and leads to trajectories  $M_X(t)$  in the space  $R$ , differing little from  $M_X^0(t)$  for  $t \geq t_0$ . (The assumption that  $\{M_X^0(t)\}$  is a process with viscoplastic deformations, is similar to the Shanley and Hill's assumption in the theory of plasticity [11]). The disturbed process  $\{M_X^*(t)\}$  as well as the basic process  $\{M_X^0(t)\}$  are really possible ones, obeying the thermodynamic principles (8). Let us introduce a fictitious process  $\{M_X^*(t)\}$ , where  $M_X^*(t) = M_X^0(t) - M_X^0(t)$  for  $X \in \Omega_0$ ,  $t \in (t_0, t_1)$ .  $M_X^*(t) = M_X^0(t)$  if  $M_X^0(t) = M_X^0(t) = 0$ . According to the second Liapunov's method we reduce the stability problem for the process  $\{M_X^0(t)\}$  to a stability problem for the fictitious equilibrium state  $\{M_X^0\}$  with disturbances given by the fictitious processes  $\{M_X^*(t)\}$  at  $t \geq t_0$ . The fictitious process  $\{M_X^*(t)\}$  is locally described by means of the following differential equations system

$$\begin{aligned} \varrho_0 \dot{\mathbf{v}}^* &= \text{Div } \mathbf{T}^*, \quad \mathbf{T}^* = \mathbf{T}' - \mathbf{T}^0, \\ \dot{\mathbf{E}}^* &= \frac{1}{2} (\dot{\mathbf{F}}^{0T} \mathbf{F}^* + \dot{\mathbf{F}}^{*T} \mathbf{F}^0 + \mathbf{F}^{0T} \dot{\mathbf{F}}^* + \dot{\mathbf{F}}^{*T} \mathbf{F}^0 + \mathbf{F}^{*T} \dot{\mathbf{F}}^*), \quad \mathbf{F}^* = \mathbf{F}' - \mathbf{F}^0, \\ \varrho_0 \dot{\eta}^* &= \text{Div} \left( \frac{1}{\theta'} \mathbf{Q}' - \frac{1}{\theta^0} \mathbf{Q}^0 \right) + \Delta \sigma, \quad \Delta \sigma = \frac{1}{\theta'} \varrho_0 \delta' - \frac{1}{\theta^0} \varrho_0 \delta^0, \\ \dot{\chi}^{(a)*} &= \Delta \Phi^{(a)}, \quad \Delta \Phi^{(a)} = \Phi^{(a)}(\mathbf{E}', \eta', \mathbf{X}^{(a)'}) - \Phi^{(a)}(\mathbf{E}^0, \eta^0, \mathbf{X}^{(a)0}) \quad (a = 1, \dots, n). \end{aligned} \quad (12)$$

The corresponding boundary conditions are

$$\begin{aligned} \mathbf{T}^* \mathbf{N} &= 0, \quad X \in S_p, \quad t \in (t_0, t_1), \\ \mathbf{v}^* &= 0, \quad X \in S_v, \quad t \in (t_0, t_1), \\ \mathbf{Q}^* \mathbf{N} &= -\theta^* h_t, \quad X \in S, \quad t \in (t_0, t_1), \end{aligned} \quad (13)$$

where

$$\mathbf{Q}^* = \mathbf{Q}' - \mathbf{Q}^0, \quad \theta^* = \theta' - \theta^0. \quad (14)$$

The initial conditions are  $M_X^*(t_0)$ . The fictitious process obeys total relations, obtained after subtracting the expressions in (7) for the disturbed and the basic processes.

$$\begin{aligned} \frac{d}{dt} \int_V \left( \varrho_0 \Delta u + \frac{1}{2} \varrho_0 \mathbf{v}^* \cdot \mathbf{v}^* + \varrho_0 \mathbf{v}^0 \cdot \mathbf{v}^* \right) dV &= \int_{S_p} \mathbf{v}^* \cdot \hat{\mathbf{T}} \mathbf{N} dS \\ &+ \int_{S_v} \hat{\mathbf{v}} \cdot \mathbf{T}^* \mathbf{N} dS - \int_S \theta^* h_t dS, \\ \frac{d}{dt} \int_V \varrho_0 \eta^* dV + \int_S \frac{\theta^*}{\tau} h_t dS &= \int_V \Delta \sigma dV, \end{aligned} \quad (15)$$

where

$$(16) \quad \Delta u = u(\mathbf{E}', \eta', X^{(\alpha)'}) - u(\mathbf{E}^0, \eta^0, X^{(\alpha)^0}), \quad \tau = \theta' \frac{\theta^0}{\hat{\theta}} > 0.$$

From (15)<sub>1</sub> and (15)<sub>2</sub> we obtain

$$(17) \quad \frac{d}{dt} \int_V \left( \varrho \Delta u + \frac{1}{2} \mathbf{v}^* \cdot \mathbf{v}^* + \varrho_0 \mathbf{v}^0 \cdot \mathbf{v}^* - \varrho_0 \hat{\theta} \eta^* \right) dV \\ = \int_{S_p} \mathbf{v}^* \cdot \hat{\mathbf{T}} \mathbf{N} dS + \int_{S_v} \hat{\mathbf{v}} \cdot \mathbf{T}^* \mathbf{N} dS + \int_S \left( \frac{\hat{\theta}}{\tau} - 1 \right) h_t \theta^* dS - \int_V \hat{\theta} \Delta \sigma dV.$$

The energy dissipation is the only heat supply in the case under consideration and therefore  $\theta' > \hat{\theta}$ ,  $\theta^0 > \theta'$  and  $\left( \frac{\hat{\theta}}{\tau} - 1 \right) \leq 0$ .

We expand the internal energy per unit mass for the disturbed process  $u(\mathbf{E}^0 + \mathbf{E}^*, \eta^0 + \eta^*, X^{(\alpha)^0} + X^{(\alpha)*})$  in Taylor's series in the neighbourhood of  $M_X^0$ . Taking into account that  $M_X^*$  were assumed to be of small values, the terms in the series consisting  $M_X^*$  of the order higher than two may be neglected and we obtain

$$(18) \quad \varrho_0 \Delta u = \text{tr} \{ \tilde{\mathbf{T}}^0 \mathbf{E}^* \} + \varrho_0 \theta^0 \eta^* - \sum_{\alpha=1}^n \text{tr} \{ \mathbf{P}^{(\alpha)^0} X^{(\alpha)*} \} + \frac{1}{2} \text{tr} \{ \Gamma_u^0 \mathbf{E}^* \mathbf{E}^* \} \\ + \frac{1}{2} v_u (\eta^*)^2 + \frac{1}{2} \sum_{\alpha=1}^n \text{tr} \{ \mathbf{B}_u^{(\alpha)^0} X^{(\alpha)*} X^{(\alpha)*} \},$$

where

$$(19) \quad \Gamma_u^0 = \varrho_0 \partial_{\mathbf{E}} (\partial_{\mathbf{E}} u) |_{M_X^0}, \quad v_u^0 = \partial_{\eta} (\partial_{\eta} u) |_{M_X^0}, \quad \mathbf{B}_u^{(\alpha)} = \varrho_0 \partial_{X^{(\alpha)}} (\partial_{X^{(\alpha)}} u) |_{M_X^0}.$$

Substituting (18) in (17) leads to

$$(20) \quad \dot{A} = \Delta D + \dot{W}^N,$$

where

$$\Delta = \frac{1}{2} \int_V \left( \text{tr} \{ \Gamma_u^0 \mathbf{E}^* \mathbf{E}^* \} + v_u^0 (\eta^*)^2 + \sum_{\alpha=1}^n \text{tr} \{ \mathbf{B}_u^{(\alpha)^0} X^{(\alpha)*} X^{(\alpha)*} \} + \varrho_0 \mathbf{v}^* \cdot \mathbf{v}^* \right) dV \\ + \int_V \left( \text{tr} \{ \tilde{\mathbf{T}}^0 \mathbf{E}^* \} + \varrho_0 \Delta \theta^0 \eta^* - \sum_{\alpha=1}^n \text{tr} \{ \mathbf{P}^{(\alpha)^0} X^{(\alpha)*} \} + \varrho_0 \mathbf{v}^0 \cdot \mathbf{v}^* \right) dV, \\ \Delta D = \int_S \left( \frac{\hat{\theta}}{\tau} - 1 \right) \theta^* h_t dS - \int_S \hat{\theta} \Delta \sigma dV, \\ \dot{W}^N = \int_{S_p} \mathbf{v}^* \cdot \hat{\mathbf{T}} \mathbf{N} dS + \int_{S_v} \hat{\mathbf{v}} \cdot \mathbf{T}^* \mathbf{N} dS, \quad \Delta \theta^0 = \theta^0 - \hat{\theta}.$$

An expression is obtained consisting the measure of alteration of the total energy of the fictitious process  $A$ , the power of the nonconservative forces

during the fictitious process  $\dot{W}^N$  and the difference  $\Delta\dot{D}$  between the dissipative and thermal terms of the basic and disturbed processes. This result is similar to the result corresponding to the case of equilibrium state stability of bodies, loaded with nonconservative forces [9]. As the load in the considered case is time-dependent, the term  $\dot{W}^N$  appears in equ. (20). All  $\{M_x^*(t)\}$  obeying conditions (13) form the manifold  $K \in R$ . We assume that the basic process  $\{M_x^0(t)\}$  is known;  $A$  is then a functional of  $M_x^*(t)$ , having the range of definition  $K$ . We consider the  $r$ -neighbourhood of the fictitious equilibrium state  $\bar{M}_x^*$ , determined by  $\|M_x^*(t)\|_R^2 < r$ . According to Liapunov's stability definition, the fictitious equilibrium state will be locally stable if for each  $0 < \varepsilon < r$  there exists a  $0 < \delta(\varepsilon) < r$  such, that  $\|M_x^*(t)\|_R^2 < \varepsilon$  if  $\|M_x^*(t_0)\|_R^2 < \delta(\varepsilon)$  at each  $t \in (t_0, t_1)$ . In that case the basic process  $\{M_x^0(t)\}$  will be stable too.

The value of  $r$  is to be chosen according to the specific character of the process. If the condition of the definition is violated with respect to the norm  $\|M_x^*(t)\|_R^2 > r$ , the process is thermo-mechanically and structurally unstable at the certain moment  $t_k$ . If  $R_I^2(v^*, v^*) > r$  at  $t_k \in (t_0, t_1)$ , the body loses its stability of motion at moment  $t_k$ ; if  $R_{II}^2(E^*, E^*) > r$ , it loses stability of the form; if  $R_{III}^2(\eta^*, \eta^*) > r$ , thermal instability occurs; if  $R_{IV}^2(X^{(a)*}, X^{(a)*}) > r$ , internal structural instability occurs. (The internal structural equilibrium state instability with constant  $E$  and  $\eta$  and  $v=0$  is investigated in [10, 7]).

In order to establish the sufficient conditions for stability of the considered process, theorem 12 [8], will be applied [9]. In order to fulfil the conditions of the theorem, the following requirements have to be proved:

a) the functional  $A(M_x^*(t))$  must be positive definite with respect to the norm  $\|M_x^*(t)\|_R$  at each  $t \in (t_0, t_1)$ , e. g.

$$(22) \quad A \geq A. \|M_x^*\|_R, \quad A > 0;$$

b) The functional  $A(M_x^*(t))$  must be decreasing with respect to  $t$  in the interval  $(t_0, t_1)$ , e. g.

$$(23) \quad \Delta\dot{D} + \dot{W}^N \leq 0.$$

c) The functional  $A(M_x^*(t))$  must be continuous and upper bounded.

If anyone of the mentioned above conditions is violated at a certain moment  $t_k \in (t_0, t_1)$ , the process is unstable at the moment  $t_k$ .

Thus an energetic criterion is obtained for the local stability of viscoplastic bodies under dynamic thermo-mechanical processes. In the particular cases of determining the stability, an essential significance has the shape of the functions  $\hat{T}(X, t)$ ,  $\hat{v}(X, t)$ , as well as the way of application of the external load and the assumed visco-plastic model.

## References

1. Pomey, J. Aperçu sur la plasticité adiabatique. — Annales du CIRP, XIII 2, 1966, 93—103.
2. Baltov, A. P. T. Vinh. Sur la formation des bades adiabatiques plastiques dans un tube mince soumis à une torsion dynamique. — C. R. Acad. Sc., Paris, 275, sér. A, 1972, 291—294.

3. Ericksen, J. L. A thermo-kinetic view of elastic stability theory. — *Int. J. Eng. Sci.*, **2**, 1966, 573—580.
4. Koiter, W. T. On thermodynamic background of elastic stability theory. *Problems of Hydrodynamics and Continuum Mechanics*. М., AS URSS, 1969, 277—285.
5. Naghdi, P. M., J. A. Trapp. On the general theory of stability for elastic bodies. — *Arch. Rat. Mech. Anal.*, **51**, 3, 1973, 165—191.
6. Коларов, Д., А. Балтов. Върху теорията на вискозопластичността при големи деформации. — *Теоретична и приложна механика*, **IV**, 2 1973, 21—30.
7. Perzyna, P. Thermodynamic theory of viscoplasticity. — *Adv. Appl. Mech.*, **11**, 1971, 313.
8. Зубов, В. И. Устойчивость движения. М., Наука. 1973.
9. Nemat-Nasser, S. On Thermomechanics of Elastic Stability. — *ZAMP*, **21**, 4, 1970, 538—558.
10. Coleman, B. D., M. E. Gurtin. Thermodynamics with Internal State Variables. — *J. Chem. Phys.*, **47**, 2, 1967, 597—613.
11. Hill, R. A general theory of uniqueness and stability in elastic-plastic solids. — *J. of Mech. Phys. Sol.*, **6**, 3, 1958, 236—249.

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