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Unsteady magnetohydrodynamic channel flow of a dusty Rivlin-Ericksen fluid

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In this paper, the unsteady flow of a Rivlin-Ericksen fluid with uniform distribution of dust particles through a channel of uniform cross-section in presence of transverse magnetic field, under the influence of variable pressure gradient has been discussed. The differential equations describing the problem are given. Analytical expressions for the velocities of the fluid and dust particles have been obtained. Several graphs have been drawn to show the effect of magnetic field and elasticity of the fluid on the velocity field.

Introduction

In a series of papers Oldroyd [1, 2, 3] proposed and studied a set of constitutive equations for elasticoviscous fluids in an attempt to explain the rheological behaviour of some non-Newtonian fluids. He found that an improvement of the constitutive equations proposed by Jeffreys [4] for the Earth lead to reasonable agreement with the experimental results (Oldroyd, Strawbridge and Toms [5]; Toms and Strawbridge [6]). Consequently a number of studies of the behaviour of non-Newtonian fluids has been carried out in the last decade (e. g. Narasimhan [7-8], Bhatnagar [9], Mishra [10-11], Thomas and Walters [12-14]). Moreover, non-Newtonian fluids are also relevant to the elastohydrodynamic regime in the theory of lubrication (see e. g. Hirst and Moore [15], Johnson and Roberts [16]). It is now generally accepted that the Oldroyed model contains a set of parameters which can be adjusted to suit a large class of fluids.

The basic theory of multiphase flows is given in a recent book by Soo [17]. To mention but a few examples of more recent work; Liu has solved the Stoke's problem [18] and attempted the Rayleigh problem [19]. Michael and Miller [20] have treated both problems approximately. Michael [21] has discussed the Kelvin-Helmholtz instability of a dusty gas. Healy [22] has proposed a different set of perturbed equations and has studied the flow past a cylinder, and a flat plate of finite width positioned normal to approach flow. Reddy [23, 24] has studied the laminar flow of an unsteady viscous liquid with uniform distribution of dust particles through a rectangular channel, and circular annulus under the influence of exponential pressure with respect to time. Yang [25] has considered the oscillatory flow of a viscous incompressible fluid

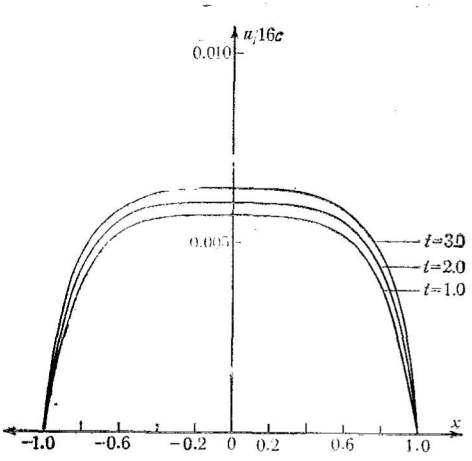


Fig. 1. Velocity profiles of liquid particles at ($l=0.2$, $f=0.8$, $M=3.0$, $\beta_0=0.2$)

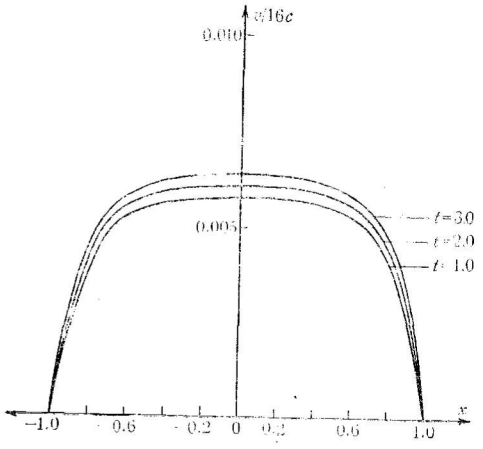


Fig. 2. Velocity profiles of dust particles at ($l=0.2$, $f=0.8$, $M=3.0$, β_0 0 2)

n a long tube with embedded, identical spherical particles, under the influence of body and pressure forces acting on the fluid. Recently Singh and Ram [26] have discussed the unsteady hydromagnetic flow of a dusty fluid through a channel under the influence of constant pressure gradient.

This paper devotes the study of unsteady flow of a dusty Rivlin-Ericksen liquid through a square channel in the presence of transverse magnetic field under the influence of periodic pressure gradient. The change in velocity profiles for liquid and dust particles for various values of Hartmann number and elastic parameter and at different times have been shown graphically.

Interest in problems of mechanics of systems with more than one phase has developed rapidly in recent years. Situations which occur frequently are concerned with the motion of liquid or gas which contains a distribution of solid particles. Such situations occur, for example, in the movement of dust laden air, in problems of fluidization, in the use of dust in gas-cooling systems to enhance heat transfer processes and in the process by which raindrops are formed by the coalescence of small droplets which might be considered as solid particles for the purpose of examining their movement prior to coalescence. Also the theoretical study of two phase systems is an important prelude to understanding such phenomena as blood flow in capillaries, pneumatic conveyance of small gram-like particles, flows in rocket tubes where small carbon or metallic fuel particles are present.

Equations of motion

The constitutive equation for a viscoelastic fluid as given by Rivlin and Ericksen has the following form

$$T_{ij}=-p\delta_{ij}+P_{ij}, \tag{1}$$

$$P_{ij}=\Phi_1 E_{ij}+\Phi_2 D_{ij}+\Phi_3 E_{im}E_{mj}, \tag{2}$$

$$E_{ij}=U_{i,j}+U_{j,i} \tag{3}$$

$$D_{ij}=A_{i,j}+A_{j,i}+2U_{m,i}U_{m,j}, \tag{4}$$

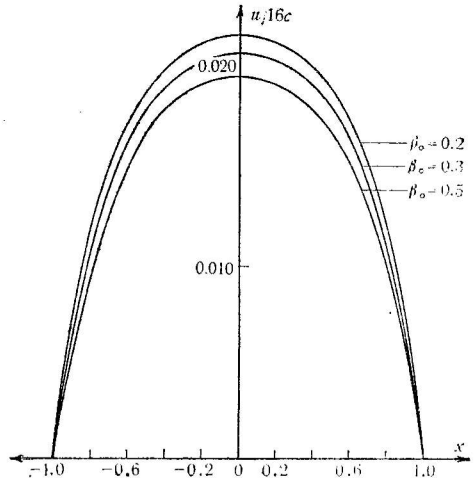


Fig. 3. Velocity profiles of liquid particles at ($l=0.2$, $f=0.8$, $t=2.0$, $\beta_0=0$)

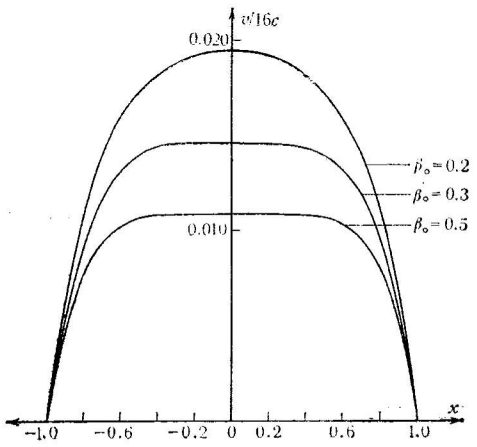


Fig. 4. Velocity profiles of dust particles at ($l=0.2$, $f=0.8$, $t=2.0$, $\beta_0=0.5$)

$$A_i=\frac{\partial U_i}{\partial T}+U_m U_{i,m}, \tag{5}$$

where a suffix following a comma denotes covarient differentiation. The equations of motion and continuity have the following form:

$$PA_i=T_{ij,j}+KN(V_i-U_i)+F_e^i, \tag{6}$$

$$m\left(\frac{\partial V_i}{\partial T}+V_m V_{i,m}\right)=K(U_i-V_i), \tag{7}$$

$$U_{i,i}=0, \tag{8}$$

$$\frac{\partial N}{\partial T}+N_i V_{i,i}=0. \tag{9}$$

In these equations U_i, V_i are velocities of liquid and dust particles respectively, p is the pressure, m — the mass of dust particle, ϱ is the liquid density, N is the number density of the dust particle, K is the Stoke's resistance coefficient (for spherical particles of radius r, it is $6\pi\Phi_1 r$), T is the time, F_e^i is the electromagnetic force, Φ_1, Φ_2, Φ_3 are coefficients of viscosity, visco-elasticity and cross-viscosity respectively.

Formulation of the problem

In the present analysis, the following important assumptions are made:

1. The flow is fully developed.
3. The dust particles are spherical in shape and uniformally distributed.
3. Chemical reaction, mass transfer and radiation between the particles and fluid are ignored.
4. The temperature is uniform within a particle.
5. Interaction between particles themselves is not considered.

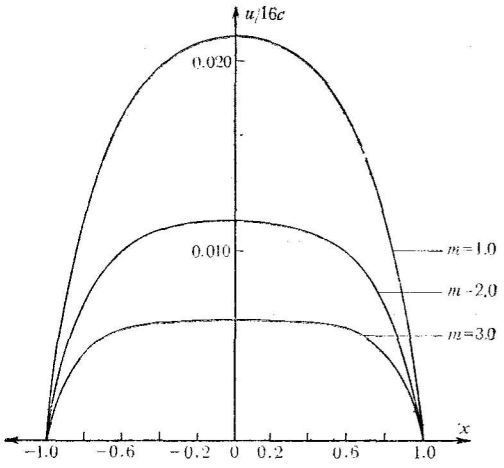


Fig. 5. Velocity profiles of liquid particles at ($l=0.2$, $f=0.8$, $M=1.0$, $t=1.0$)

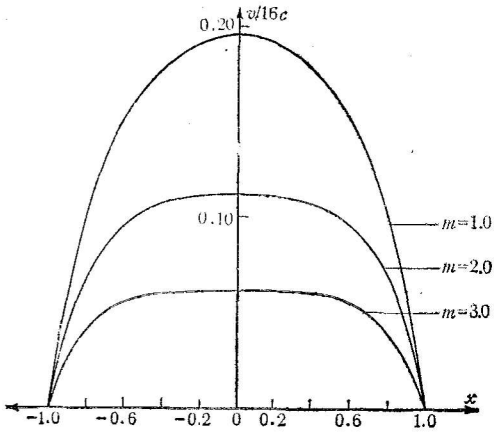


Fig. 6. Velocity profiles of dust particles at ($l=0.2$, $f=0.8$, $M=1.0$, $t=1.0$)

6. The number density of the dust particles is constant throughout the motion.

7. The displacement currents are zero since the flow velocity is small relative to the speed of light.

8. The flow induced magnetic field is neglected.

9. There is no external applied electric field.

10. The magnetic field is considered fixed relative to the channel.

11. The Hall effects are considered negligible.

12. The fluid is electrically neutral, i. e. no surplus electrical charge distribution is present in the fluid.

13. Only the electromagnetic body forces are present.

Maxwell's equations together with Ohm's law and the law of electromagnetic conservation are written in the case of zero displacement and Hall currents as:

$$\nabla \times \vec{B} = \vec{J}, \tag{10}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{11}$$

$$\vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}), \tag{12}$$

$$\nabla . \vec{B} = 0, \tag{13}$$

$$\nabla . \vec{E} = 0. \tag{14}$$

Choosing the Z-axis along the axis of the channel, the components of the velocity of the liquid and dust particles are respectively given by

$$u_1 = 0, \quad u_2 = 0, \quad u_3 = u_3(x, y, T), \tag{15}$$

$$v_1 = 0, \quad v_2 = 0, \quad v_3 = v_3(x, t, T), \tag{16}$$

where (u_1, u_2, u_3) and (v_1, v_2, v_3) denote the velocity components of liquid and dust particles respectively. Further, for $N=No$ (a constant), Eqn. (9) is satisfied throughout the motion.

In view of (15) and (16), equations (6) and (7) become

$$0=-\frac{1}{\varrho}\frac{\partial p}{\partial x}+(2\beta+r)\left[\frac{\partial}{\partial x}\left(\frac{\partial u_3}{\partial x}\right)^2+\frac{\partial}{\partial y}\left(\frac{\partial u_3}{\partial x}\cdot\frac{\partial u_3}{\partial y}\right)\right], \tag{17}$$

$$0=-\frac{1}{\varrho}\frac{\partial p}{\partial y}+(2\beta+r)\left[\frac{\partial}{\partial y}\left(\frac{\partial u_3}{\partial y}\right)^2+\frac{\partial}{\partial x}\left(\frac{\partial u_3}{\partial x}\cdot\frac{\partial u_3}{\partial y}\right)\right], \tag{18}$$

$$\frac{\partial u_3}{\partial T}=-\frac{1}{\varrho}\frac{\partial p}{\partial x}+\left(\alpha+\beta\frac{\partial}{\partial T}\right)\left(\frac{\partial^2 u_3}{\partial x^2}+\frac{\partial^2 u_3}{\partial y^2}\right)+\frac{KNo}{\varrho}(v_3-u_3)$$

$$-\frac{\sigma}{\varrho}B_0^2 u_3, \tag{19}$$

$$m'\frac{\partial v_3}{\partial T}=K(u_3-v_3). \tag{20}$$

where $\alpha=\Phi_1/\varrho$, $\beta=\Phi_2/\varrho$, $r=\Phi_3/\varrho$ are kinematical coefficients of viscosity, viscoelasticity and cross-viscosity, respectively. Eqs. (17), (18) give the variation of pressure field across the channel and that also will not be affected by the presence of the dust particles in the liquid. Eqs. (17), (18) will not be discussed further in the present investigation. Eqs. (19) and (20) determine the velocities u_3, v_3, which show that the coefficient of cross-viscosity does not affect them, as in all two-dimensional flows, but modifies the pressure field.

We now introduce the following non-dimensional quantities:

$$Z^1=\frac{Z}{a},\quad x^1=\frac{x}{a},\quad y^1=\frac{y}{a}\qquad p^1=\frac{pa^2}{\varrho\alpha^2},$$

$$t^1=\frac{\alpha T}{a^2},\quad u^1=\frac{u_3 a}{\alpha},\quad v^1=\frac{v_3 a}{\alpha},\quad l=\frac{mNo}{\varrho},$$

$$f=\frac{m\alpha_2}{Ka^2},\quad \beta_0=\frac{\beta}{a^2}\quad\text{(elastic parameter)},$$

$$M=\frac{\sqrt{\sigma}}{\Phi_1}B_0 a,$$

where M is the Hartmann number.

Eqs. (19) and (20) then become (after dropping the dashes)

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\left(1+\beta_0\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{l}{f}(v-u)-M^2 u, \tag{21}$$

$$f\frac{\partial v}{\partial t}=(u-v). \tag{2}$$

Eqs. (21) and (22) are to be solved subject to the condition

$$t=0: u=0=v \quad\text{for}\quad -1\le x\le 1,\quad -1\le y\le 1;$$

$$t>0: u=0 \quad\text{at}\quad x=\pm 1,\quad -1\le y\le 1;$$

$$u=0 \quad\text{at}\quad y=\pm 1,\quad -1\le x\le 1.$$

Method of solution

We now assume that

$$-\frac{\partial p}{\partial z}=(1-e^{-t}),$$

i. e. pressure gradient decreases exponentially with time.
We define the Laplace transform of u and v by

(23) $$U=\int_0^\infty e^{-st}u\,dt,$$

(24) $$V=\int_0^\infty e^{-st}v\,dt,$$

multiplying eqs. (21) and (22) by exp. $(-st)$ and inregrating with respect to t between the limits 0 to ∞, we have

(25) $$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}-K^2U=-\frac{1}{s(1+s)(1+\beta_0 s)},$$

and

(26) $$V=\frac{U}{1+fs},$$

where

$$K^2=\frac{ls+(1+fs)(s+M^2)}{(1+fs)(1+\beta_0 s)}.$$

The boundary conditions become

$$U=0, \quad \text{at} \quad x=\pm 1, \quad \text{and} \quad y=\pm 1.$$

The boundary conditions at $y=\pm 1$ may be satisfied by expressing U as a Fourier series in y, the coefficients of which are functions of x in the interval $[-1, 1]$ as follows

(27) $$U=\sum_{n=0}^{\infty} F(x)\cos\frac{(2n+1)}{2}\pi y.$$

This simplifies the boundary conditions at $x=\pm 1$ to

(28) $$F(\pm 1)=0.$$

Using (27) and taking $\frac{1}{s(s+1)(1+\beta_0 s)}$ as

(29) $$\frac{1}{s(s+1)(1+\beta_0 s)}=\frac{1}{\pi s(s+1)(1+\beta_0 s)}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos\frac{(2n+1)}{2}\pi y.$$

Differential equation (25) can be expressed as

(30) $$\frac{d^2F}{dx^2}-\left[K^2+\frac{(2n+1)^2\pi^2}{4}\right]F(x)=-\frac{4(-1)^n}{(2n+1)\pi}\cdot\frac{1}{s(s+1)(1+\beta_0 s)}.$$

The solution of (30) subject to (28) is

$$F(x)=\frac{4(-1)^n}{(2n+1)\pi s(s+1)(1+\beta_0 s)p^2}\left[1-\frac{ch\,px}{ch\,p}\right], \tag{31}$$

where $p^2=K^2+\frac{(2n+1)^2}{4}\pi^2$.

Substituting (31) into (27), we have

$$U=\sum_{n=0}^{\infty}\frac{4(-1)^n}{(2n+1)\pi s}\cdot\frac{1}{(s+1)(1+\beta_0 s)}\cdot\frac{1}{p^2}\left[1-\frac{chpx}{chp}\right]$$

$$\cos\left(\frac{2n+1}{2}\right)\pi y. \tag{32}$$

Hence

$$V=\sum_{n=0}^{\infty}\frac{4(-1)^n}{(2n+1)\pi s}\cdot\frac{1}{(s+1)(1+\beta_0 s)}\cdot\frac{1}{(1+fs)p^2}$$

$$\left[1-\frac{chpx}{chp}\right]\cos\frac{(2n+1)}{2}\pi y. \tag{33}$$

Now applying the Laplace inversion theorem, we have

$$\frac{u}{16}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3\pi^3}\left[1-\frac{ch\left(\sqrt{\left\{M^2+\left(\frac{2n+1}{2}\right)^2\pi^2\right\}}x\right)}{ch\left(\sqrt{\left\{M^2+\left(\frac{2n+1}{2}\right)^2\pi^2\right\}}\right)}\right]\cdot$$

$$\cos\left(\frac{2n+1}{2}\right)\pi y+\sum_{n=0}^{\infty}\frac{(-1)^n(1-f)e^{-t}}{4(2n+1)\pi(s_1-1)(s_2-1)}\left[1-\frac{ch\left(\sqrt{\{(s_1-1)(s_2-1)\}}x\right)}{ch\left(\sqrt{\{(s_1-1)(s_2-1)\}}\right)}\right]\cdot$$

$$\cos\frac{(2n+1)}{2}\pi y$$

$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+n}}{(2n+1)(2m+1)\pi^2\cdot D}\left[\frac{(1-fs_1)e^{-s_1t}}{s_1(s_1-1)}-\frac{(1-fs_2)e^{-s_2t}}{s_2(s_2-1)}\right]\cdot$$

$$\cos\left(\frac{2n+1}{2}\right)\pi y\cdot\cos\left(\frac{2m+1}{2}\right)\pi x, \tag{34}$$

and

$$\frac{v}{16}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3\pi^3}\left[1-\frac{ch\left(\sqrt{\left\{M^2+\left(\frac{2n+1}{2}\right)^2\pi^2\right\}}x\right)}{ch\left(\sqrt{\left\{M^2+\left(\frac{2n+1}{2}\right)^2\pi^2\right\}}\right)}\right]\cdot\cos\left(\frac{2n+1}{2}\right)\pi y$$

$$+\sum_{n=0}^{\infty}\frac{(-1)^n e^t}{4(2n+1)\pi(s_1-1)(s_2-1)}\left[1-\frac{\cos\left(\sqrt{\{(s_1-1)(s_2-1)\}}x\right)}{\cos\left(\sqrt{\{(s_1-1)(s_2-1)\}}\right)}\right]\cdot\cos\frac{2n+1}{2}\pi y$$

$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^{m+n}}{(2n+1)(2m+1)\pi^2\cdot D}\left[\frac{e^{-s_1t}}{s_1(s_1-1)}-\frac{e^{-s_2t}}{s_2(s_2-1)}\right].$$

(35) $$\cos\frac{(2n+1)}{2}\pi y \,.\, \cos\frac{(2m+1)}{2}\pi x,$$

$$\text{where } D^2=\left[1+l+fM^2+(\beta_0+f)\left\{\frac{(2m+1)^2}{4}\pi^2+\frac{(2n+1)^2}{4}\pi^2\right\}\right]^2$$

$$-\left[1+\beta_0\left\{\frac{(2m+1)^2}{4}\pi^2+\frac{(2n+1)^2}{4}\pi^2\right\}\right]\left[4M^2f+(2m+1)^2\pi^2f+(2n+1)^2\pi^2f\right].$$

$$2fs_1\left[1+\beta_0\left\{\frac{(2m+1)^2}{4}\pi^2+\frac{(2n+1)^2\pi^2}{4}\right\}\right]$$

$$=\left[1+l+Mf+(\beta_0+f)\left\{\frac{(2m+1)^2\pi^2}{4}+\frac{(2n+1)^2\pi^2}{4}\right\}\right]+D$$

$$2fs_2\left[1+\beta_0\left\{\frac{(2m+1)^2\pi^2}{4}+\frac{(2n+1)^2\pi^2}{4}\right\}\right]$$

$$=\left[1+l+Mf+(\beta_0+f)\left\{\frac{(2m+1)^2\pi^2}{4}+\frac{(2n+1)^2\pi^2}{4}\right\}\right]-D.$$

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