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Heat Transfer from a Porous Sphere in a Stream of Small Reynolds Number

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One of the classical problems [1, 2] with an extensive literature in fluid dynamics is the steady flow induced in an infinite incompressible viscous fluid by the slow motion of animmersed solid body. At present there appear to be no exact analytical solutions of the flow past bodies of finite size. For extreme values of the Reynolds number R_e the Navier-Stokes equations can be replaced by approximate forms which are more tractable. For small R_e the exact equations are replaced by Oseen's equations except in regions near the body, where they reduce to Stokes's equations. The nonexistence of a solution of Stokes' equation for unbounded plan flow past a body is known as Stokes' paradox and the nonexistence of a second approximation to Stokes' solution for unbounded uniform flow past a three-dimensional body is known as Whitehead's paradox. Works by Proudman and Pearson [3] and Lagestrom and Kaplun [4] have clarified the relation of these solutions to asymptotic expansions of Navier-Stokes equations.

Determination of the rate of heat transfer from a moving body is of fundamental importance in various engineering applications. There have been many investigations on the problem of heat transfer from a sphere in a low Reynolds number velocity field. The theoretical determination of the functional relation between the Nusselt number N_u and the Peklet number $P_e=\sigma R_e$ (here σ is Prandtl number) is naturally the main point of interest in these problems. There was some controversies on this topic. According to Kroning and Buijsten [5] for low values of the Peclet number

$$N_u=2+\frac{1}{2}P_e+\frac{581}{1920}P_e^2+\cdots\quad [P_e=2\sigma R_e]$$

whereas Breiman [6] concluded that

$$N_u=2+\frac{1}{2}P_e-\frac{1}{2}P_e^2\ln P_e-0.0334\,P_e^2+\cdots\quad [P_e=2\sigma R_e].$$

Two recent contributions to the subject are those of Acrivos and Taylor [7] and Rimmer [8]. They used the method of matched asymptotic expansions. The first-named authors took the velocity field given by Stokes and obtained

$$N_u=2+P_e+P_e^2\ln P_e+0.829\,P_e^2+\frac{1}{2}P_e^3\ln P_e+\cdots$$

Rimmer took into account the extra terms in the velocity field predicted by Proudman and Person. Rimmer's result can be written in the form

$$N_u = 2 + P_e + P_e^2 \ln P_e + f(\sigma) P_e^2 + \cdots$$

where

$$f(\sigma) = \frac{1}{4}\left\{(2\sigma^2 - \sigma + 4\gamma - \frac{173}{40} + 2(\sigma^3 - 3\sigma - 2)\ln \frac{\sigma}{\sigma+1}\right\}.$$

Many recent developments in boundary layer theory have been concerned with the flow over a permeable surface. In particular, Leonov [9] has considered the slow flow past a porous sphere adopting the conditions that the tangent component of the velocity on the surface of the sphere is zero and the normal component of the velocity is continuous. However, his solution is not uniformly valid in the entire flow region, because of the fact that full Navier—Stokes equations have not been taken into account. Gupta [10] extending Leonov's work has solved this problem on the basis of the Navier—Stokes equations, using the method of matched asymptotic expansions.

In the present article we have found the temperature distribution in a low Reynolds number flow past a porous sphere by using the velocity field from [10]. Consequently, the main purpose of our paper is the effect of the permeability on the heat transfer from a sphere.

2. Formulation of the problem

We consider the steady flow of an incompressible viscous fluid past a porous sphere. Introduced are spherical polar coordinates (r, θ, λ) with axis $\theta=0$ in the direction of the free stream, and it is supposed that u and v are the velocity components in the r and θ directions. From [10] we find

(1) $$\psi_{st} = \psi_1(r, \theta) + R_e \psi_2(r, 0) + \cdots$$

where

(1a) $$\psi_1(r, \theta) = \frac{1}{4}\left[2r^2 - (3-m_0)r + \frac{m_0+1}{r}\right]\sin^2\theta$$

(1b) $$\psi_2(r, \theta) = \frac{3-m_0}{32}\left[2r^2 - (3-m_0)r + \frac{m_0+1}{r}\right]\sin^2\theta$$

$$-\frac{3-m_0}{32}\left[2r^2 - (3-m_0) - \frac{1+m_0}{r} + \frac{1+m_0}{r^2} + 1 + \frac{64}{3}M_1(3-m_0)\right]\sin^2\alpha\cos\theta$$

and

(2) $$\psi_{0\,\mathrm{seen}} = \frac{1}{\sigma^2 R^2}\psi_1(\varrho, \theta) + \frac{1}{\sigma R_e}\psi_2(\varrho, \theta) + \cdots$$

where

(2a) $$\psi_1(\varrho, \theta) = \frac{1}{2}\varrho^2\sin^2\theta$$

(2b) $$\psi_2(\varrho, \theta) = -\frac{3-m_0}{2}(1+\cos\theta)\left[1 - e^{-\frac{1}{\alpha}\frac{\varrho}{\sigma}(1-\cos\theta)}\right].$$

Here m_0 is a constant of range $0 \le m_0 \le \frac{1}{7}$ which is called **permeability.**

The heat transfer equation in terms of non-dimensional variables (the sphere radius a and the stream velocity U_∞ being the length and velocity scales, respectively) is

$$\nabla_r^2 h = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \mu}\left[(1-\mu^2)\frac{\partial h}{\partial \mu}\right] = \sigma R_e \mathbf{q}\,.\,\mathrm{grad}\,h \tag{3}$$

where

$$h=\frac{T-T_\infty}{T_w-T_\infty}, \quad \mu=\cos\theta \tag{4}$$

$\mathbf{q}$ — velocity, T_w — temperature at the surface of the sphere and T_∞ — temperature at infinity. This equation must be solved subject to these conditions

$$h=1 \quad \text{at} \quad r=1, \quad h\to 0 \quad r\to\infty. \tag{5}$$

Now we will solve equation (3) with boundary conditions (5) by means of matched asymptotic expansions of h in each of the two regions, the inner one, near to and the outer, far from the sphere. For the inner region equation (3) at boundary condition $h=1$ at $r=1$ is used. For the outher region we introduce new variables

$$\varrho=\sigma R_e\,.\,r,\ \mu; \quad \underset{\varrho\to 0}{H(\varrho,\mu)}=\underset{r\to\infty}{h(r,\mu)} \tag{6}$$

The outer expansion $H(\varrho,\mu)$ must satisfy the equation

$$\nabla_\varrho^2 H=\mathbf{q}\,.\,\mathrm{grad}\,H \tag{7}$$

and boundary condition $H\to 0$ as $\varrho\to\infty$. The two expansions match identically at some arbitrary distance from the surface, and both remain bounded as $P_e\to 0$. This helps us to find the constants appearing in the h and H. The inner and outer expansions are assumed to be of the form

$$h(r,\mu)=\sum_{n=1}^{\infty} f_n(R_e)\,h_n(r,\mu), \quad f_1(R_e)=1 \tag{8}$$

and

$$H(\varrho,\mu)=\sum_{n=1}^{\infty} F_n(R_e)\,H_n(\varrho,\mu) \tag{9}$$

where

$$\lim_{R_e\to 0}\frac{f_{n+1}}{f_n}=0, \quad \lim_{R_e\to 0}\frac{F_{n+1}}{F_n}=0.$$

The boundary conditions for $h_n(r,\mu)$ and $H_n(\varrho,\mu)$ are

$$h_1(1,\mu)=1 \quad \text{and} \quad h_n(1,\mu)=0 \quad \text{for} \quad n\geq 2 \tag{10}$$

and

$$H_n(\infty,\mu)=0 \quad \text{for} \quad n\geq 1. \tag{11}$$

3 Construction of the solution

A. First expansion term

Substituting (8) and (9) in equations (3) and (7) respectively and equating the terms free of P_e and containing $F_1(R_e)$ on both sides we have:

(12) $$\nabla_r^2 h_1(r, \mu)=0$$

and

(13) $$\nabla_\varrho^2 \bar{H}_1(\varrho, \mu)=\mu\frac{\partial \bar{H}}{\partial \varrho}+\frac{1+\mu^2}{\varrho}\frac{\partial \bar{H}_1}{\partial \mu}$$

The general solution of equation (12) which satisfies condition $h_1(1, \mu)=1$ and remains bounded for all θ is

(14) $$h_1=1-A_0+\frac{A_0}{r}+\sum_{k=1}^{\infty} A_k(r^{-k-1}-r^k)P_k(\mu)$$

where $P_k(\mu)$ are the Legendre polynomials and A_k — constants. In order to solve equation (13) we use the substitution

$$\bar{H}_1=G_1 e^{\frac{1}{2}\varrho\mu}$$

and it is obtained equation

(15) $$\nabla_\varrho^2 G_1-\frac{1}{4}G_1=0.$$

Then the appropriate general solution of equation (13) which subject to the condition $H_1(\infty, \mu)=0$ is

(16) $$\bar{H}_1(\varrho.\mu)=F_1(\varepsilon)\sqrt{\frac{\pi}{\varrho}}\, e^{\frac{1}{2}\varrho\mu}\sum_{k=0}^{\infty} C_k K_{k+\frac{1}{2}}\left(\frac{\varrho}{2}\right)P_n(\mu)$$

where $K_{k+\frac{1}{2}}$ is a modified Bessel function defined as

(17) $$K_{n+\frac{1}{2}}\left(\frac{t}{2}\right)=\sqrt{\frac{\pi}{\varrho}}\, e^{-\frac{\varrho}{2}}\sum_{s=0}^{n}\frac{(n+s)!}{(n-s)!\, s!\, \varrho^s}$$

and C_k are constant.
By matching (14) as $r\to\infty$ and (16) as $\varrho\to 0$, we obtain

$$A_0=1,\quad A_k=0 \quad \text{for} \quad k\geq 1, \quad F_1(\varepsilon)=P_e$$

$$C_0=\frac{1}{\pi},\quad C_k=0 \quad \text{for} \quad k\geq 1.$$

Therefore

(18) $$h_1=\frac{1}{r},\quad H_1=\frac{1}{\varrho}e^{\frac{\varrho}{2}(\mu-1)}.$$

B. Second expansion term

From the form of $H_1(\varrho, \mu)$ for small ϱ we conclude that $f_1(R_e)=\sigma R_e$. Thus, if the velocity in the Stokes region is

$$\mathbf{q}=\mathbf{q}_1(r, \mu)+R_e\,\mathbf{q}_2(r, \mu)+O(R_e^2 \ln R_e)$$

from equation (3) it will be deduced that

(19) $$\nabla_r^2 h_2=\mathbf{q}_1 \cdot \operatorname{grad} h_1.$$

Using (1a) and (18) we obtain

$$\nabla_r^2 h_2=\left(-\frac{1}{r^2}+\frac{3-m_0}{2r^3}-\frac{1+m_0}{2r^5}\right)\mu.$$

In view of (14) the general solution of the equation satisfying the boundary condition (10), is

$$h_2=A_0\left(\frac{1}{r}-1\right)+\left[\left(\frac{3-m_0}{8}-A_1\right)r+\frac{A_1}{r^2}+\left(\frac{1}{2}-\frac{3-m_0}{4r}-\frac{1+m_0}{8r^3}\right)\right]\mu$$

$$+\sum_{k=2}^{\infty} A_k(r^{-k-1}-r^k)P_k(\mu).$$

Matching $h_1+\sigma R_e h_2$ with $\sigma R_e H_1$ we find that

$$A_0=\frac{1}{2}, \quad A_1=\frac{3-m_0}{8}, \quad A_k=0 \quad \text{for} \quad k\geq 2.$$

Hence

(20) $$h_2=\frac{1}{2}\left(\frac{1}{r}-1\right)+\left(\frac{1}{2}-\frac{3-m_0}{4r}+\frac{3-m_0}{8r^2}-\frac{1+m_0}{8r^3}\right)\mu.$$

Now, from the form of $h_1+\sigma R_e h_2$ for small ϱ we conclude

$$F_2(R_e)=\sigma^2 R_e^2.$$

Thus, if the velocity in the Oseen region is

$$\mathbf{q}=Q_1(\varrho, \mu)+R_e\,Q_2(\varrho, \mu)+O(R_e^2)$$

we deduce from equation (7) and (18) that

(21) $$\nabla_\varrho^2 H_2-\mu\frac{\partial H_2}{\partial \varrho}+\frac{\sqrt{1-\mu^2}}{\varrho}\cdot\frac{\partial H_2}{\partial \mu}=D(\varrho, \mu, \sigma, m_0)$$

where

$$D(\varrho, \mu, \sigma, m_0)=-\frac{3-m_0}{4}\varrho^{-4}e^{-\frac{1}{2}\varrho}[2\sigma+\sigma\varrho-\sigma\varrho\mu-$$

$$-e^{-\frac{1}{2}\varrho\sigma-1}(\sigma\varrho+2\sigma+\varrho)-(\sigma-1)\varrho\mu e^{\frac{1}{2}\varrho\mu\sigma-1}].$$

The transformation

$$H_2(\varrho, \mu)=e^{\frac{1}{2}\varrho\mu}G_2^*(\varrho, \mu)$$

reduce the equation (21) to

$$\left(\nabla_{\varrho}^{2}-\frac{1}{4}\right) G_{2}^{*}=D(\varrho, \mu, \sigma, m_{0})$$

Further, since

$$e^{\mu z}=(2\pi)^{\frac{1}{2}} \sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right) z^{-\frac{1}{2}} I_{n+\frac{1}{2}}(z) P_{n}(\mu)$$

$$\mu\, e^{\mu z}=(2\pi)^{\frac{1}{2}} \sum_{n=0}^{\infty} n\left(n+\frac{1}{2}\right) z^{-\frac{2}{3}} I_{n+\frac{1}{2}}(z) P_{n}(\mu)+(2\pi)^{\frac{1}{2}} \sum_{n=0}^{\infty}\left(n+\frac{1}{2}\right) z^{-\frac{1}{2}} I_{n+\frac{3}{2}}(z) P_{n}(\mu)$$

where $I_{n+\frac{1}{2}}(z)$ is a modified Bessel function we can put

$$G_{2}(\varrho, \mu)=\sum_{n=0}^{\infty} g_{n}(\varrho) P_{n}(\mu). \tag{22}$$

After some algebra it ts obtained

$$g_{0}(\varrho)=A_{\varrho}^{-1} e^{-\frac{1}{2} \varrho}+\frac{3-m_{0}}{4\varrho}\left(\frac{1}{3} \sigma^{3} \varrho^{-2}-\frac{1}{3} \sigma^{3} \varrho^{-2}\right) e^{-\frac{1}{2} \varrho}$$

$$+\frac{3-m_{0}}{4\varrho}\left[M(\sigma, \varrho)\, e^{-\frac{1}{2} \varrho-\varrho \sigma^{-1}}+N(\sigma, \varrho)\right]$$

where

$$M(\sigma, \varrho)=-\frac{1}{3} \sigma^{3} \varrho^{-2}-\frac{1}{3} \sigma^{2} \varrho^{-1}+\frac{1}{3} \sigma^{3} \varrho^{-1}$$

$$N(\sigma, \varrho)=-\sigma\left(1-\frac{\sigma^{2}}{3}\right) E_{1}(\varrho)\, e^{\frac{1}{2} \varrho}-\frac{2}{3} E_{1}(\varrho \sigma^{-1})\, e^{-\frac{1}{2} \varrho}-\frac{(1+\sigma)^{2}}{3}(\sigma-2)\, e^{\frac{\varrho}{2}} E_{1}\left[\frac{(1+\sigma) \varrho}{\sigma}\right]$$

and

$$E_{1}(\varrho)=\int_{\varrho}^{\infty} \frac{e^{-\varrho}}{\varrho}\, d\varrho.$$

For small ϱ

$$g_{0}(\varrho) \approx \varrho^{-1}\left\{A+\frac{3-m_{0}}{4}\left[\frac{\sigma}{6}(1-2\sigma)+\frac{1}{3}(\sigma+1)^{2}(\sigma-2) \ln(\sigma+1)-\frac{1}{3} \sigma(\sigma^{2}-3) \ln \sigma\right]\right\}+$$

$$+\left\{-\frac{1}{2} A+\frac{3-m_{0}}{4}\left[-\frac{2}{3}(\gamma+\ln \varrho)-\frac{1}{6}(\sigma^{3}-3\sigma-4) \ln \sigma+\right.\right.$$

$$\left.\left.+\frac{1}{6}(\sigma+1)^{2}(\sigma-2) \ln(\sigma+1)-\frac{1}{36}(6\sigma^{2}+3\sigma-56)\right]\right\}+O(\varrho)$$

where γ is the familiar Euler constant

$$g_{1}(\varrho) \approx-\frac{3-m_{0}}{4} \varrho^{-1}+O(1)$$

and

$$g_{n}(\varrho)=O(\varrho^{n-2}) \quad n \geq 2$$

Matching $h_{1}+\sigma R_{e} h_{2}$ with $\sigma R_{e} H_{1}+\sigma^{2} R_{e}^{2} H_{2}$ in an intermediate region we find that

$$A=\frac{1}{2}-\frac{3-m_0}{4}\left[\frac{\sigma}{6}(1-2\sigma)+\frac{1}{3}(\sigma+1)^2(\sigma-2)\ln(\sigma+1)-\frac{1}{3}\sigma(\sigma^2-3)\ln\sigma\right].$$

C. Higher-order expansion terms

Since $H_2(\varrho,\mu)$ contains the term $\sigma^2 R_e^2 \ln\varrho$ where $\varrho=\sigma R_e . r$ we conclude that $f_3=\sigma^2 Re^2 \ln \sigma R_e$ and $f_4=\sigma^2 R_e^2$.
Hence

(23) $$\nabla_\varrho^2 h_3=0$$

and

(24) $$\nabla_\varrho^2 h_4=\mathbf{q}_1 . \operatorname{grad} h_2+\sigma^{-1}\mathbf{q}_2 . \operatorname{grad} h_1.$$

Further, it is found the solution of equation (23) which satisfies the boundary condition $h_3(1,\mu)=0$ and the appopriate matching condition

$$h_3=-\frac{3-m_0}{6}\left(1-\frac{1}{r}\right).$$

Equation (24), in view of equations (2), (2a), (2b), (18) and (20) may be rearranged into

$$\nabla_\varrho^2 h_4=\sum_{k=0}^{2} G_k(r)P_k(\mu)$$

where

$$G_0(r)=\frac{1}{3r}-\frac{3-m_0}{6r^2}+\frac{m_0^2-8m_0+7}{48r^4}+\frac{(1+m_0)(3-m_0)}{24r^5}-\frac{(3-m_0)(1+m_0)}{16r^6}+\frac{(1+m_0)^2}{12r^7}$$

$$G_1(r)=\left[-\frac{1}{2r^2}+\frac{3-m_0}{4r^3}-\frac{1+m_0}{4r^5}\right]\left[1+\frac{3-m_0}{4}\sigma^{-1}\right]$$

$$G_2(r)=\left[-\frac{1}{3r}+\frac{5(3-m_0)}{12r^2}-\frac{3m_0^2-24m_0+45}{24r^3}+\frac{5m_0^2-10m_0+65}{48r^4}\right.$$
$$\left.-\frac{5(1+m_0)(3-m_0)}{48r^5}-\frac{(1+m_0)(3-m_0)^2}{16r^6}+\frac{5(1+m_0)^2}{48r^7}\right]$$
$$+\sigma^{-1}\frac{3-m_0}{32}\left[\frac{2}{r^2}-\frac{3-m_0}{r^3}-\frac{m_0+1}{r^5}+\frac{m_0+1}{r^6}+\frac{1}{r^4}+\frac{64}{3r^4}\cdot\frac{M_1}{3-m_0}\right]$$

where M_1 is a permeability constant of second order.

The general solution of this equation which ramains bounded for all $\mu\in[-1,1]$ is:

$$h_4=\sum_{k=0}^{2}\Phi_k(r)p_k(\mu)+\sum_{n=0}^{\infty}(A_n r^{-n-1}+B_n r^n)P_n(\mu)$$

where the A_n and B_n are constants and

$$\Phi_0(r)=\frac{1}{6}r+\frac{3-m_0}{6}-\frac{3-m_0}{6}\ln r+\frac{m_0^2-8m_0+7}{96r^2}+$$
$$+\frac{(1+m_0)(3-m_0)}{144r^3}-\frac{(3-m_0)(1+m_0)}{192r^4}+\frac{(1-m_0)^2}{240r^5}-\frac{(3-m_0)}{24}\left(1-\frac{1}{r}\right)$$

$$\Phi_1(r)=\frac{1}{16}\left[4+(3-m_0)\,\sigma^{-1}\right]\left[1-\frac{3-m_0}{2r}-\frac{1+m_0}{4r^3}\right]$$

$$\begin{aligned}\Phi_2(r)=&\frac{r}{12}-\frac{5(3-m_0)}{72}+\frac{m_0^2-8m_0+15}{48r}-\frac{5\,(m_0^2-2m_0+13)}{192r^2}\\&+\frac{(1+m_0)(3-m_0)}{240r^3}+\frac{(1+m_0)(3-m_0)}{48r^3}\ln r-\frac{(1+m_0)(3-m_0)}{96r^4}\\&+\frac{5\,(1+m_0)^2}{672r^5}+\frac{3-m_0}{16}\,\sigma^{-1}\left[-\frac{1}{3}+\frac{3-m_0}{6r}+\frac{m_0+1}{25r^3}\right.\\&\left.+\frac{m_0+1}{5r^3}\ln r+\frac{1+m_0}{6r^4}-\frac{1}{4r^2}\left(1-\frac{64}{3}\cdot\frac{M_1}{3-m_0}\right)\right]\end{aligned}$$

Using the condition $h_4(1,\mu)=0$ and matching $h(r,\mu)$ with $H(\varrho,\mu)$ we obtain

$$\begin{aligned}A_0=&-\frac{3-m_0}{6}-\frac{1}{4}+\frac{3-m_0}{8}\left[\frac{\sigma}{6}(1-2\sigma)+\frac{1}{3}(\sigma+1)^2(\sigma-2)\ln(\sigma+1)\right.\\&\left.-\frac{1}{3}\sigma(\sigma^2-3)\ln\sigma\right]+\frac{3-m_0}{4}\left[-\frac{2}{3}\gamma-\frac{1}{6}(\sigma^3-3\sigma-4)\ln\sigma\right.\\&\left.+\frac{1}{6}(\sigma+1)^2(\sigma-2)\ln(\sigma+1)-\frac{1}{36}(6\sigma^2-3\sigma-56)\right]\end{aligned}$$

$$\begin{aligned}A_1=-\frac{1}{4},\quad A_n=0\quad n\geq 2-(A_0+B_0)=&\frac{1}{6}+\frac{3-m_0}{6}+\frac{m_0^2-8m_0+7}{96}\\&+\frac{(1+m_0)(3-m_0)}{144}-\frac{(3-m_0)(1+m_0)}{192}+\frac{(1+m_0)^2}{240}\end{aligned}$$

$$B_1=-\frac{1}{4}-\frac{1}{16}\left[4+(3-m_0)\,\sigma^{-1}\right]\left[1-\frac{3-m_0}{2}-\frac{1+m_0}{4}\right]$$

$$\begin{aligned}B_2=&-\frac{1}{12}-\frac{m_0^2-8m_0+15}{48}+\frac{5\,(m_0^2-2m_0+13)}{192}-\frac{(1+m_0)(3-m_0)}{240}+\frac{(3-m_0)(1+m_0)}{96}\\&-\frac{5\,(1+m_0)^2}{672}-\frac{3-m_0}{16}\,\sigma^{-1}\left[-\frac{1}{3}+\frac{3-m_0}{6}+\frac{1+m_0}{25}+\frac{1+m_0}{6}-\frac{1}{4}\left(1-\frac{64}{3}\cdot\frac{M_1}{3-m_0}\right)\right]\end{aligned}$$

$$B_n=0\quad\text{for}\quad n\geq 3$$

The method of matched asymptotic expansions is systematic and we are not going to consider higher orders in this paper. The total flux from the surface of the sphere is

$$Q=-\int_0^{\pi}\lambda\left(\frac{1}{a}\frac{\partial T}{\partial r}\right)_{r=1}2\pi a^2\sin\theta d\theta$$

where λ is the thermal conductivity. Then Nusselt number defined as $N_u=\frac{Q}{4\pi a^2}\cdot\frac{\lambda\,(T_w-T_\infty)}{2a}$ is given by $N_u=\int_{-1}^{1}\left(\frac{\partial h}{\partial r}\right)_{r=1}d\mu.$

In view of the orthogonality relation

$$\int_{-1}^{1}P_i(\mu)d\mu=0\quad\text{for}\quad i\neq 0$$

We obtain

$$N_u = 2 + P_e + \frac{3-m_0}{3}\left(P_e^2 \ln P_e + P_e^2 \varphi(\sigma)\right) \tag{25}$$

where

$$\varphi(\sigma) = -\frac{173}{160} + \frac{174}{1440} m_0 + \frac{83}{1440} m_0^2 + \frac{3-m_0}{12}\left[(2\sigma^2 - \sigma + 4\gamma) + 2(\sigma^3 - 3\sigma - 2)\ln\frac{\sigma}{\sigma+1}\right] =$$
$$= -\frac{345}{1440} m_0 + \frac{83}{1440} m_0^2 + \frac{3-m_0}{12}\left[\left(2\sigma^2 - \sigma - 4\gamma - \frac{173}{40}\right) + 2(\sigma^3 - 3\sigma - 2)\ln\frac{\sigma}{\sigma+1}\right] \tag{26}$$

4. Conclusion

The present paper considers the problem of the heat transfer from a sphere in flows which have small Reynolds number. The principal results of this investigation are given in formulas (25) and (26). Formula (25) for the Nusselt number is applicable under the provision $R_e < 1$ and $\sigma R_e < 1$. Our analysis of the heat transfer from a porous sphere in a low Reynolds number flow includes Rimmer's analysis as a particular case — when the permeability constants m_0 and M_1 are equal to zero. Values of the Nusselt number calculated from (25) are found to differ significantly from the results calculated in the case of an impervious sphere. The effect of permeability consists in the fact that the gradient of the temperature and of the heat transfer from a sphere decreases. This can be seen from formula (25) and Table 1, where $m_0 = \frac{1}{10}$.

Table 1

σ	$f(\sigma)$	$\varphi(\sigma)$	σ	$f(\sigma)$	$\varphi(\sigma)$
0.50	1.3499	1.2815	1.05	1.1203	1.0593
0.55	1.3144	1.2472	1.10	1.1092	1.0487
0.60	1.2836	1.2176	1.15	1.0990	1.0389
0.65	1.2566	1.1913	1.20	1.0895	1.0298
0.70	1.2328	1.1681	1.25	1.0806	1.0215
0.75	1.2115	1.1476	1.30	1.0723	1.0130
0.80	1.1924	1.1293	1.35	1.0646	1.0056
0.85	1.1752	1.1125	1.40	1.0574	0.9986
0.90	1.1596	1.0976	1.45	1.0506	0.9920
0.95	1.1453	1.0836	1.50	1.0441	0.9858
1.00	1.1323	1.0709			

Here for comparison the Rimmer's calculations for $f(\sigma)$ are given.

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