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Changes of elastic modulus in surface layer and its thickness evaluation

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In numerous problems of Mechanics of Solid Bodies the values of elastic moduli are assumed to be constant.

Actually, however, various actions can be followed by significant variations of these values due to variations in internal equilibrium of a body. The elastic moduli variation has to result in important consequences, as e.g. the fallacy of the hypothesis of convex and gradiental character of plasticity conditions. Elastic moduli are to be regarded as a function of thermomechanical state of a material.

One of the problems worked on was the effect of plastic prestrain and that of specimen geometry on the value of elastic modulus.

Elastic modulus of solid steel specimens prestrained in torsion was found to depend on their diameter [1]. The variations of the modulus are thought to result from various strain gradient values, depending on the diameter of a specimen. This conclusion was confirmed by the results of material density measurements carried out several times after etching of consecutive surface layers [2].

Different strain gradients and strain distribution are not the only factors involved in the effect of specimen geometry on its elastic modulus value. The surface itself and the state of surface layer are of importance as well. The measurements of elastic moduli of non-deformed brass specimens with various ratios of specimen surface F to its volume V have shown that the value of elastic modulus E depends linearly on the ratio F/V [3, 4]

$$E=a-b\frac{F}{V}.$$

As can be seen in Fig. 1 the variation of elastic modulus E is equal 6.5 % within the investigated range of F/V ratio, 95 % confidence range was assumed in calculation of modulus variation value. The variation may result from a considerable change of modulus value in the surface layers of specimens due to machining. Solid and hollow cylindrical specimens were tested, both of the same cross-section surface area value. External surfaces of specimens were precision-turned on the lathe and ground, while their internal surfaces underwent reaming.

When we assume the same strain of the surface layer of thickness δ and elastic modulus E_1 and of the specimen core of modulus E_2 as well as the

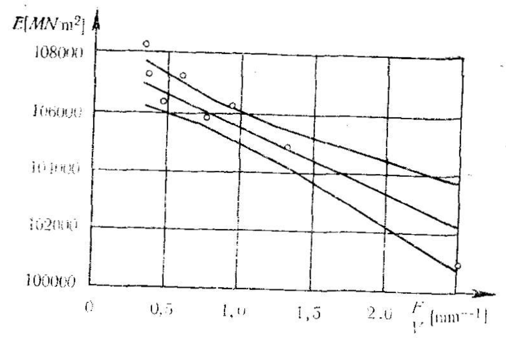

Fig. 1. The dependence of the elastic modulus of solid and hollow specimens on the surface volume ratio of the specimen

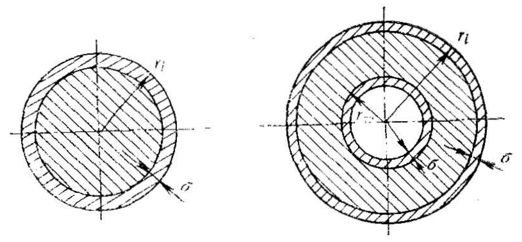

Fig. 2. The cross-sections of solid and hollow specimens

proportionality between load and deformation then from the condition of equilibrium we obtain

(2) $$E_1F_{1i}+E_2F_{2i}=E_iF_i,$$

where:

E_i — elastic modulus of i-th specimen;

F_i — total surface area of its cross-section;

$F_{1i}=2\pi r_i\delta,$

$F_{2i}=\pi(r_i-\delta)^2$, for solid specimens.

and

$F_{1i}=2\pi\delta(r_i+r_w),$

$F_{2i}=\pi[(r_i-\delta)^2-(r_w+\delta)^2]$, for hollow specimens.

The cross-section of solid and hollow specimens can be seen on Fig. 2. The same thickness δ for both external and internal layers have been assumed.
To cover a possibly large range of F/V ratio values the measurements were carried out on 4 solid and 4 hollow cylindrical specimens. 2.500 measurement points were obtained for $i=8$ values of F/V ratio.

The overdetermined system of non-linear equations (2) with 3 unknowns E_1, E_2 and δ was obtained.

We have obtained unreal solution for this system, then we must find another way for solving this problem.

Möhler and Roth [5] found the thickness of the layer deformed during rough turning of low-carbon steel to be equal to 0.3 *mm*, while precision turning resulted in 0.05 mm thick deformed layer. According to Maslow [6] grinding results in 6 μm thick deformed layer.

Taking these results into account, E_1 and E_2 values were computed (on Odra 1304 computer) from the overdetermined system of eq (2) for some values of δ.

The solution obtained for $\delta = 0.01$ *mm* being unreal, the remaining results are tabularized in Table 1.

Table 1

The values of elastic modulus obtained as solution equation 2

Assumed value [mm]	Calculated values of elastic modulus [MN/m²]	
	in surface layer E_1	in specimen core E_2
0.02	81	10766
0.03	3648	10766
0.04	5434	10766
0 05	6506	10766
0.06	7222	10765
0.07	7734	10764
0.08	8119	10764
0.09	8419	10763
0.10	8659	10762

In many investigations (e. g. Zukow [7], Bastun and Czerniak [8]) plastic prestrain to $\varepsilon = 0.2\,\%$ results in the decrease of elastic modulus by 20%.

Our observations of cross-sections on an optical microscope have shown that slip bands reach the depth of 0.07 mm. When the range of elastic deformations is also considered, as well as the results presented in papers [7, 8], it seems that the thickness of deformed surface layer can be assumed to be equal to 0.07-0.1 mm. It would be in good agreement with the results presented by Möhler and Roth [5]. Resulting change in elastic modulus value would then be equal to 20-28%.

It seems worth mentioning that this variation of elastic modulus is very important with regard to friction and wear problems and investigation of thin-walled specimens. We are still working on these problems.

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