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Chain solution bifurcation of offshore scaffold bridge hydroelastic problem*

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The solution of the problem on the scattering of a plane wave by an arbitrary configuration of parallel cylinders is specialized to consider the case of an impulsive loading. The problem of a row of piles when all axes lie in one and the same plane under an impulsive loading is considered in details. The scheme for construction of the solution and the convergence of the method are shown. For several time increments the elastic responses of the piles are determined during the process.

Introduction

Circular cylindrical structural components are widely used in offshore and other structures. The design of such structures is always connected with an estimation of the hydrodynamical loadings, acting on the components of the structure.

When the problem is to calculate the wave forces on a group of cylinders with sufficiently large radii, it is necessary to take into account the effect of the velocity field modification caused by the presence of the cylinders and also the interaction between the velocity field. Diffraction theory takes full account of these effects.

The classical method for wave force calculations involves the simultaneous solution of an infinite number of linear equations with Bessel-function coefficients. This is obviously very cumbersome and impractical, especially if the structure examined has more than two cylinders. Therefore it is necessary to develop methods for the solution simplification.

In the present paper a method consisting of successive approximations for calculation of the forces acting on a group of cylinders is represented.

Assumptions

It is assumed that a group of vertical parallel cylinders vibrates under the wave loading in shallow water with constant depth h (Fig. 1). The water is assumed to be compressible with sound velocity c. The purpose of

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the investigation is to determine the wave loading acting on the cylinders by a simple method.

Solution

Here the linear hydroelasticity will be considered. Therefore the full hydrodynamic pressure can be represented as an algebraic sum of the pressures acting on a stationary rigid cylinder: $p_{s,0}$ — the pressure due to the incident field, $p_{s,d}$ — the pressure due to the scattered field and the pressure $p_{s,f}$ due to the additional field caused by the elastic motion of the cylinders.

The pressure on the s^th^ cylinder of the group can be written in the form

$$p_s = p_{s,0} + p_{s,d} + p_{s,f} \tag{1}$$

and correspondingly — the velocity potential will be

$$\Phi_s = \Phi_{s,0} + \Phi_{s,d} + \Phi_{s,f}. \tag{2}$$

For the velocity potentials $\Phi_{s,d}$ and $\Phi_{s,f}$ it is enough to satisfy the wave equations

$$\Delta\Phi_{s,d} - c^{-2}\ddot{\Phi}_{s,d} = 0 \quad \text{and} \quad \Delta\Phi_{s,f} + c^{-2}\ddot{\Phi}_{s,f} = 0, \tag{3,4}$$

and the corresponding boundary conditions on the surfaces of the cylinders

$$(\partial\Phi_{s,0}/\partial n)_{\Sigma_s} + (\partial\Phi_{s,d}/\partial n)_{\Sigma_s} = 0, \tag{5}$$

$$(\partial\Phi_{s,f}/\partial n)_{\Sigma_s} = \partial w_s/\partial t, \tag{6}$$

where **n** is the normal vector to the surface Σ_s and w_s are the transversal displacements $w_s(y, z)$.

1. Method of Solution

To determine the pressure $p_{s,d}$ a method of successive examination of the influence of the surrounding cylinders on any other cylinder is applied. The method is called "method of successive approximations".

As a first approximation the velocity potential created by a single cylinder without taking into account the influence of the neighbouring cylinders is accepted. The scattered field by every cylinder is considered as an incident field on the stage of the second approximation. At the third approximation the second interaction between the cylinders is taken into account, etc.

This method enables us to write the solution in the definite form independently of the disposition of the cylinders. It is important to note that at every approximation the problem of single cylinder is a subject of solution, and this gives a possibility to consider a simple arborescent structure of the whole solution, called a "bifurcation chain".

2. The Velocity Potential

2.1. The Incident Field. Under consideration is a linear plane wave of amplitude a_0 and frequency ω irradiating a group of vertical parallel cylinders of radii R_s. They are cantilevered in the see bed which is flat and impermeable. The wave is travelling at angle Θ to the positive y-axis (Fig. 1).

The potential due to the incident plane wave, in terms of *Bessel* — functions, is [1]:

$$\Phi_{s,0}=\frac{iga_0}{\omega}X(x)\sum_{n=0}^{\infty}\varepsilon_n(-i)^n J_n(\psi r_s)\exp[-in(\theta-\Theta)]\exp(i\omega t), \tag{7}$$

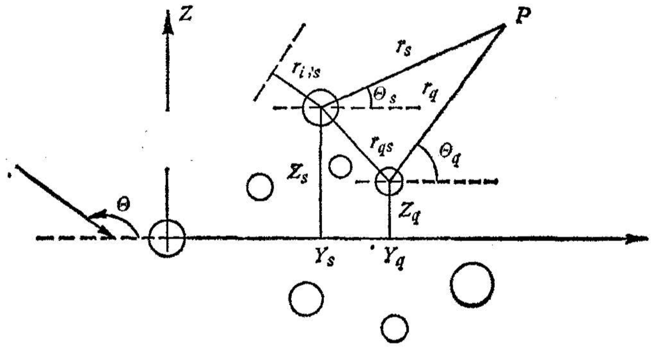

Fig. 1. An arbitrary configuration of parallel cylinders

where (θ, r) are cylindrical co-ordinates, ψ is the general wave number $\psi^2=[(\omega^2/g)-(\omega/c)^2]$.

The phase of the incident wave at the s^th^ cylinder is $\gamma_{i,s}=\psi r_{o,s}$ while the phase difference between the waves originating at the cylinders s and q is $\gamma_{d,sq}=\psi r_{sq}$. This phases can be easy calculated in every specific case.

2.2. *Scattered Field.* The problem of interaction between the liquid and the single cylinder under wave loading is solved. The total velocity potential in the near vicinity of every cylinder is sum of the potentials of the incident and scattered fields

$$\Phi_{1,s}=\Phi^{(1)}_{s,o}+\Phi^{(1)}_{s,d}. \tag{8}$$

The potential of the scattered field is obtained in the form

$$\Phi_{s,d}=\frac{iga_0}{\omega}X(x)\sum_{n=0}^{\infty}A^{(1)}_n H^{(2)}_n(\psi r_s)\exp(-in\theta)\exp(i\omega t), \tag{9}$$

where the constants $A^{(1)}_n$ are determined using the boundary conditions (5) on a stationary cylinder

$$A^{(1)}_n=\varepsilon_n(-i)^n\exp(in\theta)J'_n(\psi R_s)/\overline{H}^{(2)}_n(\psi R_s), \tag{10}$$

where $\varepsilon_n=1$ for $n=0$, $\varepsilon_n=2$ for $n\neq 0$; J'_n and $\overline{H}^{(2)}_n$ are derivatives of the cylindrical functions, given by the formulas

$$J'_n(\psi R_s)=\psi^{-1}[\partial J_n(\psi r_s)/\partial r_s]_{r_s=R_s},\quad \overline{H}^{(2)}_n(\psi R_s)=\psi^{-1}[\partial H^{(2)}_n(\psi R_s)/\partial r_s]_{r_s=R_s},$$

$X(x)=cosh[k(x+h)]/cosh(kh)$, k is the wave number along the x-axis given by the equation $(kh).th(kh)=\omega^2h/g$.

At the second approximation under consideration is the effect, which the presence of all cylinders has on every one of them. The scattered field propagates to the neighbouring cylinders and upon reaching their rigid surfaces gives rise to a secondary scattering.

Here the incident field is [2]:

$$\Phi^{(2)}_{s,0}=\sum_{q\neq s}\Phi^{(2)}_{o,(q\to s)}=\sum_{q\neq s}\Phi^{(1)}_{d,(q\to s)},$$

(11)
$$\Phi^{(2)}_{s,o}=\frac{iga_0}{\omega}X(x)\sum_{q\neq s}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\varepsilon_m A^{(1)}_n[(-1)^m(Z^{nm}_{qs})_1 e^{im\theta_s}+$$
$$+(Z^{nm}_{qs})_2 e^{-im\theta_s}]J_m(\psi r_s)\,exp\,(i\omega t+i\gamma_{i,q}),$$

where $(Z^{nm}_{qs})_1=H^{(2)}_{n+m}(\psi r_{qs})e^{-i\beta_{qs}(n-m)}$, $(Z^{nm}_{qs})_2=H^{(2)}_{n-m}(\psi r_{qs})e^{i\beta_{qs}(n+m)}$ and β_{qs} characterizes the mutual disposition of the cylinders.

The secondary scattering is defined in an analogous form

(12)
$$\Phi^{(2)}_{s,d}=\frac{iga_0}{\omega}X(x)\sum_{m=0}^{\infty}[A^{(2)}_m e^{-im\theta}+B^{(2)}_m e^{im\theta}]H^{(2)}_m(\psi r_s)\,exp\,(i\omega t),$$

where the unknown $A^{(2)}_m$ and $B^{(2)}_m$ are obtained from the conditions (5) on the rigid surface of s^{th} cylinder

(13)
$$A^{(2)}_m=\sum_{q\neq s}\sum_{n=0}^{\infty}\varepsilon_m A^{(1)}_n(Z^{nm}_{qs})_2\,exp\,(i\gamma_{i,q})J'(\psi R_s)/\overline{H}^{(2)}_m(\psi R_s),$$
$$B^{(2)}_m=\sum_{q\neq s}\sum_{n=0}^{\infty}\varepsilon_m A^{(1)}_n(Z^{nm}_{qs})_1\,exp\,(i\gamma_{i,q})J'(\psi R_s)/\overline{H}^{(2)}_m(\psi R_s).$$

The total velocity potential after the second approximation, which takes into account the interaction between all cylinders, is a sum of the velocity potential obtained at the first approximation $\Phi_{1,s}$ and the potential obtained at the second approximation

(14)
$$\Phi_{2,s}=\Phi_{1,s}+\Phi^{(2)}_{s,0}+\Phi^{(2)}_{s,d}.$$

As an incident field at the third approximation the secondary scattering is taken. It acts analogously on the rigid surfaces of the cylinders.

The velocity potential of the "third" incident field is

(15)
$$\Phi^{(3)}_{s,0}=\sum_{q\neq s}\Phi^{(3)}_{0,(q\to s)}=\sum_{q\neq s}\Phi^{(2)}_{d,(q\to s)}.$$

The new scattered field is in the same form:

(16)
$$\Phi^{(3)}_{s,d}=\frac{iga_0}{\omega}X(x)\sum_{k=0}^{\infty}[A^{(3)}_k e^{-ik\theta_s}+B^{(3)}_k e^{ik\theta_s}]H^{(2)}_k(\psi r_s)\,exp\,(i\omega t),$$

where $A^{(3)}_k$ and $B^{(3)}_k$ are defined from the same boundary conditions.

After the third approximation the total velocity potential in the s^{th} coordinate system is

(17)
$$\Phi_{3,s}=\Phi_{2,s}+\Phi^{(3)}_{s,0}+\Phi^{(3)}_{s,d}.$$

The approximations can be continued until it is necessary to take into account the successively obtained scattered field

$$(18)\qquad \Phi_{j,s}=\sum_{\varkappa=1}^{j}(\Phi^{(\varkappa)}_{s,0}+\Phi^{(\varkappa)}_{s,d}).$$

In this method of successive approximations, it is necessary to calculate only two types of constants $A^{(\varkappa)}_k$ and $B^{(\varkappa)}_k$. They are obtained directly from the boundary conditions. It is proved [3] that the method used is convergent.

3. *The Bifurcation Chain*

The method for the velocity potential calculation described above can be easy generalized by a bifurcation chain from which the unknown constants at every approximation can be calculated. Such bifurcation chain is given in Fig. 2. By A the coefficient in front of the negative exponent degree and by B the coefficient in front of the positive exponent degree are marked. This bifurcation chain is valid for any two cylinders of the group. In that way, for calculation of the unknown coefficients A and B, it is enough to trace the chain from the level of a certain approximation to its beginning. The factors before the positive and negative exponents have to be grouped and multiplied by the corresponding factors

$$\prod_{\varkappa=1}^{j-1}(J'_m/\overline{H}^{(2)}_m).$$

The number of approximations sufficient to obtain results with a satisfactory precision can be estimated by the values of the coefficients at every approximation.

It is necessary to mention that in the summation of all velocity fields the phase difference between the cylinders has to be taken into account.

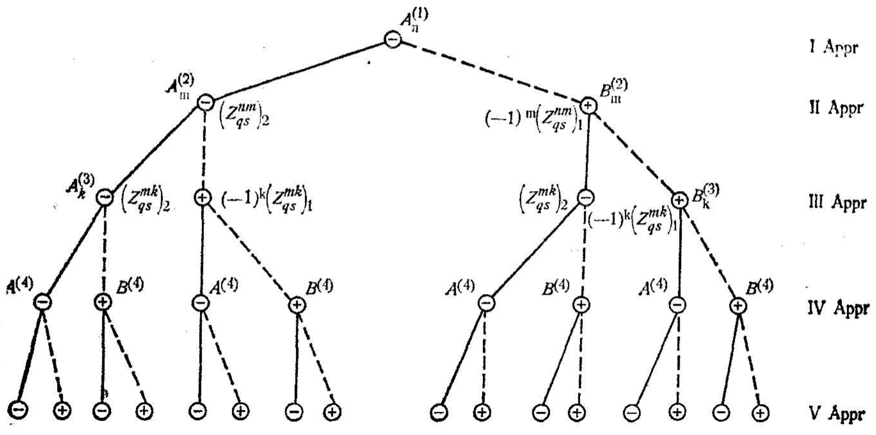

Fig. 2. A bifurcation chain

4. The System Response to an Impulsive Loading

After decomposition the impulse, the general solution can be obtained as a sum of system responses to all harmonics or to several of them, thus providing the main part of the solution

$$\Phi_s = \sum_{m=1}^{\infty} \Phi_{s,m} \tag{19}$$

where m is the number of the harmonic.

It is no difficult to calculate the force acting on every cylinder by velocity potential Φ_s as

$$\vec{F}_x(x, t) = \sum_m \bar{F}_s(x, \omega_m, t) = -\sum_m \int_\Sigma p_m \bar{n} d\Sigma, \tag{20}$$

and the force per unit length at depth x is

$$\vec{F}_s(x, \omega_m, t) = \frac{\cosh[k(x+h)]}{\cosh(kh)} \rho \int_0^{2\pi} \frac{\partial \Phi_{s,m}}{\partial t} \vec{n} R_s d\theta_s. \tag{21}$$

Example

A scaffold bridge in shallow water with constant depth ***h*** is considered, Fig. 3. It is represented by a row of long cylindrical piles of identical radii R. The distance between two neighbouring cylinders is d.

1. The Loading

At angle Θ to the positive y-axis the impulsive loading $p(x, t)$ acts on the row of piles

$$p(x, t) = \begin{cases} X(x) p_0 \sum_{m=1}^{\infty} \frac{m\pi[1-(-1)^m e^{-1}]}{1+m^2\pi^2} \sin\frac{m\pi}{t_1}, & \gamma_{i,s}/\omega_m \leq t \leq t_1 + \gamma_{i,s}/\omega_m \\ 0, & t > t_1 + \gamma_{i,s}/\omega_m, \end{cases} \tag{22}$$

where $X(x)$ is an above mentioned form, t_1 is the duration of the impulse.

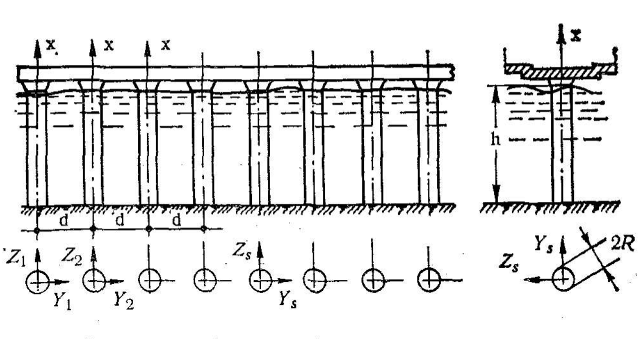

Fig. 3. Scaffold bridge in shallow water

2. The Forces

At the first approximation the force on every single cylinder is calculated in the form

$$(23)\qquad F_{1,s}(x,\omega_m,t)=X(x)\rho\int_0^{2\pi}\frac{\partial\Phi_{1,s}}{\partial t}\bar{n}R_s d\theta_s\exp(i\omega_m t)$$

$$=X(x)\rho g p_0 R\left[J_1(\psi R)+\frac{J_1'(\psi R)}{\bar{H}_1^{(2)}(\psi R)}H_1^{(2)}(\psi R)\right]\exp[i(\omega_m t+\gamma_{i,s}]=X(x).K.\exp(i\omega_m t),$$

where $K=\rho g p_0 R[J_1(\psi R)+J_1'(\psi R).H_1^{(2)}(\psi R)/\bar{H}_1^{(2)}(\psi R)]\exp(i\gamma_{i,s})$.

At the second approximation the additional force on every pile due to the presence of other cylinders is obtained

$$(24)\qquad F_s^{(2)}(x,\omega_m,t)=X(x)p\int_0^{2\pi}\frac{\partial\Phi_s^{(2)}}{\partial t}\bar{n}Rd\theta_s\exp(i\omega_m t)=F_{1,s}(x,\omega_m,t).C_{2,s,m}$$

$$=X(x)K\int_0^{2\pi}\sum_{q\neq s}\sum_{n=0}^{\infty}(-i)^n\varepsilon_n\frac{J_n'}{\bar{H}_n^{(2)}}[(-1)^k(Z_{qs}^{nk})_1e^{ik\theta_s}+(Z_{qs}^{nk})_2e^{-ik\theta_s}]\begin{pmatrix}\cos\theta_s\\ \sin\theta_s\end{pmatrix}e^{i(\omega_m t+\gamma_{i,s})}$$

The next additional force taking into account the secondary scattering is

$$(25)\qquad F_s^{(3)}(x,\omega_m,t)=F_{1,s}(x,\omega_m,t).C_{3,s,m}.$$

Thus, the total force acting on an arbitrary rigid pile is a sum of the forces calculated at every approximation, and it can be represented as

$$(26)\qquad F_s(x,\omega_m,t)=F_s^{(1)}(x,\omega_m,t)+F_s^{(2)}(x,\omega_m,t)+F_s^{(3)}(x,\omega_m,t)+\ldots$$

$$=F_s^{(1)}(x,\omega_m,t)(1+C_{2,s,m}+C_{3,s,m}+\ldots).$$

The moduli of the coefficients $C_{j,s,m}$ are called "influence coefficients" and they strongly decrease with increasing the distance. In Fig. 4 the influ-

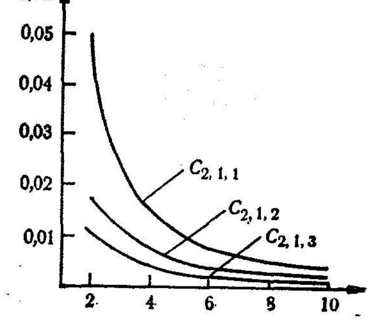

Fig. 4. The influence coefficients (first scattering)

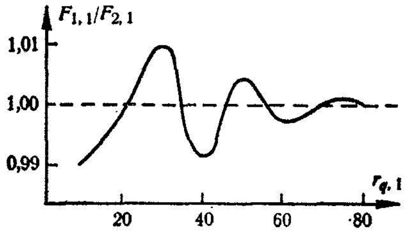

Fig. 5. The interaction beween two cylinders

ence coefficients as functions of pile disposition are shown for the first three frequencies ω_m. In Fig. 5 the ratio $F_{2,s}/F_{1,s}$ for $s=1$ is given as the same function. As can be seen from Fig. 5 the interaction effect decreases with increasing the distance between the piles examinated but not monotonously. Analo-

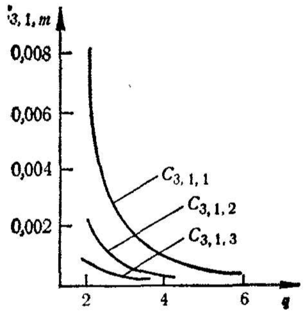

Fig. 6. The influence coefficient (secondary scattering)

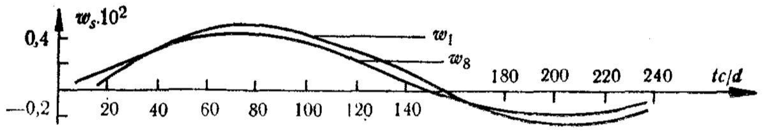

Fig. 7. The elastic response of the system

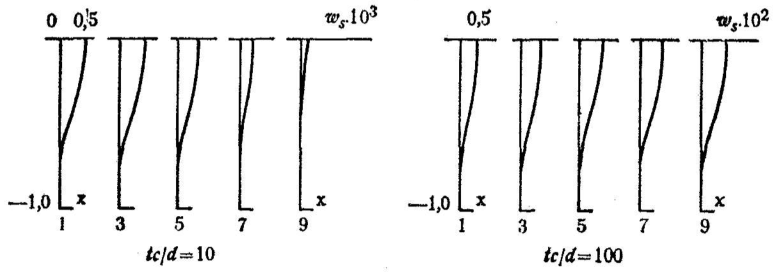

Fig. 8. The mode shapes

gous influence coefficients are obtained for the secondary scattering. They are shown in Fig. 6. It can be seen that the moduli of $C_{j,s}$ decrease with increasing the number of approximation.

3. The Elastic Motion

The equilibrium equation for the s^{th} pile is

$$A\rho_1\frac{\partial^2 w_s}{\partial t^2}+EI\frac{\partial^4 w_s}{\partial x^4}=F_s(x,\ \omega_m,\ t)+F_{s,f}(x,\ \omega_m,\ t), \tag{27}$$

where $F_s(x, \omega_m, t)$ and $F_{s,f}(x, \omega_m, t)$ are the applied forces along the pile correspondingly the force on the rigid pile and one due to the elastic pile motion: I is the moment of inertia, A is the cross-sectional area; ρ_1 is the material density, E — the elasticity modulus.

The boundary conditions are

$$x=-h:\ w_{s,m}=0,\quad (\partial w_{s,m}/\partial x)=0;\tag{28}$$

$$x=0:\ (\partial w_{s,m}/\partial x)=0,\quad (\partial^3 w_{s,m}/\partial x^3)=0;$$

and the initial ones are

$$w_{s,m}(x,\gamma_{i,s}/\omega_m)=0,\quad [\partial w_{s,m}(x,t)/\partial t]_{t=\gamma_{i,s}/\omega_m}=0.\tag{29}$$

The system response to the given loading for $x=0$ is shown in Fig. 7 and mode shapes of piles are represented in Fig. 8 for two time increments. The computational procedure is realized on HP-1000.

Conclusions

1. By the method described when the geometry and the characteristics of the system are known the solution can be determined without solving an infinite system of equations.
2. As can be seen from the solution structure, the bifurcation chain is very convenient for computing.
3. The method is convergent and the solution can be calculated with the necessary precision.
4. By the influence coefficients and sufficient number of approximations can be determined as well as the domain in which the interaction between the cylinders has to be taken into account.
5. Because of its simplicity and rapid convergence the method is very convenient to solve similar problems for groups of many cylinders. The same method is applied by *V. Dzhupanov* in the inside problem of the transient pressure waves in a vessel in which a tube array is placed [2].

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