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Heat transfer from two spherical particles in an oscillating flow

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I. Introduction

The heat and mass transfer between particles and oscillating fluids is a topic of great interest, an understanding of which is essential for predicting the magnitude of the beneficial effects and also for the design of oscillatory process equipment. There is experimental evidence (Al Taweel and Landau [1]) that oscillatory flows are beneficial in a wide variety of processes, e. g. extraction, drying, absorption, crystallization, dissolution and leaching, etc.

It is well known that the main bearer of utility of the oscillatory flows in these processes is the steady streaming developing in such flows. When periodic disturbances are imposed on a fluid otherwise at rest, they generate oscillations in the fluid. Then the most interesting effect of viscosity is the fact that the flow oscillations do not average to zero, but rather, a net steady flow is generated conventionally labelled "steady streaming". In a theoretical work, Schlichting [2] found a periodic solution for the problem of an oscillating cylinder in an unbounded viscous fluid by the method of two-dimensional nonsteady boundary-layer equations.

Historically, the topic has interested investigators long before the development of boundary-layer concepts. Experimental evidence of steady streaming was reported by Faraday [3] and Dvorak [4]. The case of an oscillating circular cylinder has also been investigated experimentally by Andres and Ingard [5] and theoretically by Stuart [6], Riley [7] and Wang [8]. The most comprehensive and inclusive account of earlier experimental works as well as theoretical analyses can be found in Riley [9].

Because of the difficulties associated with the three-dimensional analysis, steady streaming around spheres received much less attention (see Wang [10] and Riley [11]). An examination of the flow field between two concentric spheres when the inner one is forced to execute torsional oscillations while the outer sphere remains at rest was undertaken by Zapryanov and Stoyanova (Tabakova) [12] and Zapryanov and Christov [13].

In an recent investigation Tabakova and Zapryanov [14] studied the hydrodynamic interaction of two solid spheres immersed in a viscous incompressible fluid, oscillating at infinity in a direction parallel to their line of centres. The steady streaming generated both inside and ouside of the formed boundary layer around the spheres was studied analytically and some special pictures were found and discussed.

The enhancement of heat transfer by oscillating flows was discussed by Keil and Baird [15]. A critical review of the subject was given by Richardson [16].

A systematic investigation of the heat transfer from a circular cylinder oscillating in an unbounded viscous fluid which is otherwise at rest has been given by Davidson [17]. The described analysis leads to a more satisfactory understanding of the rôle played by the steady streaming velocity field in heat transfer from an oscillating cylinder in a viscous fluid. Heat transfer on a rotationally oscillating cylinder in a crossflow has recently been considered by Childs and Mayle [18]. Steady streaming and thermal effects in pipe flow were studied by Thomann [19]. For the case of high viscosity, the leading terms of the second order solution (steady streaming) were given. They were used to explain the large differences between the thermal effects (heat transfer to the wall) in an incompressible fluid and in an ideal gas.

Natural convection around a horizontal circular isothermal cylinder was considered by Vahl Davis and Richardson [20]. The problem was treated within the framework of boundary-layer theory with the formulation including creation or modification of the steady convective motion by the Reynolds stesses associated with the oscillations.

The beneficial effect of pulsating flow on mass transfer in liquid-liquid extraction apparatus was studied by Krasuk and Smith [21]. It has been shown (Jameson [22]), that in certain circumstances the mechanism of the mass transfer is similar to that of heat transfer from a hot body in a fluctuating stream, in which the bulk of the transfer takes place in a steady streaming flow. The rate of mass transfer of a diffusing substance along a pipe was augmented by an oscillatory flow (Watson [23]). The increase of the flux was evaluated for the cases of a circular pipe and of a two-dimensional channel.

The objective of the present paper is to solve the problem of the heat transfer from two spheres immersed in a viscous fluid, oscillating in a direction parallel to their line of centres. Following [14] we assume that the fluid is homogeneous, incompressible, Newtonian and with constant physical properties. The amplitude of the oscillations is small compared with the characteristic length of the flow. The analysis is based upon the use of the steady streaming flow field and consideration is given to the case of small streaming Reynolds numbers. The results are obtained with the Prandtl number of order unity. The method of matched asymptotic expansions is exploited in the analysis.

II. Formulation of the problem

Let two spherical particles of radii a and b are immersed in a viscous, incompressible fluid, oscillating at infinity in a direction parallel to their line of centres with velocity $U_0 \cos \omega t$, where U_0 is a typical speed and ω — the frequency. Suppose that the temperatures of the particles are Θ_1 and Θ_2, respectively, and the temperature at infinity is Θ_∞. We shall consider the case in which the ratios $\frac{\Theta_1-\Theta_\infty}{\Theta_\infty}$ and $\frac{\Theta_2-\Theta_\infty}{\Theta_\infty}$ are small compared with unity, so that the dependence of the density and diffusivities upon temperature may be ignored, and the Grashof number is so small, that natural convection effects may also be neglected. In contrast to the forced convection problem with buoyancy effect, for the problem at hand, the momentum equations and energy equation are uncoupled and the flow problem can be solved independently [14]. That is why in order to solve the problem of particle-to-particle heat transfer, we shall use the velocity field predicted in [14].

At first a cylindrical polar coordinates are introduced (ρ, φ, z), connected with the spheres, with origin on the line of centres so that the sphere of radius a is lying in the positive half-space $(z>0)$, and the sphere of radius b in the negative one $(z<0)$ (see Fig. 1). With L as a typical length, $\Theta_1-\Theta_\infty$ as a typical temperature and ν as a typical kinematic viscosity of the fluid, one can construct the following dimensionless parameters which characterize the motion and the heat transfer:

$$\varepsilon=\frac{U_0}{\omega L}, \quad M^2=\frac{\omega L^2}{\nu}, \quad Re_s=\frac{U_0^2}{\omega\nu}, \quad Re=\frac{U_0 L}{\nu}, \quad Pr=\frac{\nu}{\lambda},$$

where λ is the thermal conductivity. Then the governing equations of motion and the energy equation in dimensionless form are

$$\frac{\partial \bar{v}}{\partial \tau}+\varepsilon(\bar{v}.\nabla)\bar{v}=-\operatorname{grad} p+\frac{1}{M^2}\nabla^2\bar{v}, \tag{1}$$

$$\nabla.\bar{v}=0, \tag{2}$$

$$\frac{\partial T}{\partial \tau}+\varepsilon(\bar{v}.\nabla)T=\frac{\varepsilon^2}{PrRe_s}\nabla^2 T, \tag{3}$$

where τ is the dimensionless time and $\bar{v}(v_\rho, v_\varphi, v_z)$, p, $T=\frac{\Theta-\Theta_\infty}{\Theta_1-\Theta_\infty}$ are the local fluid velocity, the pressure and the dimensionless temperature.

The boundary conditions are

$$v_\rho=v_\varphi=0,\quad v_z=\cos\tau,\quad T=0 \quad \text{at infinity,} \tag{4}$$

$$\bar{v}=0 \qquad \text{on the spheres,} \tag{5}$$

$$T=1 \qquad \text{on the sphere of radius } a, \tag{6}$$

$$T=\frac{\Theta_2-\Theta_\infty}{\Theta_1-\Theta_\infty}=t_2 \qquad \text{on the sphere of radius } b. \tag{7}$$

We restrict ourselves entirely to the situations in which

$$M\gg 1,\ Re_s\ll 1\ \left(\varepsilon\ll\frac{1}{M}\right) \text{ and } Pr\sim O(1).$$

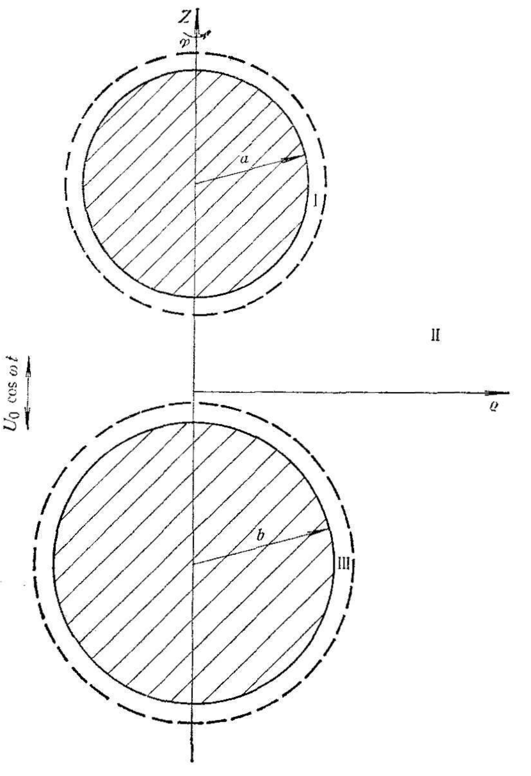

Fig. 1. The flow geometry and notation

For the problem of particle-to-particle heat transfer we use, as in [14], bispherical coordinates (ξ, η, φ), fixed in the particles. In the meridian plane one has conformal transformation

$$z=\frac{\operatorname{sh}\xi}{\operatorname{ch}\xi-\cos\eta},\quad \rho=\frac{\sin\eta}{\operatorname{ch}\xi-\cos\eta},\quad L=c>0,$$

where $-\infty<\xi<\infty$, $0\leq\eta\leq\pi$, $0\leq\varphi<2\pi$. The surfaces obtained by rotating the curves $\xi=\text{const}$ about the axis of z are a family of spheres having $z=0$ (or $\xi=0$) for a common radial plane. So, the two spheres of radii a and b can be defined by $\xi=\alpha_1>0$ ($a=c$ cosech α_1) and $\xi=\alpha_2<0$ ($b=-c$ cosech α_2).

Due to the symmetry of the problem one may introduce a stream function Ψ.

In order to solve the problem by means of the method of matched asymptotic expansions the space can be devided into three separate but overlapping regions:

i) two Stokes' layers adjacent to the spheres with thickness $\frac{\sqrt{2}}{M}$ (here the thickness of dynamic δ_D and the thickness of the thermal boundary layer δ_T are of the same order $\delta_D\sim\delta_T$);

ii) an intermediate region outside the two boundary layers (see Fig. 1).

In regions I and III we use the following stretched variables

(8) $$\zeta=\frac{(\alpha_1-\xi)M}{\sqrt{2}},$$

(9) $$\bar{\zeta}=\frac{(\xi-\alpha_2)M}{\sqrt{2}}.$$

For small ε and large M the unknown functions were expanded [14] in ε and in inverse powers of M, for example

(10) $$F=F_{00}+M^{-1}.F_{01}+\cdots+\varepsilon[F_{10}+M^{-1}.F_{11}+\cdots]+\cdots,$$

where $F\equiv(\psi,\ \Psi,\ \bar{\psi})$.

By introducing non-dimensional variables

(11) $$T=\frac{\Theta-\Theta_\infty}{\Theta_1-\Theta_\infty}\ \text{and}\ \tau=\omega t,$$

it is readily seen that the energy equation (3) in bispherical coordinates has the form

$$\frac{\partial T}{\partial\tau}+\varepsilon\left\{\frac{(\operatorname{ch}\xi-\cos\eta)^3}{\sin\eta}\cdot\left[\frac{\partial\Psi}{\partial\eta}\cdot\frac{\partial T}{\partial\xi}-\frac{\partial\Psi}{\partial\xi}\cdot\frac{\partial T}{\partial\eta}\right]\right\}=\frac{\varepsilon^2}{PrRe_s}\cdot\nabla^2T,$$

where

$$\nabla^2\equiv L_0^2\equiv\frac{(\operatorname{ch}\xi-\cos\eta)^3}{\sin\eta}\cdot\left[\frac{\partial}{\partial\xi}\left(\frac{\sin\eta}{\operatorname{ch}\xi-\cos\eta}\cdot\frac{\partial}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\frac{\sin\eta}{\operatorname{ch}\xi-\cos\eta}\ \frac{\partial}{\partial\eta}\right)\right].$$

The boundary conditions (4), (6), (7) can be rewriten as follows:

(13) $$T=0 \qquad \text{as } \xi^2+\eta^2\to0\quad(z^2+\rho^2\to\infty),$$

(14) $$T=t_1=1 \qquad \text{at } \xi=\alpha_1,$$

(15) $$T=\frac{\Theta_2-\Theta_\infty}{\Theta_1-\Theta_\infty}=t_2 \qquad \text{at } \xi=\alpha_2.$$

We seek a solution of the problem (12)—(15) in the form

(16) $$T=T_{00}^0+(PrRe_s)T_\infty^1+\cdots+M^{-1}T_{01}^0+\varepsilon[T_{10}^0+(PrRe_s)T_{1\infty}^1+\cdots+M^{-1}T_{11}^0]+\cdots$$

III. Solution of the problem

The Kaplun's matching principle is used in order to find an approximate solution uniformly valid everywhere in the region Ω: $\{-\infty<\xi<\infty,\ 0\le\eta\le\pi,\ 0\le\varphi<2\pi\}$. This leads us to impose additional boundary conditions on the boundaries between the different regions. This procedure is quite standard and it will not be presented here in details. We shall give for each region only the corresponding differential equations with their boundary conditions and the obtained solutions.

III.1. First approximation in ε

From (12) after taking into account (8), (9) and (16) for the terms of equal powers in M and independent on ε and $(PrRe_s)$, we get:

(17) $$\frac{\partial\hat{t}_{ij}}{\partial\tau}-\frac{1}{2Pr}\cdot(\operatorname{ch}\alpha_k-\cos\eta)^2\cdot\frac{\partial^2\hat{t}_{ij}}{\partial\hat{\zeta}^2}=\hat{G}_{ij},$$

where $j=0,\ 1,\ldots;\ i=0;\ \hat{G}_{00}=0;$

$$\widehat{G}_{01}=\pm\frac{\sqrt{2}}{2Pr}(\operatorname{ch}\alpha_k-\cos\eta)\operatorname{sh}\alpha_k\cdot\left(\frac{\partial\widehat{t}_{00}}{\partial\widehat{\zeta}}-2\widehat{\zeta}\cdot\frac{\partial^2\widehat{t}_{00}}{\partial\widehat{\zeta}^2}\right);\quad k=1,\ 2.$$

We use the symbol $\widehat{f}$, which means a function or a variable, related to the region I at $k=1$ and to the region III — at $k=2$. The boundary conditions (14) and (15) give appropriate conditions for $\widehat{t}_{0j}$:

(18) $$\widehat{t}_{00}=t_k \quad\text{and}\quad \widehat{t}_{01}=0 \quad\text{at}\quad \widehat{\zeta}=0.$$

When the upper conditions are fulfilled, the solution of (17) is stationary:

(19) $$\widehat{t}_0=\widehat{t}_0^{(s)}=t_k\mp\frac{\widehat{\zeta}\sqrt{2}}{M}\widehat{\gamma}_{00}^k+0(M^{-2}),$$

where $\widehat{\gamma}_{00}^k$ will be determined after matching $\widehat{t}_0$ with T_0.

Again using (12) the outer solution T_{0j} must satisfy the equation

(20) $$\nabla^2T_{0j}=0,\quad (j=0,\ 1,\dots)$$

with boundary conditions (13) and those obtained after matching with the inner solutions. So the solution of the upper equation with the corresponding boundary conditions is:

$$T_{0j}(\xi,\eta)=T_{0j}^{(s)}(\xi,\eta)=(\operatorname{ch}\xi-\cos\eta)^{1/2}\sum_{n=0}^{\infty}[A_{0j}\operatorname{sh}(n+1/2)\xi+B_{0j}\operatorname{ch}(n+1/2)\xi]P_n(\cos\eta),$$

where $P_n(\cos\eta)$ are the Legendre polinomials of order n,

(21) $$A_{0j,n}=\frac{\beta^{(j)}(\alpha)\operatorname{ch}(n+1/2)\alpha_2-\beta^{(j)}(\alpha_2)\operatorname{ch}(n+1/2)\alpha_1}{\operatorname{sh}[(n+1/2)(\alpha_1-\alpha_2)]},$$

$$B_{0j,n}=\frac{\beta^{(j)}(\alpha_2)\operatorname{sh}(n+1/2)\alpha_1-\beta^{(j)}(\alpha_1)\operatorname{sh}(n+1/2)\alpha_2}{\operatorname{sh}[(n+1/2)(\alpha_1-\alpha_2)]},$$

$$\beta^{(1)}(\alpha_k)=\sqrt{2}\exp[\mp(n+1/2)\alpha_k].t_2,\ \beta^{(2)}(\alpha_k)=0;\quad k=1,\ 2.$$

From here we find $\widehat{\gamma}_{00}^k=\mp\sqrt{2}\left.\frac{\partial T_{00}}{\partial\xi}\right|_{\xi=\alpha_k}$.

III.2. Second approximation in ε

In the boundary layers the temperature terms of order ε^1 have to satisfy the equations (17) at $i=1$, where

(22) $$\widehat{G}_{1j}=\pm\frac{(\operatorname{ch}\alpha_k-\cos\eta)^3}{\sin\eta}\left[\frac{\partial\widehat{\psi}_{00}}{\partial\eta}\cdot\frac{\partial\widehat{t}_{0j}}{\partial\widehat{\zeta}}-\frac{\partial\widehat{\psi}_{00}}{\partial\xi}\cdot\frac{\partial\widehat{t}_{0j}}{\partial\eta}\right]\quad(j=0,\ 1)$$

with boundary conditions $\widehat{t}_{1j}=0$ at $\widehat{\zeta}=0$. Because of the unstationary character of the righthand side $\widehat{G}_{1j}$, we seek $\widehat{t}_{1j}$ in the form $\widehat{g}_{1j}(\widehat{\zeta},\eta)e^{it}$. The solutions of the upper equations are the following:

$$\widehat{t}_{10}=0,$$

$$\widehat{t}_{11}=\widehat{t}_{11}^{(u)}=\widehat{g}_{11}(\widehat{\zeta},\ \eta)\,e^{it},$$

i) at $Pr\neq 1$

$$\widehat{g}_{11}(\widehat{\zeta},\ \eta)=(1-\widehat{E}^{\sqrt{Pr}})\,\widehat{c}_{11}-(\widehat{E}^{\sqrt{Pr}}-\widehat{E})\,\widehat{e}_{11}+\widehat{d}_{11}\widehat{\zeta}+\widehat{f}_{11}\widehat{\zeta}\widehat{E},$$

where $\widehat{E}^{\sqrt{Pr}}=\exp\left[-\frac{(1+i)}{\gamma}\sqrt{Pr}\,\widehat{\zeta}\right]$, $\widehat{E}=\exp\left[-\frac{(1+i)}{\gamma}\widehat{\zeta}\right]$,

(23) $$\gamma=(\operatorname{ch}\alpha_k-\cos\eta),$$

$$\widehat{c}_{11}=\mp\frac{(1+i)}{\sqrt{2}}\gamma^3\left\{\frac{\operatorname{sh}\alpha_k\cdot t_k}{2\gamma^{3/2}}\sum_{n=1}^{\infty}\widehat{\lambda}_{00}^n\left[\frac{1}{2}C_{n+1}^{-1/2}(\cos\eta)+\gamma P_n(\cos\eta)\right]\right.$$

$$\left.\pm\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}\widehat{\lambda}_{00}^n\widehat{\delta}_{00}^m P_m(\cos\eta)\left[\frac{1}{4}C_{n+1}^{-1/2}(\cos\eta)+\gamma P_n(\cos\eta)\right]\right\},$$

$$\widehat{d}_{11}=\mp i\sqrt{2}\sum_{n=1}^{\infty}\widehat{\lambda}_{00}^n\left\{C_{n+1}^{-1/2}(\cos\eta)\left[-\frac{\operatorname{sh}\alpha_k}{4}\cdot t_k\cdot\gamma^{1/2}\pm\sum_{m=0}^{\infty}\widehat{\delta}_{00}^m\left[P_m(\cos\eta)\gamma^2\right.\right.\right.$$

$$\left.\left.\left.-P'_m(\cos\eta)\gamma^3\right]\right]-P_n(\cos\eta)\left[\frac{\operatorname{sh}\alpha_k}{2}t_k\gamma^{3/2}\pm\gamma^3\sum_{m=0}^{\infty}\widehat{\delta}_{00}^m P_m(\cos\eta)\right]\right\},$$

$$\widehat{f}_{11}=-\frac{i\sqrt{2}\,Pr}{(1-Pr)}\cdot\gamma^2\cdot\widehat{H}_{11}^1,$$

$$\widehat{e}_{11}=\frac{\sqrt{2}(1+1)}{(1-Pr)}\gamma^3\left[\frac{\widehat{H}_{11}^2}{2}-\frac{Pr}{(1-Pr)}\widehat{H}_{11}^1\right],$$

$$\widehat{H}_{11}^1=\sum_{n=1}^{\infty}\widehat{\lambda}_{00}^n C_{n+1}^{-1/2}(\cos\eta)\sum_{m=0}^{\infty}\widehat{\delta}_{00}^m\left[-\frac{3}{2}P_m(\cos\eta)+P'_m(\cos\eta)\cdot\gamma\right],$$

$$\widehat{H}_{11}^2=\sum_{n=1}^{\infty}\widehat{\lambda}_{00}^n\left[-\frac{3}{4}C_{n+1}^{-1/2}(\cos\eta)+\gamma P_n(\cos\eta)\right]\left[\frac{\operatorname{sh}\alpha_k\cdot t_k}{2\gamma^{3/2}}\pm\sum_{m=0}^{\infty}\widehat{\delta}_{00}^m P_m(\cos\eta)\right]$$

$$\widehat{\lambda}_{00}^n=\beta(\alpha_k)-\frac{\beta(\alpha_e)+\beta(\alpha_k)\operatorname{ch}[(n+1/2)(\alpha_1-\alpha_2)]}{\operatorname{sh}[(n+1/2)(\alpha_1-\alpha_2)]},$$

$$\beta(\alpha_k)=\mp\frac{(2n+1)}{\sqrt{2}}n(n+1)\exp[\mp(n+1/2)\alpha_k],$$

$$\widehat{\delta}_{00}^m=\frac{(2m+1)}{2}\cdot\frac{\beta^{(1)}(\alpha_k)\operatorname{ch}[(n+1/2)(\alpha_1-\alpha_2)]-\beta^{(1)}(\alpha_l)}{\operatorname{sh}[(n+1/2)(\alpha_1-\alpha_2)]};\qquad(l,\ k=1,\ 2;\ k\neq l),$$

$C_{n+1}^{-1/2}(\cos\eta)$ are the Gegenbauer polinomials of order $(n+1)$ and degree $(-1/2)$, $P'_m(\cos\eta)$ are the derivatives of Legendre polinomials of order m;

ii) at $Pr=1$

$$\widehat{g}_{11}(\widehat{\zeta},\eta)=(1-\widehat{E})\,\widehat{c}_{11}+\widehat{d}_{11}\widehat{\zeta}+\widehat{e}_{11}\widehat{\zeta}\,\widehat{E}+\widehat{f}_{11}\widehat{\zeta}^2\widehat{E},$$

where $\widehat{d}_{11}$ and $\widehat{c}_{11}$ are given by (23)

$$\widehat{f}_{11}=\frac{\sqrt{2}}{4}(1-i)\gamma^2\cdot\widehat{H}_{11}^1$$

(24) $$\widehat{e}_{11}=\mp i\frac{\sqrt{2}}{2}\gamma^3[\widehat{H}_{11}^2\pm\gamma\widehat{H}_{11}^1].$$

In all expressions shown by now, we have used the sign (+) or (—) in the case of $k=1$ or $k=2$, respectively.

From (12) after equating the terms of order ε^1, we get the following equation for the region II:

$$\frac{\partial T_1}{\partial \tau} - \frac{1}{PrM^2}(\nabla^2 T_1) = -\left\{\frac{(\mathrm{ch}\,\xi - \cos\eta]^3}{\sin\eta}\left[\frac{\partial \Psi_0}{\partial \eta}\cdot\frac{\partial T_0}{\partial \xi} - \frac{\partial \Psi_0}{\partial \xi}\,\frac{\partial T_0}{\partial \eta}\right]\right\} \tag{25}$$

and suppose, that T_1 can be devided into two different parts — stationary and unstationary one, i. e.:

$$T_1 = T_1^{(s)} + T_1^{(u)}.$$

Because the righthand side of (25) is only unstationary, then

$$\nabla^2 T_1^{(s)} = 0$$

and the boundary conditions of $T_1^{(s)}$ are zeroes at $\xi = \alpha_1$ and $\xi = \alpha_2$ ($\hat{t}_1^{(s)} = 0$). This means that the solution $T_1^{(s)}$ is the trivial one $T_1^{(s)} \equiv 0$.

Expanding $T_1^{(u)}$ in a power series in M^{-1}, we find the equations for $T_{1j}^{(u)}$ ($j = 0, 1$)

$$\frac{\partial T_{1j}}{\partial \tau} = -\Phi_{1j}\,.\,e^{i\tau}, \tag{26}$$

where $\Phi_{1j} = \frac{(\mathrm{ch}\,\xi - \cos\eta)^3}{\sin\eta}\cdot\left[\frac{\partial \Psi_{0j}}{\partial \eta}\cdot\frac{\partial T_{00}}{\partial \xi} - \frac{\partial \Psi_{0j}}{\partial \xi}\,\frac{\partial T_{00}}{\partial \eta}\right].e^{-i\tau}$

and then their solutions are:

$$T_{10}^{(u)} = i\,e^{i\tau}\,\Phi_{10}, \quad T_{11}^{(u)} = i\,e^{i\tau}\,\Phi_{11}. \tag{27}$$

III.3. Third approximation in ε

Using the expression $\frac{\varepsilon^2}{PrRe_s} = \frac{1}{PrM^2}$, the term of power in equation (12) is:

$$\frac{\partial T_2}{\partial \tau} + \frac{(\mathrm{ch}\,\xi - \cos\eta)^3}{\sin\eta}\left[\frac{\partial \Psi_1}{\partial \eta}\cdot\frac{\partial T_0}{\partial \xi} - \frac{\partial \Psi_1}{\partial \xi}\cdot\frac{\partial T_0}{\partial \eta} + \frac{\partial \Psi_0}{\partial \eta}\cdot\frac{\partial T_1}{\partial \xi} - \frac{\partial \Psi_0}{\partial \xi}\cdot\frac{\partial T_1}{\partial \eta}\right] = \frac{1}{PrM^2}\nabla^2 T_2.$$

Assuming that T_2 has the structure

$$T_2 = PrM^2\,.\,(T_{20}^{(s),1} + T_{20}^{(u),1}) + (T_{20}^{(s),0} + T_{20}^{(u)0}) + \cdots \equiv \frac{(PrRe_s)}{\varepsilon^2}\,T_{00}^1 + T_{20}^0 + \cdots$$

then a term from the expansion in $(PrRe_s)$ is obtained.

After some algebra, we reach to the equations for $T_{20}^{(s),1}$ and $T_{20}^{(u),1}$:

$$\frac{\partial T_{20}^{(u),1}}{\partial \tau} = 0, \tag{28}$$

$$\nabla^2 T_{20}^{(s),1} = \tilde{\Phi}(\xi, \eta), \tag{29}$$

$$\tilde{\Phi}(\xi, \eta) = \frac{(\mathrm{ch}\,\xi - \cos\eta)^3}{\sin\eta}\left[\frac{\partial \Psi_{10}}{\partial \eta}\cdot\frac{\partial T_{00}}{\partial \xi} - \frac{\partial \Psi_{10}}{\partial \xi}\cdot\frac{\partial T_{00}}{\partial \eta}\right].$$

Therefore $T_{20}^{(u),1} \equiv 0$ in the three regions.

We shall concentrate our attention on the equation (29), because it represents the convective supplement to the stationary part of the temperature function and it is based on the stationary part of the hydrodynamic flow, i. e. of the stream function.

In the region II the solution of (29) is:

$$T_{20}^{(s),1} \equiv T_{00}^{1} = (\operatorname{ch}\xi - \cos\eta)^{1/2} \sum_{m=1}^{\infty} \{[C_{1,m} + \widetilde{C}_{1,m}(\xi)] \operatorname{ch}(m+1/2)\xi$$

$$+ [C_{2,m} + \widetilde{C}_{2,m}(\xi)] \operatorname{sh}(m+1/2)\xi\} P_m(\cos\eta),$$

(30) where
$$\widetilde{C}_{1,m}(\xi) = -\frac{2}{(2m+1)} \int \Phi_m(\xi) \operatorname{sh}(m+1/2)\xi \, d\xi$$

$$\widetilde{C}_{2,m}(\xi) = \frac{2}{(2m+1)} \int \Phi_m(\xi) \operatorname{ch}(m+1/2)\xi \, d\xi,$$

$$\Phi_m(\xi) = -\frac{(2m+1)}{2} \int_{-1}^{1} \frac{\sin^2\eta}{(\operatorname{ch}\xi - \cos\eta)^{5/2}} \widetilde{\Phi}(\xi, \eta) . P_m(\cos\eta) \, d\eta$$

and it satisfies the conditions at infinity (13).

In the boundary layers the corresponding solutions $\widehat{t}_{20}^{(s),1}$ that are obtained must satisfy the equations:

$$\frac{\partial^2 t_{20}^{(s),1}}{\partial \widehat{\xi}^2} = 0$$

and the boundary conditions (22) at $i=2$, $j=0$. In this way for the regions I and II we have the stationary part of the temperature:

(31)
$$\widehat{t}_{00}^{1} \equiv \widehat{t}_{20}^{(s),1} = \widehat{b}_{20}^{(s),1} . \widehat{\zeta},$$

where $\widehat{b}_{20}^{(s),1} = \mp\sqrt{2} \left. \frac{\partial T_{20}^{(s),1}}{\partial \xi} \right|_{\xi=\alpha_k}$, $(k=1, 2)$.

III.4. Higher order approximations in ε

Equating the expressions of order ε^4 in (12) we find the term of power $(\varepsilon Pr\, Re_s)$ in the expansion (16), that is only an unstationary function. So, the terms of even powers in ε are the stationary parts of the temperature function, while the terms of odd powers in ε are the unstationary ones. By the upper described procedure, we have obtained some more terms of the expansion (16), but in sake of briefness, we restrict ourselves with the given by now terms. The found solution of the temperature problem (12)–(15) is uniformly valid in the whole region Ω, it has accuracy $O(M^{-2}, (Pr\, Re_s)^2)$ and holds formally for $d > 2\,\delta_D \approx 2\delta_T$ $(d = c \coth\alpha_1 - c\coth\alpha_2 - a - b)$.

III.5. Heat transfer from the spheres

The full heat flux per unit length from the surface of anyone of the spheres is given by the expression

(32)
$$Q = -\lambda \int_S \left(\frac{\partial\Theta}{\partial\eta}\right) ds,$$

where λ is the fluid thermal conductivity, S — the spherical surface.

Then the Nusselt number based upon the sphere radius a_k is:

$$Nu_{,k} = \frac{Q}{4\pi a_k \lambda(\Theta_k - \Theta_\infty)} = \mp \frac{\operatorname{sh}\alpha_k}{2t_k} \int_{-1}^{1} \frac{1}{(\operatorname{ch}\alpha_k - \mu)} \left(\frac{\partial T}{\partial\xi}\right)_{\xi=\alpha_k} d\mu,$$

$(k=1, 2$ and $a_1 = a$, $a_2 = b)$

where $\mu = \cos\eta$ and the sign $(-)$ corresponds to $\alpha_1 > 0$, while $(+)$ — to $\alpha_2 < 0$.

If we average by time the heat flux, we get

$$\overline{Q}=\frac{1}{2\pi}\int\limits_{0}^{2\pi} Q\,d\tau$$

and then the time average (the stationary) Nusselt number for each of the spheres is the following:

$$\overline{Nu}_{,k}=\frac{1}{2}\pm\frac{\operatorname{sh}\alpha_k}{t_k}\cdot\sqrt{2}\sum_{n=0}^{\infty}{}' \exp[\mp(n+1/2)\alpha_k]\,[\beta^1(\alpha_k)\operatorname{th}[(n+1/2)(\alpha_1-\alpha_2)]$$

$$-\frac{\beta^1(\alpha_l)}{\operatorname{sh}[(n+1/2)(\alpha_1-\alpha_2)]}\Big]+\frac{\operatorname{sh}\alpha_k}{t_k}\cdot\sqrt{2}\,Pr\,Re_s\sum_{n=0}^{\infty}\frac{\exp[\mp(n+1/2)\alpha_k]}{(2n+1)\operatorname{sh}[(n+1/2)(\alpha_1-\alpha_2)]}$$

$$\times\Big[\operatorname{ch}(n+1/2)\alpha_k\int\limits_{\alpha_2}^{\alpha_1}\Phi_n(\xi)\operatorname{sh}(n+1/2)\xi\,d\xi-\operatorname{sh}(n+1/2)\alpha_k \tag{33}$$

$$\times\int\limits_{\alpha_2}^{\alpha_1}\Phi_n(\xi)\operatorname{ch}(n+1/2)\xi\,d\xi\Big]=\overline{Nu}^0_{,k}+(Pr\,Re_s)\,\overline{Nu}_{,k}+O((Pr\,Re_s)^2),$$

$$(k,\ l=1,\ 2,\quad l\neq k).$$

The corresponding Nusselt number for an oscillating single sphere in an unbounded fluid [11] is calculated and it's value is:

$$\overline{Nu}_{,\infty}=1+0.01461\,.(Pr\,Re_s)^2. \tag{34}$$

IV. Analysis of the results and conclusions

The obtained temperature function expressions (19), (21), (23), (24), (27) (30), (31) give us the opportunity to analyze the effects due to the diffusion — the first order approximation in ε, and those due to the convection — the remaining terms in the asymptotic expansion (16). Moreover, some of the solutions due to the convection and induced by the secondary stationary flow. The stationary heat flow is

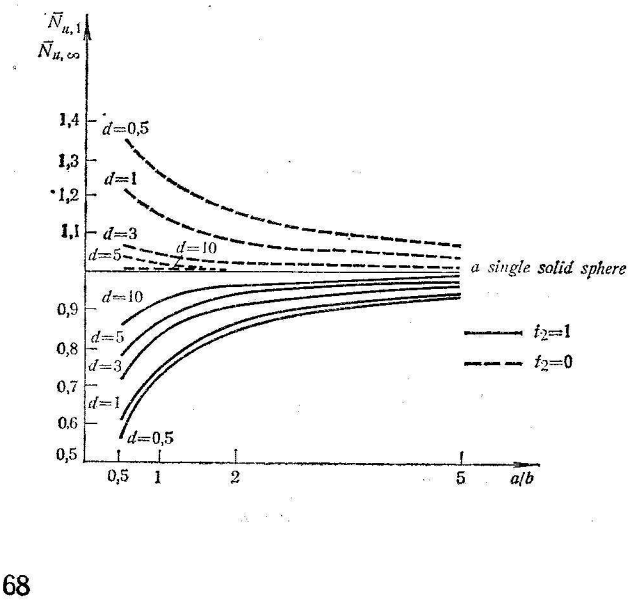

Fig. 2. Ratio of the time average Nusselt number $\overline{N}_{u,1}$ on a sphere of radius a (due to the present analysis (33)) and the time average Nusselt number $\overline{N}_{u,\infty}$ on a single sphere (see ormula (34)) as a function of a/b at different d, $PrRe_s=0.3$ for the cases $t_2=1$ and $t_2=0$

also divided into diffusive and convective part. In the here examined approximation $(Pr Re_s) \ll 1$, $Re_s \ll 1$ this convective heat flow and the stationary fluid flow have Stokes' character. The Nusselt number for the whole body surface is the main characteristics of the temperature function. Formula (33) gives us a good vizualization of the influence of the secondary stationary flow on the thermal flux, induced by the sphere. Comparing the two formulaes (33) and (34) we conclude that the presence of second sphere in the fluid intensifies the influence of the secondary stationary flow on the temperature distribution in the whole region Ω. The convective part of the time average Nusselt number is of one order higher in $(Pr Re_s)$ than that in the case of a single sphere.

On Fig. 2 the relation $\frac{\overline{Nu}_{,1}}{\overline{Nu}_{,\infty}}$ is shown as a function of the spherical radii ratio a/b and the distance d between the spheres at different t_2 ($t_2=1$, and $t_2=0$). In the first case this function is increasing with an asymptote 1 — this means that the second sphere diminishies the heat flux from the first sphere, but when decreasing it's radius and increasing the distance d, this effect is gradually decaying. While in the case $t_2=0$ ($\tilde{\Theta}_2=\Theta_\infty$) the second sphere presence intensifies the heat flux from the first sphere. This process decreases when increasing each of the two parameters a/b and d.

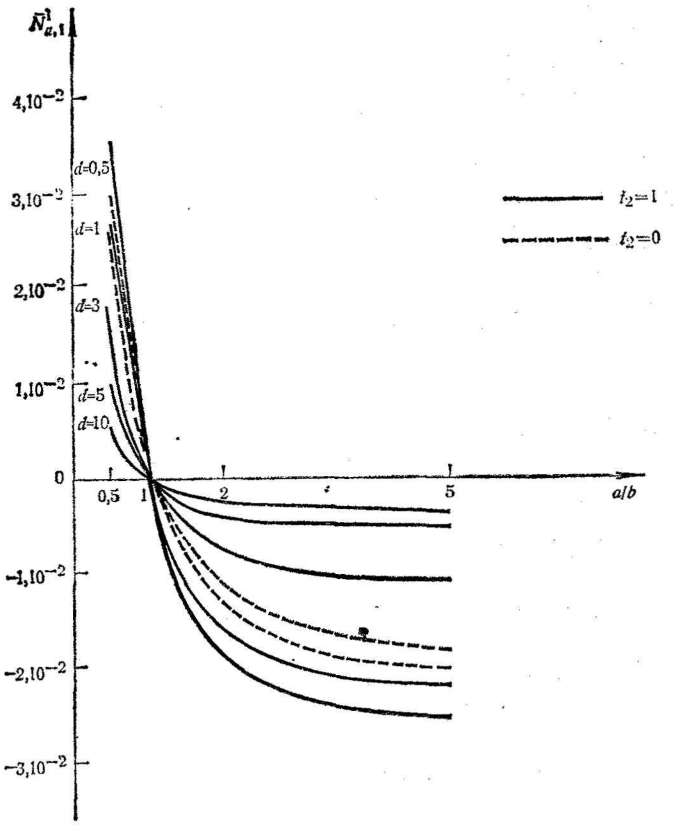

Fig. 3. The perturbation part of the time average Nusselt number $\overline{Nu}_{,1}$ (see formula (33)) on a sphere of radius a for different d, a/b and $t_2=1$; $t_2=0$

The secondary stationary streaming effect on the heat transfer is represented by $\overline{Nu}^1_{,1}$, $t_2=0$ and $t_2=1$ are shown on Fig. 3. Here, it is interesting to note, that the presence of second particle in the fluid gives rise to a different perturbation in the heat flux from the first particle and this is caused by the different hydrodynamic secondary flows at different a/b. At $a/b\leq 1$ this result is positive, independently on the sphere's temperature t_2, i. e. this secondary convective heat flux coincides in sign with the pure diffusive heat flux. But when $a/b>1$, this secondary transport of heat counteracts those arising from pure diffusion. Increasing the distance d between the spheres, the upper effects become less significant.

As a conclusion, we can say, that after analyzing the heat transfer from two spherical particles with different temperatures, radii, and distances between them, oscillating parallel to their line of centres, we reach to the fact that the steady streaming gives rise to a perturbation of a higher order than that in the case of single sphere. This perturbation term intensifies the heat transport from one of the spheres only when the other one has an equal or a greater radius.

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