

## Rarefied Gas Flow during Evaporation and Filtration in High-porous Media

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In the present paper evaporation from a buried boundary of phase transition and gas filtration in a high-porous layer, which is given in the form of a dusty gas model, are studied by solving a corresponding boundary-value problem of the kinetic theory of gases [1—3].

The knowledge of the laws of the gas flow through the layer at different values of the Knudsen number is necessary, for example, when describing the sublimation drying of materials that in the majority of cases is performed under the conditions when water steam flows through the material in the transient regime [4]. Besides, in the sublimation drying of thin films, when the mean free path of molecules is comparable to the film thickness [5], the use of the phenomenological theory is incorrect.

The dusty gas model was previously applied to the description of gas filtration in an unbounded porous body, i. e. without taking into account the conjugation effects of the body with the surrounding medium. There the expression for the gas flow, applicable over the whole range of the Knudsen number, was given in the superposition form of the Knudsen and viscous flows.

Consider the slow isothermal flow of a vapour through a porous layer with the thickness  $L$ , bounded from one side by the surface of the evaporating fluid and from the other by its vapour, the pressure of which,  $p_L$ , at  $x=L$  is set ( $p_L < p_0$ ).

The distribution function of gas molecules,  $f$ , inside a porous body satisfies the equation

$$(1) \quad \xi_x \frac{\partial f}{\partial x} + \frac{F}{m} \frac{\partial f}{\partial \xi_x} = 0,$$

where the "external" force,  $F$ , is the result of the collective interaction of gas molecules with particles. In equation (1), the term with the effective "external" force,  $F$ , is introduced instead of the collision integral. This force is determined in the following way. The force,  $F_d$ , affecting a separate spherical particle in the case of diffusive reflection of molecules from the surface, is calculated on the basis of the solution of the kinetic theory boundary-value problem for the gas flow around this particle (6)

$$F_d = F_s A_1(Kn),$$

where

$$Kn = \lambda/r, \quad F_s = 3\pi\lambda\rho\bar{c}v,$$
$$A_1 = \frac{15 - 3Kn + 3(8 + \pi)Kn^2}{15 + 12Kn + 18Kn^2 + 54Kn^3}.$$

The value of  $F$  is obtained from the relation

$$\psi n F = -n_d F_d,$$

where

$$(2) \quad n_d = (1 - \psi) / \frac{4}{3} \pi r^3.$$

In the boundary-value problem considered here the unknown distribution function  $f$ , is approximated by the two-sided Maxwell distribution

$$f = \begin{cases} f_1 = n_1 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left\{ -\frac{m}{2kT} [(\xi_x - v_1)^2 + \xi_y^2 + \xi_z^2] \right\}, & \xi_x > 0 \\ f_2 = n_2 \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left\{ -\frac{m}{2kT} [(\xi_x - v_2)^2 + \xi_y^2 + \xi_z^2] \right\}, & \xi_x < 0 \end{cases}$$

To determine  $n_1, n_2, v_1, v_2$  we make use of the following four moment equations obtained from (1) [7]:

$$\frac{d}{dx} \int \xi_x \varphi f d\vec{\xi} - \frac{F}{m} \int f \frac{d\varphi}{dx} d\vec{\xi} = 0,$$

where  $1, \xi_x, \xi_x^2, \xi_x^3$  are taken to be  $\varphi$ .

The boundary conditions have the form at  $x=0$

$$f_1 = n_0 [\psi + (1 - \psi)(1 + v_{r1})] \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \{-c^2\}, \quad c_x > 0$$

at  $x=1$

$$f_2 = \psi f_L + (1 - \psi)n_0(1 + v_{r2}) \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \{-c^2\}, \quad c_x < 0,$$

where the values of  $v_{ri}$  are determined from the condition of the impermeability of solid particles and  $f_L$  is the Maxwell function with the parameters  $n_L, u_L$  ( $u_L$  is the unknown dimensionless gas velocity above the porous body).

The solution of the considered problem on the evaporation in the porous layer is presented in [8, 9]. As a result

$$(3) \quad u_L = - \frac{\psi^2 v_L}{(4 - 2\psi - \psi^2 + A)\pi^{1/2}},$$

where

$$(4) \quad v_L = (n_L - n_0)/n_0, \quad A = \frac{9(1 - \psi)}{\pi} \frac{L}{r} Kn A_1 (Kn).$$

Note that in the given simplified statement the presence of the Knudsen layer near the boundary  $x=1$  from the side of the surrounding medium is not taken into account. The presence of the non-equilibrium layer near this boundary from the side of the porous body leads, in particular, to the gas density jump,  $n(1) - n_L$ :

$$(5) \quad \frac{n(1) - n_L}{n_0 - n_L} = \frac{v_L - v(1)}{v_L} = 1 + \frac{\pi^{1/2} u_L}{\psi^2 v_L} \left[ A + 2 - \psi - \frac{\psi^3}{2} \left( 1 - \frac{2}{\pi} \right) \right].$$

This density (pressure) jump is the analogue of the pressure loss at the outlet of the porous body (10) and is connected with the fact that two one-sided distribution functions (one of which allows for hydrodynamic resistance of motionless particles) "meet" each other in the above-mentioned non-equilibrium layer. In the table the results are pre-

Table 1

The dependence of  $u_L$  and  $[n(1)-n_L]/(n_0-n_L)$  on  $Kn$  in the evaporation from a porous body for  $\varphi=0.9$ ,  $L/r=100$ ,  $(n_L-n_0)/n_0=-0.1$

$Kn$	$u_L \cdot 10^2$	$\frac{n(1)-n_L}{n_0-n_L}$	$Kn$	$u_L \cdot 10^2$	$\frac{n(1)-n_L}{n_0-n_L}$
$\infty$	0.239	0.0221	5	0.256	0.0238
500	0.239	0.0221	1	0.314	0.0292
100	0.240	0.0222	0.5	0.412	0.0377
50	0.241	0.0223	0.1	1.139	0.105
30	0.242	0.0224	0.01	2.732	0.252
20	0.244	0.0225	0.001	3.221	0.298
10	0.248	0.0230	0.0001	3.281	0.303

sented of the calculation of one of the variants of the problem of evaporation from a porous layer.

The problem on gas filtration through a porous body with the set gas density at the inlet ( $x=0$ )  $n_0$  and outlet ( $x=1$ )  $n_L$  is solved analogously. Having performed corresponding calculations, we obtain the expressions for the velocity

$$(6) \quad u_L = -\frac{\psi^2 v_L^2}{(4-2\psi-2\psi^2+A)\pi^{1/2}}$$

and for the density jumps at the inlet of, and outlet from a porous body

$$(7) \quad \frac{n_0-n(0)}{n_0-n_L} = -\frac{\pi^{1/2} u_L}{\psi^2 v_L} (2-\psi-\psi^2),$$

$$(8) \quad \frac{n(1)-n_L}{n_0-n_L} = 1 + \frac{\pi^{1/2} u_L}{\psi^2 v_L} (A+2-\psi-\psi^2).$$

In the case of evaporation from the porous body with the growing  $1/Kn$  the velocity  $u_L$  first increases slowly from the free-molecular value, then, at approximately  $Kn=1$  according to (3, 4),  $u_L$  increases more sharply and passes into the linear dependence  $u_L \sim 1/Kn$ , characteristic of a viscous gas flow. Finally, with a further decrease of  $Kn$  in the case of evaporation  $u_L$  acquires a constant value, depending only on  $\psi$  and  $v_L$  ( $u_L = -\psi^2 v_L / [(4-2\psi-\psi^2)\pi^{1/2}]$ ) which with  $\psi \rightarrow 1$  approaches  $u_{L0} = -v_L / \pi^{1/2}$  what corresponds to the evaporation from the open surface.

In the problem of filtration the function  $u_L$  depending on  $1/Kn$  behaves in a similar way (see Eq. (6)) and, when  $A \ll 1$  (but  $\psi \neq 1$ ), it acquires a constant value, in this case the initial density drop ( $v_L$ ) becomes equal to the sum of density jumps at the inlet (7) and outlet (8). However, if the above density jumps are neglected, then with  $Kn \rightarrow 0$  the linear dependence  $u_L \sim 1/Kn$  remains, that is characteristic for the Darsi law of filtration. The mentioned relation  $u_L(Kn)$  for all  $Kn$  is qualitatively confirmed by the experimental data obtained for  $\psi \approx 0.4$  (11). At the same time in the problem of evaporation, when the density jump at the boundary  $x=1$  is neglected, a free term (depending only on  $\psi$ ) in the expression  $u_L(Kn)$  does not disappear and as  $Kn \rightarrow 0$   $u_L$  acquires a constant value.

Compare the obtained values of  $u_L$  in the free-molecular regime with the available theoretical results. If one determines the permeability  $\kappa$  from the expression

$$n u_L \left( \frac{2kT}{m} \right)^{1/2} = -\kappa \frac{dn}{dx},$$

then from (3) and (6) for  $Kn \rightarrow \infty$  and at  $A \gg 1$  we have

$$\kappa_\infty = a \left( \frac{2kT}{m} \right)^{1/2} \frac{\psi^2 d}{1-\psi},$$

where  $d=2r$ , the value of the factor is  $a=\pi^{1/2}/(8+\pi)=0.159$ . The analogous expression for  $\kappa_\infty$  follows from [1] with  $a=0.171$  and from [11] where, with the account for the correction factor (the tortuosity factor  $q=1.4$ ) introduced when comparing with the experimental data in the case of  $\psi \approx 0.4$ ,  $a=0.122$  (without this factor  $a=0.172$ ). Note also that the method of the mean free path of molecules [12] gives  $a=0.251$ . The same result also follows from [1] in the case of specular reflection of molecules from the surface of particles.

The results obtained were also compared with the experimental data presented in [13, 14], where the permeability of porous catalysts is measured for the sake of their characteristics determination. From the comparison made whose technique is presented in [8, 9] there follows a satisfactory agreement between the theoretical and experimental values of  $\kappa(Kn)$  within the range  $1 \leq Kn \leq \infty$ . As to the region of small values of  $Kn$ , then generally speaking, in the linear portion of the function  $u_L(1/Kn)$  the permeability  $\kappa$  necessitates the introduction of the correction factor  $W_0(\psi)$  [15], which depends on  $\psi$  and allows the account for the flow "confinement" in a porous medium. As far as the permeability on the basis of (3) and (6) in the continuum medium regime is equal to  $\kappa_0 = \psi^2 d^2 p / 18 (1 - \psi) \eta$  with account for  $W_0(\psi)$ , we have

$$\kappa_0 = \frac{\psi^2 d^2 p}{18(1-\psi)\eta} W_0(\psi)^2,$$

where, for example, for  $\psi=0.9$  the value of  $W = \psi^2 W_0 = 0.32$ . The table for  $W(\psi)$  values, which were compared with the experimental results, is given in [15]. It is shown, particularly, that the Karman-Kozeny expression for permeability (here it has the form  $\psi^3 d^2 p / 180 (1 - \psi^2) \eta$ ) is good for the range of  $0.26 < \psi < 0.7$ . In view of this, note that in case the formula for  $A_1(Kn)$  is written in the form

$$A_1 = \frac{15 - 3Kn + 3(8 + \pi)Kn^2}{15 W_0(\psi) + 12Kn + 18Kn^2 + 54Kn^3},$$

then, as is shown by estimations, the above expressions for the rates of evaporation and filtration can be used within the entire range of the  $Kn$  variation.

### Nomenclature

$F_d$  — force affecting a particle,  $F_s$  — Stokes force,  $r$  — particle radius  
 $n_d$  — number of particles in porous body per unit volume,  $\lambda$  — mean free path of molecules,  $Kn$  — Knudsen number,  $\xi$  — molecular velocity,  $\bar{\xi} = (8kT\pi m)^{1/2}$  — mean velocity of molecules,  $x = X/L$  — dimensionless coordinate,  $p_0$  — saturated vapour pressure,  $\psi$  — porosity.

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