Journal of Theoretical
and Applied Mechanics
Sofia 1993, Year XXIV No 1

FLUID MECHANICS

NUMERICAL MODELLING OF GAS-LIQUID FLOW

I. BOZDOUGANOV, Z. ZAPRYANOV, YU. BUYEVICH

Gas-liquid flows consist of two phases which do not mix on a molecular level. In chemical and process engineering these flows are used to contact the gas and the liquid in order to carry out a chemical or a phisical process in which one or more components have to be transferred from one phase to another or(and) these components are involved in a chemical reaction.

Most of the gas-liquid systems and apparatus used to handle them have been studied extensively and many empirical relations have been derived to describe their behaviour as well as mass and heat transfer, mixing etc.

Any theoretical treatment of gas-liquid flow which predicts the details of the macroscopic process requires constitutive equations relating fluxes of mass, momentum and energy to gradients in the local state variables[1-4].

In the present paper we assume that the gas-liquid mixture consists of small gas bubbles dispersed in a continuous phase. The liquid is supposed to be incompressible, while the gas behaves as an ideal gas. In view of the large heat capacity of the liquid we shall assume that the temperature is uniform and constant. Volume fraction occupied by the gas bubbles is denoted by φ. Mass densities d_0 and d_1 of the continuous and dispersed phases are given by, regarding,

$$d_0 = (1 - \varphi)d_l,$$

$$d_1 = \varphi d_g.$$

Here d_l is constant mass density of the pure liquid and d_g- mass density of the pure gas. Total mass density d is expressed by

$$d = d_0 + d_1 = (1 - \varphi)d_l + \varphi d_g.$$

We shall also assume that the gas-liquid mixture can be considered within the framework of a model of two interacting and interpenetrating continuous media. Then the equations of conservation of mass and momentum of the two components are

$$\frac{\partial \varphi}{\partial t} + \nabla(\varphi \boldsymbol{W}) = 0, \tag{1}$$

$$\frac{\partial}{\partial t}(1 - \varphi) + \nabla[(1 - \varphi)\boldsymbol{U}], \tag{2}$$

$$\varphi d_1 \Big(\frac{\partial}{\partial t} + \boldsymbol{W}\nabla\Big)\boldsymbol{W} = \boldsymbol{f} - \varphi d_1 \nabla \Pi, \tag{3}$$

$$(1 - \varphi)d_0\Big(\frac{\partial}{\partial t} + \boldsymbol{V}\nabla)\boldsymbol{V} = \nabla p + \nabla(\mu \nabla \boldsymbol{V}\Big) - \boldsymbol{f} - (1 - \varphi)d_0 \nabla \Pi, \tag{4}$$

where $\boldsymbol{V}$ is liquid velocity, $\boldsymbol{W}$ - gas velocity, μ - liquid dynamic viscosity and Π - gravity force potential.

The interaction force between the liquid and the gas has the following components:

Archimedt force

$$\boldsymbol{f}_A = \varphi d_0 \Big[\nabla \Pi + \Big(\frac{\partial}{\partial t} + \boldsymbol{W}\nabla\Big)\boldsymbol{W}\Big], \tag{5}$$

Averaging Stokes force

$$\boldsymbol{f}_S = \frac{9}{2}\varphi M(\varphi)\frac{\mu}{a^2}(\boldsymbol{V} - \boldsymbol{W}), \tag{6}$$

Force of inertia

$$\boldsymbol{f}_I = \frac{3}{2}\varphi(1 - \varphi)\Big[1 - \Big(\frac{1}{2} + \frac{15}{4}M(\varphi)\Big)\varphi\Big] d\Big(\frac{\partial}{\partial t} + \boldsymbol{W}\nabla\Big)(\boldsymbol{V} - \boldsymbol{W}), \tag{7}$$

Basset force

$$\boldsymbol{f}_B = \varphi \left[\frac{(1 - \varphi)M(\varphi)}{\pi}\right]^{0.5} \left[\frac{\mu d_0}{a^2}\right]^{0.5} \frac{9}{2} \int_{-\infty}^{t} (\frac{d}{dt} + \boldsymbol{W})(\boldsymbol{V} - \boldsymbol{W})\frac{dt}{\sqrt{t - \tau}}, \tag{8}$$

Faxen force

$$f_F = \frac{3}{4}\varphi\eta_0\nabla[M(\varphi)\nabla V], \tag{9}$$

Nonlinear force

$$f_N = 6.46\frac{3\varphi d_0}{4\pi a}\left[\mu M(\varphi)\left|\frac{\partial V}{\partial y}\right|\right]^{0.5}(V - W)\mathrm{sign}\Big(\frac{\partial V}{\partial y}\Big). \tag{10}$$

Therefore

$$f = f_A + f_S + f_I + f_B + f_F + f_N. \tag{11}$$

The concentration constant $M(\varphi)$ is:

$$M(\varphi) = \frac{1}{1 - 2.5\varphi}, \tag{12}$$

for small bubbles with solidify surface and

$$M(\varphi) = \frac{1}{1 - \varphi}. \tag{13}$$

for small bubbles with free surface.

Formulation of the problem.

Let us consider a circular cylindrical vessel of radius R, filled with a viscous fluid. We suppose that at the bottom of the vessel there is a gas feeder of radius $a < R$ which originates a flow of bubble in the vessel liquid (Fig.1).

The bubbles travel under buoyancy and initial velocity V_0 at the bottom. Once and all in the vessel are formed two zones

(i) Zone I – a moving pure liquid,

(ii) Zone II– a moving gas-liquid suspension.

If U is the velocity field in zone I then the liquid motion equations are continuity and Navier-Stokes equations:

$$\frac{\partial\varphi}{\partial t} - \mathrm{div}[(1 - \varphi)U] = 0, \tag{14}$$

$$(1 - \varphi)\rho\Big(\frac{\partial}{\partial t} + U\nabla\Big)U = -\nabla p + \mu\Delta U. \tag{15}$$

The governing equations for zone II are (1) – (4).

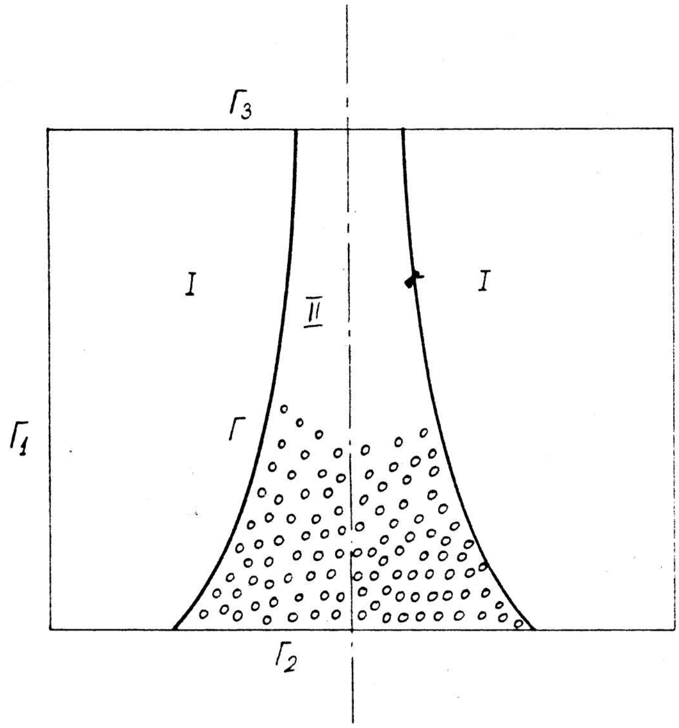

Fig.1

Boundary conditions.

In order to model the considered apparatus in Fig.1 it is necessary to write down the appropriate boundary conditions for Zone I and Zone II.

Let us denote by Γ_1 the apparatus walls for zone I, — by Γ_2 the apparatus walls for zone II, by Γ_3 — the free interface and by Γ — the joint boundary of two zones. Then the boundary conditions can be writen as follows:

$$\boldsymbol{U}|_{\Gamma_1} = \mathbf{O}, \tag{16}$$

$$\boldsymbol{V}|_{\Gamma_2} = \boldsymbol{V}_0, \tag{17}$$

$$u_n|_{\Gamma_3} = v_n|_{\Gamma_3}, \tag{18}$$

$$v_t|_{\Gamma} = u_t|_{\Gamma}, \quad v_n|_{\Gamma} = (1 - \varphi)u_n|_{\Gamma}. \tag{19}$$

The formulated boundary value problem (1)–(4), (14), (15), (16)–(19) is unsteady and of an elliptical type. The solution of this problem, even numerical, is very difficult. In order to simplify this problem we shall assume that the concentration of the feeded gas is very small and the gas-liquid flow is steady i.e. $R >> a$. Then we can use Euler equations or boundary layer equations. When the partial differential equations are parabolic one can uses the marching method which is much simplier than the methods for solving elliptic equations.

Inviscid fluid

When the fluid is inviscid the problem can be formulated as follows:
Continuity equations

$$\operatorname{div}(\varphi \boldsymbol{W}) = 0, \tag{20}$$

$$\operatorname{div}[(1-\varphi)\boldsymbol{V}] = 0. \tag{21}$$

Equations of motion:

$$(1-\varphi)d_0(\boldsymbol{V}\nabla)\boldsymbol{V} = -\nabla p - \boldsymbol{f}, \tag{22}$$

$$\varphi d_1(\boldsymbol{W}\nabla)\boldsymbol{W} = \boldsymbol{f} - \varphi d_g. \tag{23}$$

Since gas density d_1 is very small in comparison with liquid density d_0, we can write down equation (23) in the form

$$\boldsymbol{f} = 0. \tag{24}$$

We suppose that the bubble motion is steady and of a small velocity. Then velocity $\mathbf{W}_0$ calculated by Stokes formula is

$$\frac{9}{2}\frac{ga^2d_0}{\mu}. \tag{25}$$

The interaction between the bubbles and the liquid can be calculated by quasi stokes force

$$\frac{9}{2}\varphi M(\varphi)\ \frac{\mu}{a^2}(\boldsymbol{U} - \boldsymbol{W}). \tag{26}$$

The assumption of small relative velocity of the gas bubbles and the phase medium, as a whole, gives us arguments to take only the Archimedt and quasistokes forces, i.e.

$$f = f_A + f_{QS}. \tag{27}$$

Boundary conditions for inviscid fluid flow.

We suppose that there is a flow through the boundary Γ from Zone I to Zone II, i.e.

$$v_n|_\Gamma = (1-\varphi)u_n|_\Gamma. \tag{28}$$

The boundary condition at Γ_2 has the same form as before

$$\boldsymbol{V}|_{\Gamma_2} = \boldsymbol{W}|_{\Gamma_2} = \boldsymbol{V}_0. \tag{29}$$

Since the continuous phase is an inviscid liquid, the gas-liquid flow in Zone II does not act upon the liquid in Zone I. Therefore we have to calculate only the velocity field and the concentration field in Zone II.

Numerical implementation.

In two dimentional formulation the system (20)–(25) has the form

$$\varphi\Big(\frac{\partial w_r}{\partial r} + \frac{\partial w_z}{\partial z}\Big) + w_r\frac{\partial \varphi}{\partial r} + w_z\frac{\partial \varphi}{\partial z} = 0, \tag{30}$$

$$(1-\varphi)\Big(\frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z}\Big) - u_r\frac{\partial \varphi}{\partial r} - u_z\frac{\partial \varphi}{\partial z} = 0, \tag{31}$$

$$(1-\varphi)d_0\Big(u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z}\Big) = -\frac{\partial p}{\partial r} - f_r, \tag{32}$$

$$(1-\varphi)d_0\Big(u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\Big) = -\frac{\partial p}{\partial z} - f_z - \rho(1-\varphi)g, \tag{33}$$

$$f_r = \varphi d_0\left[w_r\frac{\partial w_r}{\partial r} + w_z\frac{\partial w_r}{\partial z}\right] + \frac{9}{2}\varphi\frac{1}{1-2.5\varphi}\frac{\mu}{a^2}(u_r - w_r), \tag{34}$$

$$f_z = \varphi d_0\left[-g + w_r\frac{\partial w_z}{\partial r} + w_z\frac{\partial w_z}{\partial z}\right] + \frac{9}{2}\varphi\frac{1}{1-2.5\varphi}\frac{\mu}{a^2}(u_z - w_z). \tag{35}$$

In order to approximate these equations we use a grid shown in Fig.2.

The grid points are uniformly spaced in direction z and non uniformly spaced in direction r. The calculation procedure is based on the numerical solution of the following finit difference approximation of equations (30) – (35)

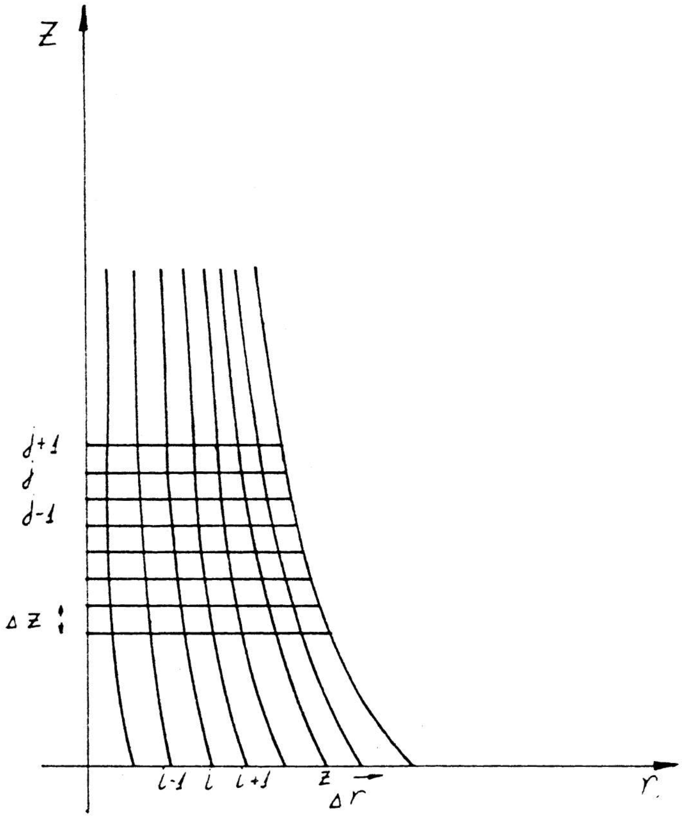

Fig.2

$$\varphi_{i,j}\left[\frac{(w_r)_{i+1,j}-(w_r)_{i-1,j}}{2\Delta r}+\frac{(w_z)_{i,j+1}-(w_z)_{i,j}}{\Delta z}\right]+ \tag{36}$$

$$(w_r)_{i,j}\frac{\varphi_{i+1,j}-\varphi_{i-1,j}}{2\Delta r}+(w_z)_{i,j}\frac{\varphi_{i,j+1}-\varphi_{i,j}}{\Delta z}=0,$$

$$(1-\varphi_{i,j})\left[\frac{(u_r)_{i+1,j}-(u_r)_{i-1,j}}{2\Delta r}+\frac{(u_z)_{i,j+1}-(u_z)_{i,j}}{\Delta z}\right]- \tag{37}$$

$$(u_r)_{i,j}\frac{\varphi_{i+1,j}-\varphi_{i-1,j}}{2\Delta r}+(u_z)_{i,j}\frac{\varphi_{i,j+1}-\varphi_{i,j}}{\Delta z}=0,$$

$$(1-\varphi_{i,j})d_0\Big[(u_r)_{i,j}\frac{(u_r)_{i+1,j}-(u_r)_{i-1,j}}{2\Delta r}+ \tag{38}$$

$$(u_z)_{i,j}\frac{(u_r)_{i,j+1}-(u_r)_{i,j}}{\Delta z}\Big]=-f_r,$$

$$(1-\varphi_{i,j})d_0\Big[(u_r)_{i,j}\frac{(u_z)_{i+1,j}-(u_z)_{i-1,j}}{2\Delta r}+ \tag{39}$$

$$(u_z)_{i,j}\frac{(u_z)_{i,j+1}-(u_z)_{i,j}}{\Delta z}\Big] = -f_z - d_0(1-\varphi_{i,j})g,$$

(40)
$$(f_r)_{i,j} = \varphi_{i,j}d_0\Big[(w_r)\frac{(w_r)_{i+1,j}-(w_r)_{i-1,j}}{2\Delta r}+$$
$$(w_z)_{i,j}\frac{(w_r)_{i,j+1}-(w_r)_{i,j}}{\Delta z}\Big] + \frac{9}{2}\varphi_{i,j}\frac{1}{1-2.5\varphi_{i,j}}\frac{\mu}{a^2}\left[(u_r)_{i,j}-(w_r)_{i,j}\right],$$

(41)
$$(f_z)_{i,j} = \varphi_{i,j}d_0\Big[(w_r)\frac{(w_z)_{i+1,j}-(w_z)_{i-1,j}}{2\Delta r}+$$
$$(w_z)_{i,j}\frac{(w_z)_{i,j+1}-(w_z)_{i,j}}{\Delta z}\Big] + \frac{9}{2}\varphi_{i,j}\frac{1}{1-2.5\varphi_{i,j}}\frac{\mu}{a^2}\left[(u_z)_{i,j}-(w_z)_{i,j}\right].$$

From these equations one can calculate the unknown quantities at $j+1$ layer using the following formulas:

(42)
$$(u_r)_{i,j+1} = \frac{\Delta z}{(u_z)_{i,j}}\Big[-\frac{f_r}{(1-\varphi_{i,j})d_0} + (u_r)_{i,j}\frac{(u_z)^2_{i,j}}{(\Delta z)^2} -$$
$$(u_r)_{i,j}\frac{(u_r)_{i+1,j}-(u_r)_{i-1,j}}{2\Delta r}\Big],$$

(43)
$$(u_z)_{i,j+1} = \frac{\Delta z}{(u_z)_{i,j}}\Big[-\frac{f_z}{(1-\varphi_{i,j})d_0} + (u_z)_{i,j}\frac{(u_z)^2_{i,j}}{(\Delta z)^2} -$$
$$(u_r)_{i,j}\frac{(u_z)_{i+1,j}-(u_z)_{i-1,j}}{2\Delta r} - d_0(1-\varphi_{i,j})g\Big],$$

(44)
$$(w_r)_{i,j+1} = \frac{\Delta z}{(w_z)_{i,j}} - \Big\{\Big[-\frac{9}{2}\frac{1}{1-2.5\varphi_{i,j}}\frac{\mu}{a^2}((u_r)_{i,j}-(w_r)_{i,j})+$$
$$(f_r)_{i,j}\Big]\frac{1}{\varphi_{i,j}d_0} - (w_r)_{i,j}\frac{(w_r)_{i+1,j}-(w_r)_{i-1,j}}{2\Delta r} + \frac{(w_z)_{i,j}(w_r)_{i,j}}{\Delta z}\Big\},$$

(45)
$$(w_z)_{i,j+1} = \frac{\Delta z}{(w_z)_{i,j}}\Big\{\Big[-\frac{9}{2}\frac{1}{1-2.5\varphi_{i,j}}\frac{\mu}{a^2}((u_z)_{i,j}-(w_z)_{i,j})+$$
$$(f_z)_{i,j}\Big]\frac{1}{\varphi_{i,j}d_0} - (w_r)_{i,j}\frac{(w_z)_{i+1,j}-(w_z)_{i-1,j}}{2\Delta r} + \frac{(w_z)_{i,j}(w_z)_{i,j}}{\Delta z}\Big\},$$

(46)
$$\varphi_{i,j+1} = \varphi_{i,j} - \Big\{\varphi_{i,j}\Big[\frac{(w_r)_{i+1,j}-(w_r)_{i-1,j}}{2\Delta r}+$$
$$\frac{(u_r)_{i+1,j}-(u_r)_{i-1,j}}{2\Delta r} + \frac{(u_z)_{i+1,j}-(u_z)_{i-1,j}}{2\Delta r} + \frac{(w_z)_{i+1,j}-(w_z)_{i-1,j}}{2\Delta r} -$$

$$\frac{(u_r)_{i+1,j}-(u_r)_{i-1,j}}{2\Delta r}-\frac{(u_z)_{i,j+1}-(u_z)_{i,j}}{\Delta z}+$$
$$\left((u_r)_{i,j}-(u_r)\right)\frac{\varphi_{i+1,j}-\varphi_{i-1,j}}{2\Delta r}\Big\}\frac{\Delta z}{(u_z)_{i,j}-(w_z)_{i,j}}.$$

The running radius of the bubble liquid suspension is calculated using the equation of conservation of the quantity of a two-phase medium passing through the unit area.

Numerical experiment.

On the basis of the above numerical procedure we have calculated velocity. Conservation field of the two phase flow is induced when the dispersed phase is moved with respect to the continuous one.

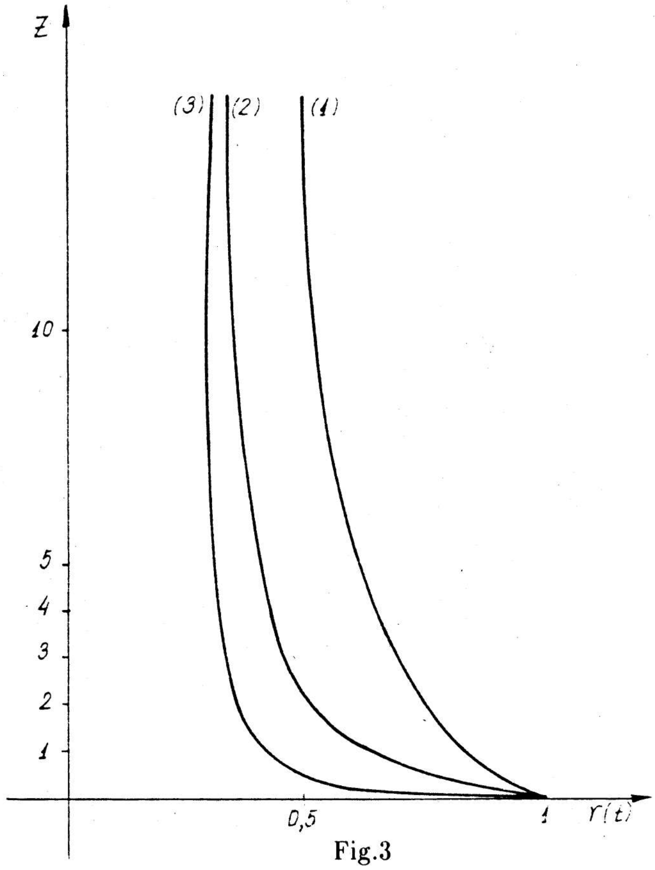

Fig.3

Calculations were carried out to find the influence of the feeded bubbles on the sizes and on the shape of the two phase flow, as well as on the velocity field.

Figure 3 shows the size of the two phase flow for different values of the dispersed phase: $\varphi = 0.01$ for curve (1); $\varphi = 0.02$ for curve (2); $\varphi = 0.05$ for curve (3). The

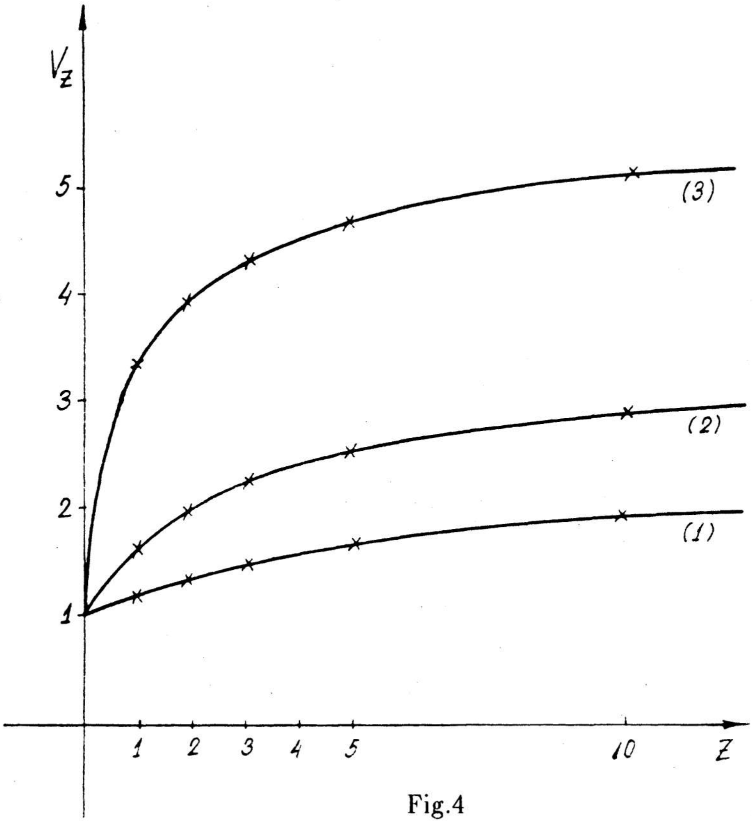

Fig.4

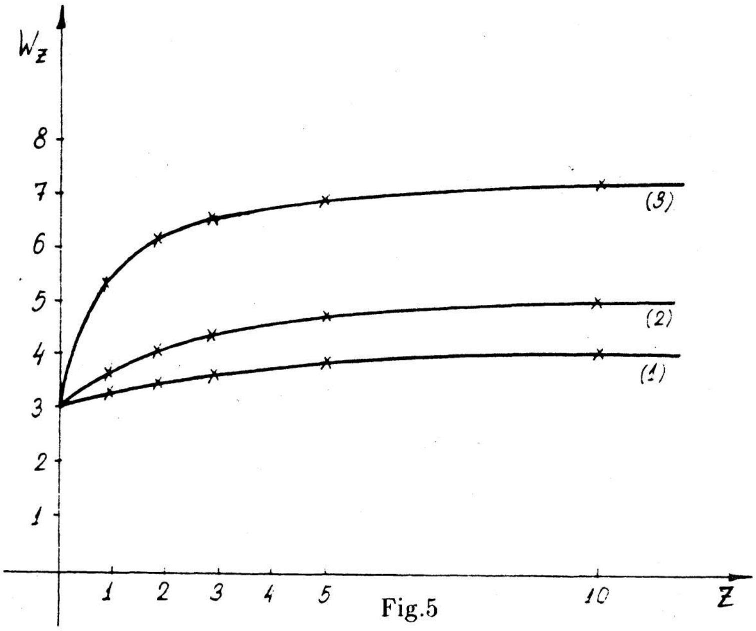

Fig.5

radius of the feeded bubbles is equal to 0.05 sm, the liquid viscosity is 0.25 mPa and the liquid density $d_0 = 1.0$. Reynolds number is $\mathrm{Re} = 0.436 < 1/2$ and relative velocity of the bubbles $W_0 = 2.18$ sm/s.

It is seen from Fig.3 that when the concentration of the dispersed phase is increased the rising flow gets narrower.

The velocity fields of the two phases are shown in Fig.4 and Fig.5. The values of the caracteristic quantities are the same as before. One can see that the concentration influences strongly the velocity field due to the increasing of the quasistokes force.

Conclusions.

The model of inviscid liquid for continuous phase used describes the process of suspension motion qualitatively very well. A quantitative evaluation is not possible however, because experimental data are absent so far.

When the concentration of the dispersed phase is larger, the interaction between the two zones will be stronger and one has to use the model of viscous liquid for the continuous phase. In this case it is possible to use the boundary layer equations. We shall apply this model in a next investigation.

This work was supported by grant MM 72/91 from the Ministry of Education, Science and Culture.

REFERENCES

[1] D. A. Drew, Mathematical modeling of two-phase flow, *Ann. Rev. Fluid Mech.*, **15** (1983), 261-291.

[2] L. V. Wijngaarden, Some problems in the formulation of the equations for gas-liquid flows, Theor. and Appl.Mech. North-Holland Publishing Company, 1976, 249-260.

[3] K. Rietema, Science and technology of dispersed two-phase systems, *Chem. Engin. Sci.* **37** (1982), 1125-1150.

[4] Ю. А. Буевич, А. Зубарев, А. Исаев О хидромеханике суспензии, Инж. физ. Журнал 57(3), 1989.

Received on May 20, 1993.