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EXPERIMENTAL AND NUMERICAL STUDY OF A HORIZONTAL HEATED ROUND JET

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Horizontal heated jets have found a wide application in combustion and ventilation technology in food processing and chemical engineering. Their prediction is important in solving problems of present interest, concerning decrease of the environmental pollution caused by waste gases and liquids discharged from power plants. Jets, which issue horizontally from a round nozzle into a denser quiescent ambient fluid follow a curved trajectory in response to the influence of buoyant forces. In most of the published mathematical models of such jets the flow is assumed to be axisymmetrical. The analysis methods, proposed for horizontal heated, jets have generally been integral [1]. In [2] a differential method and a simple mean flow turbulence model for non-vertical jets have been described. In [3] a $k - \varepsilon$ model with algebraic stress/flux expressions for horizontal free shear flows was developed. The available experimental studies of horizontal heated jets are restricted to measurements of the jet centre-line trajectory and the centre-line temperature. In the publications there is not enough experimental data supporting the assumption of axial symmetry.

In the present paper two turbulence models are employed to study a horizontal heated round jet: a $k - \varepsilon - t'$ model and a second order model. The predictions of these models are compared with our own measurements and experimental data of other authors [4] in order to determine the constants in the models, which best match the experimental jet trajectory, mean velocity distribution and mean temperature distribution. Since our measurements are taken as a basis for improvement of theoretical models, it is necessary to calculate the measurement errors and thus to evaluate the data reliability.

An experimental setup of the Dept. of Hydroareodynamics at the Technical University – Sofia is used for the investigation of horizontal heated round air jets. Single measurements of mean velocity and mean temperature are performed. The random errors are not determinative for the measurement accuracy. More details about the experiment conditions are presented in [5]. The mean temperature is measured using 6 thermocouples copper-constantan and a self-balancing potentiometer KSP-4. The maximal absolute measurement error is $\pm 2.5^0 C$. It is determined taking into consideration the basic admissible error of the potentiometer and the admissible calibration error of the thermocouples [6]. The mean velocity is measured using a 3mm diameter Prandtl tube and a differential micromanometer. The maximal relative error of velocity measurements is $\pm 1.4\,\%$. It is calculated taking into consideration the basic admissible error of the micromanometer and the error of calculating the air density [7], which is obtained by using the state equation for ideal gas. The Prandtl tube is unsensitive to deviation from the flow direction. The measured and the real dynamic pressure are equal up to a deviation angle of $\alpha = 16^0$. When $\alpha = 20^0$ the measured dynamic pressure is 5 % lower than the real one [6]. When Re= $Vd/\nu < 700$ ($V < 3.5$ m/s) the tube coefficient ζ (from the equation for velocity calculation $V = \zeta(2p_d/\rho)^{1/2}$) decreases and when Re= $300(V \approx 1.5$ m/s) ζ is 6% lower than 1 [6].

The conservation equations for an axisymmetric horizontal buoyant jet in terms of the boundary layer approximation can be written as follows:
continuity:

$$\partial(\rho u)/\partial x + (1/y)\partial(y\rho v)/\partial y = 0 \tag{1}$$

x – momentum:

$$\rho(u\partial u/\partial x + v\partial u/\partial y) = -(1/y)\partial(y\rho\overline{u'v'})/\partial y + (\rho_a - \rho)g\sin\theta, \tag{2}$$

y – momentum:

$$(\partial\theta/\partial x)\int_0^\infty \rho u^2 y\,dy = g\cos\theta\int_0^\infty(\rho_a - \rho)y\,dy, \tag{3}$$

energy:

$$\rho(u\partial t/\partial x + v\partial t/\partial y) = -(1/y)\partial(y\rho\overline{v't'})/\partial y \tag{4}$$

The system (1)-(4) is closed using the state equation for ideal gas and the turbulence models equations, which can be written as:

$$\rho(u\partial\phi/\partial x + v\partial\phi/\partial y) = (1/y)\partial(y\Gamma_\phi\partial\phi/\partial y)/\partial y + S_\phi. \tag{5}$$

The values of ϕ, Γ_ϕ and S_ϕ are shown in Table 1. The constants in the models are shown in Table 2, 3. The governing equations are solved numerically by means of a modification of the Patankar-Spalding finite-difference procedure [8]. The equations (1)-(5) are solved at the following boundary and initial conditions: at the jet boundary $x > 0$, $y \to \infty$, $t = t_a$, $u = 0$, $\phi = 0$; initial conditions $x = 0$, $t = t_0$, $u = V_0$, $k = k_0 = 0.008V_0^2$, $\varepsilon = \varepsilon_0 = 0.002V_0^3/D_0$, $\overline{t'^2} = (\overline{t'^2})_0 = 0.008(t_{0c} - t_a)^2$. For the cases of our own experiment, the initial temperature profiles are not uniform in conformity with the experiment. Initial distributions of the Reynolds stresses and the turbulent heat fluxes are provided for the second order model [5]. The calculated and measured iso-velocity and isothermal lines for a jet $Fr_0 = 447$, $t_{0c} - t_a = 57^0C$, $t_a = 21^0C$, $V_0 = 6.25\,\mathrm{m/s}$ are compared in Fig. 1–4. The maximal relative measurement error of $(t - t_a)/(t_{0c} - t_a) = 1$ is $\pm 9\%$ and of $(t - t_a)/(t_{0c} - t_a) = 0.5$ is $\pm 13\%$. The maximal relative error of V/V_0 is $\pm 2\,\%$ except for the outer region. The models results are in good agreement with the experiment, except for the higher values of velocity and temperature along the jet centre-line. The calculated centre-lines for the regimes $Fr_0 = 447$ and 60 lie higher than the experimental ones – Fig. 5.

In summary, it was found that the results obtained with the models in this work are in conformity with the experimental data but discrepancies exist, which cannot be eliminated only by changing the constants. Further research is needed after supplying the missing experimental information about the turbulent characteristics of the flow.

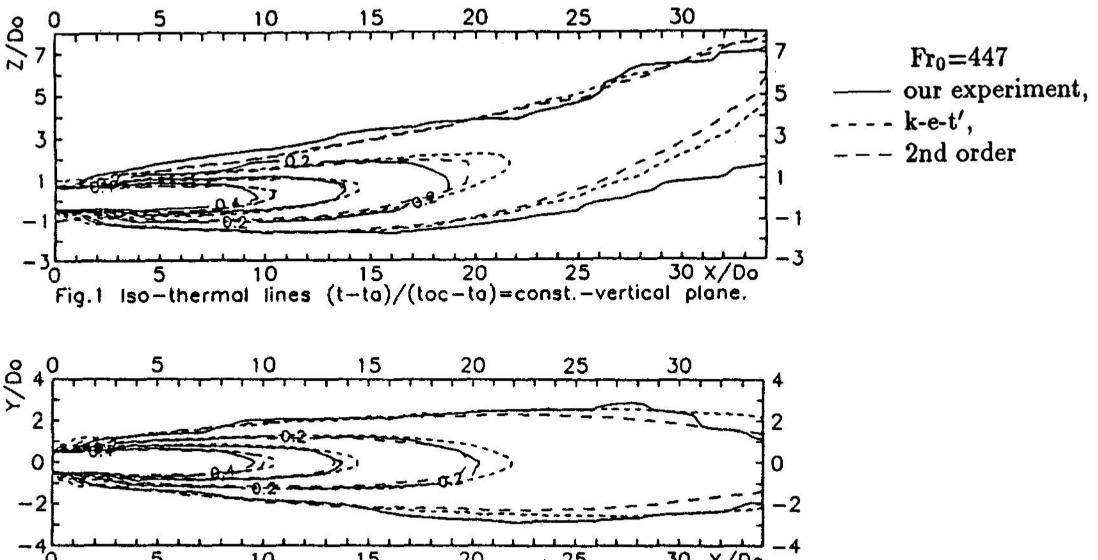

Fig. 1 Iso-thermal lines (t−ta)/(toc−ta)=const.–vertical plane.

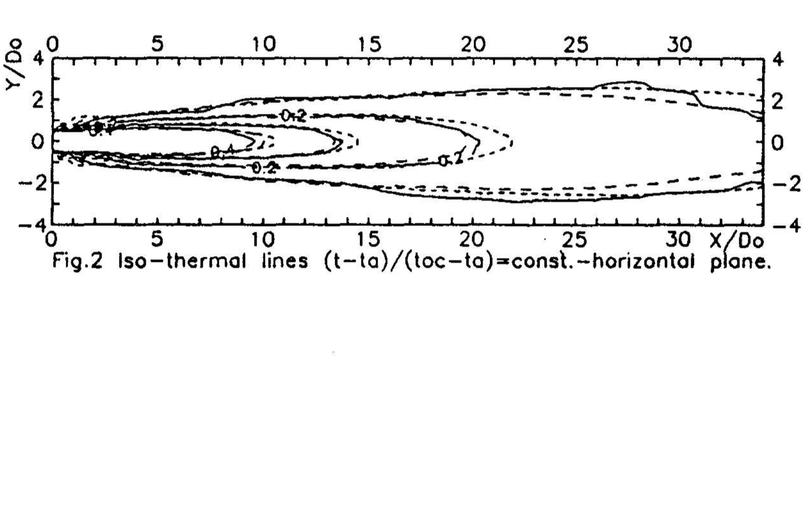

Fig. 2 Iso-thermal lines (t−ta)/(toc−ta)=const.–horizontal plane.

$Fr_o=447$

—— our experiment,----- k-e-t',- - - 2nd order

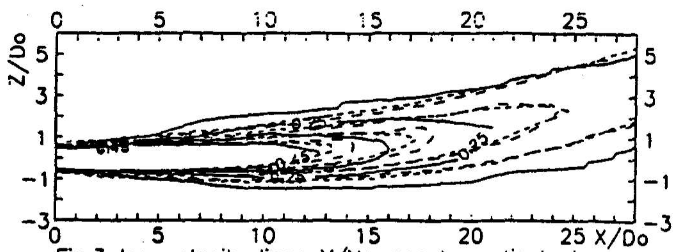

Fig.3 Iso-velocity lines V/Vo=const.-vertical plane.

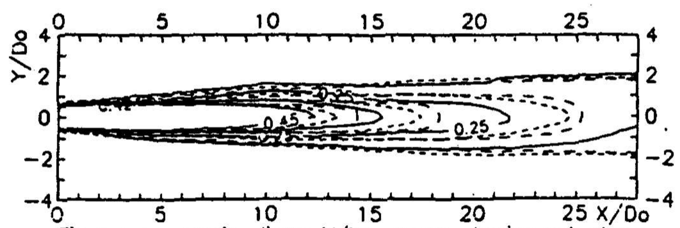

Fig.4 Iso-velocity lines V/Vo=const.-horizontal plane.

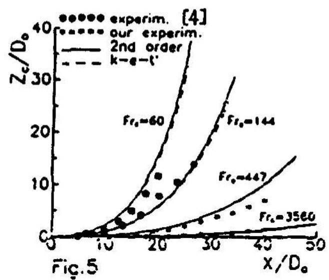

Fig.5

Table 1

Equation	ϕ	Γ_ϕ	S_ϕ
turbulent kinetic energy [a]	k	$\frac{\mu_T}{\sigma_k}$	$\mu_T(\partial u/\partial y)^2 + \beta[\rho g_x C_4(\overline{t'^2}k)^{1/2} - g_y(\mu_T/Pr_T)\partial t/\partial y] - \rho\varepsilon$
dissipation rate [a]	ε	$\frac{\mu_T}{\sigma_\varepsilon}$	$\{C_{1\varepsilon}\mu_T(\partial u/\partial y)^2 + C_{3\varepsilon}\beta[\rho g_x C_4(\overline{t'^2}k)^{1/2} - g_y(\mu_T/Pr_T)\partial t/\partial y] - C_{2\varepsilon}\rho\varepsilon\}\varepsilon/k$
temperature fluctuation [a]	$\overline{t'^2}$	$\frac{\mu_T}{\sigma_t}$	$C_{1q}\mu_T(\partial t/\partial y)^2 - \rho\varepsilon\overline{t'^2}/(Rk)$
turbulent kinetic energy [b]	k	$C_s\rho\overline{v'^2}k/\varepsilon$	$\rho[-\overline{u'v'}\partial u/\partial y + \beta(g_x\overline{u't'} + g_y\overline{v't'}) - \varepsilon]$
dissipation rate [b]	k	$c_\varepsilon\rho\overline{v'^2}k/\varepsilon$	$\rho[-C_{1\varepsilon}\overline{u'v'}\partial u/\partial y + C_{3\varepsilon}\beta(g_x\overline{u't'} + g_y\overline{v't'}) - C_{2\varepsilon}\varepsilon]\varepsilon/k$
Equation	ϕ	Γ_ϕ	S_ϕ
temperature fluctuation [b]	$\overline{t'^2}$	$C_q\rho\overline{v'^2}k/\varepsilon$	$\rho[-2\overline{v't'}\partial t/\partial y - \varepsilon\overline{t'^2}/(Rk)]$
Reynolds stresses [b]	$\overline{u'v'}$	$C_S\rho\overline{v'^2}k/\varepsilon$	$\rho[-(1-C_2)\overline{v'^2}\partial u/\partial y + (1-C_3)\beta(g_x\overline{v't'} + g_y\overline{u't'}) - C_1\overline{u'v'}\varepsilon/k]$
	$\overline{v'^2}$	$C_S\rho\overline{v'^2}k/\varepsilon$	$\rho[(1-2C_3/3)\beta 2g_y\overline{v't'} - C_1(\overline{v'^2} - 2k/3)\varepsilon/k - (2C_2/3)\overline{u'v'}\partial u/\partial y + C_3\beta 2g_x\overline{u't'}/3 - 2\varepsilon/3]$
heat fluxes [b]	$\overline{v't'}$	$C_t\rho\overline{v'^2}k/\varepsilon$	$\rho[-\overline{v'^2}\partial t/\partial y + (1-C_{3t})g_y\overline{t'^2} - C_{1t}k\overline{v't'}/\varepsilon]$
	$\overline{u't'}$	$C_t\rho\overline{v'^2}k/\varepsilon$	$\rho[-\overline{u'v'}\partial t/\partial y - (1-C_{2t})\overline{v't'}\partial u/\partial y + (1-C_{3t})\beta g_x\overline{t'^2} - C_{1t}k\overline{u't'}/\varepsilon]$

$a - k - \varepsilon - t'$ model, b – 2nd order model. $g_x = g\sin\theta$, $g_y = d\cos\theta$

$k - \varepsilon - t'$ model Table 2

Ref	C_μ	$C_{1\mu}$	$C_{2\varepsilon}$	$C_{3\varepsilon}$	C_{1q}	C_4
[5]	0.09(1-0.465 F)	1.43	1.92(1-0.035 F)	1.43	2.22	0.5
this work	0.09(1-0.465 F)	1.44	1.92(1-0.035 F)	1.44	2.22	0.5
Ref	R	σ_k	σ_ε	σ_t	Pr_T	
[5]	0.8	1	1.3	0.9	0.5	
this work	0.8	1	1.3	0.9	0.6	

$F = |(y_{1/2u}/2u_c)(du/dx - |du_c/dx|)|^{0.2}$, $y_{1/2u}$ – jet half width

2nd order model Table 3

Ref	C_s	C_1	C_2	C_3	C_t	C_{1t}	C_{2t}	C_{3t}
[9]	0.22	3	0.3	0.3	0.2	2.85	0.55	0.55
[5]	0.22	3	0.53	0.53	0.2	3.2	0.55	0.55
this work	0.22	3	0.53	0.53	0.2	3.2	0.55	0.55
Ref	C_ε	$C_{1\varepsilon}$	$C_{2\varepsilon}$	$C_{3\varepsilon}$	C_q	R		
[9]	0.15	1.4	1.8	0.98	0.11	0.56		
[5]	0.15	1.4	1.8	1.4	0.11	0.56		
this work	0.15	1.43	1.8	1.43	0.11	0.56		

Nomenclature: D_0–jet diameter at exit; d – Prandtl tube diameter; Fr_0 – Froude number at exit $Fr_0 = \dfrac{V_0^2}{gD_0(\rho_a - \rho)/\rho_0}$ in case of uniform profiles of velocity and temperature; g – acceleration of gravity; p_d – dynamic pressure; Pr_T – turbulent Prandtl number, t – time mean temperature; u – x-component of time mean velocity; V – time mean velocity; v – y-component of time mean velocity; x – distance from nozzle exit along jet centre-line; X – horizontal distance from exit along nozzle axis; y – radial distance from jet centre-line; Y – horizontal distance from X–axis; Z – vertical distance from X-axis; β – isobaric volume expansivity = $-(\partial\rho/\partial t)/\rho$; θ – angle between x-axis and a horizontal; μ_T – turbulent dynamic viscosity; ν – kinematic viscosity; ρ – density. Subscripts: a – ambient condition; c – centre-line condition; o – exit condition. Superscripts: $(\ )'$ – fluctuating quantities; $\overline{(\ )}$ – time mean quantities.

REFERENCES

[1] Бруяцкий, Е. В. Турбулентные стратифицированные струйные течения, Киев, Наукова думка, 1986

[2] Madni, I. K., R. H. Pletcher. Buoyant jets discharging nonvertically into a uniform, quiescent ambient – a finite-difference analysis and turbulence modeling. *Trans ASME, J.Heat Transfer*, **99**, 1977, 641-646

[3] Gibson, M. M., B. E. Launder. On calculation of horizontal turbulent free shear flows under gravitational influence. *Trans. ASME, J.Heat Transfer*, **Feb.** 1976, 81-87

[4] Омельчук, В. С., В. Х. Посохин. Экспериментальное исследование неизотермических струй, истекающих горизонтально. *Водоснабжение и санитарная техника*, **11**, 1970, 26-29

[5] Джонова-Атанасова, Д. Б. Топлообменни процеси при двумерни свободни турбулентни струи. Дисертация, София, 1992

[6] Преображенский, В. П. Теплотехнические измерения и приборы. Москва, Энергия, 1975

[7] Пенев, Д., А. Господинов. Топлотехнически измервания и уреди, София, Техника, 1983

[8] Патанкар, С. В., Д. Б. Сполдинг. Тепло– и массообмен в пограничных слоях, Москва, Энергия, 1971

[9] Malin, M. R., B. A. Younis. Calculation of turbulent buoyant plumes with Reynolds stress and heat flux transport closure. *Int. J. Heat Mass Transfer*, **33**, No. 10, 1990, 2247-2264

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