Journal of Theoretical
and Applied Mechanics
Sofia 1993, Year XXIV No 2

IDENTIFICATION OF RHEOLOGY PARAMETERS OF RUBBER USING DIGITAL SPECKLE PATTERN INTERFEROMETRY*

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1. Introduction

Rubber structure elements are widely applied in engineering. They are most often used as seals in machines and pipelines. Rubber is significantly resistant to mechanical and thermal effects and to corrosion, but it manifests viscous properties too, which could cause seal weakening and pipeline failure. Hence, the study of rubber rheological properties is of essential practical interest. A lot of research has been done in this field – [1] – [4] and many others. It must be noted, however, that a number of difficulties arize when parameters of elastic-viscous models of rubber are to be identified. Hence, most authors use results of one-dimensional experiments. The aim of this paper is to outline a method for the identification of the rheology parameters of rubber in the case of plane stress.

We consider a circular plate, loaded by a force P at its center. The instantaneous plate deflections, due to P, and the viscous plate deflections developing in time and due to a constant load, are found by using digital speckle pattern interferometry (DSPI).

2. Methods for the Identification of the Rheology Parameters of Rubber

In what follows we consider two identification stages:

STAGE 1. Consider a thin, circular, rubber plate, clamped along the edge and loaded by a force P applied at its centre – Fig.1. The plate is loaded by means of a screw and the screw edge is round, so that plate damage be avoided. The load P is

*This work was supported by the Bulgarian Foundation for Scientific Research, Grant No MM 16/91

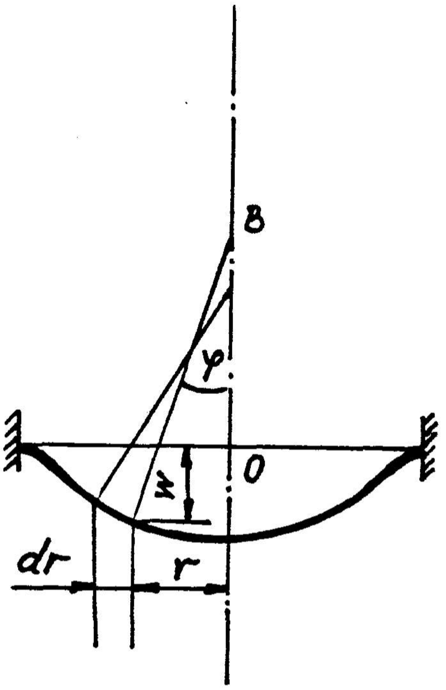

Fig. 1.

applied for a short time and the instantaneous plate deflections are experimentally obtained using DSPI. We are to identify the unknown load P for known deflections. We assume that the material instantaneous response is purely elastic. Its mechanical characteristics are found by performing uniaxial tension of flat rubber specimens, after a subsequent linearization at each loading interval – Fig.2. The circular plate is considered thin and elastic and we use the mechanical model, given in detail in [1]. There the radial bending moment M_r and the peripheral one M_t are expressed by the subsequent curvatures κ_r and κ_t according to Hooke's law:

$$
\begin{aligned}
M_r &= -D(\kappa_r + \nu\kappa_t) \\
M_t &= -D(\kappa_t + \nu\kappa_r)
\end{aligned}
\tag{1}
$$

where $D = \dfrac{Eh^3}{12(1-\nu^2)}$ is the bending stiffness and $\kappa_t = -\dfrac{d^2w}{dr^2}$, $\kappa_t = -\dfrac{1}{r}\dfrac{dw}{dr}$, while E and ν – material Young's modulus and Poisson coefficient, determined from the initial region OA_1 of the stress-strain diagramme – Fig. 2, h – plate thickness, r – radius-vector with an origin at the plate center ($0 \leq r \leq a$) and a – plate radius – see Fig. 1. Since loading is symmetrical with respect to the normal axis z, we assume that the normal instantaneous deflection w is a function of r only – $w = w(r)$. We obtain the following expression for w using the solution for a thin elastic plate, symmetrically loaded by a force P

$$w = P\overline{w} \tag{2}$$

where

$$\overline{w} = \frac{r^2}{\pi D}\ln\frac{r}{a} + \frac{1}{16\pi D}(a^2 - r^2) \tag{3}$$

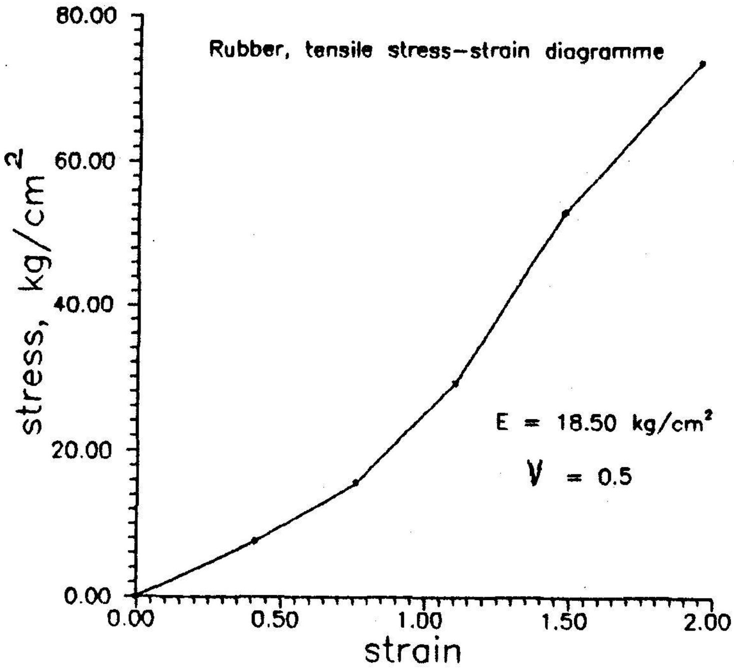

Fig. 2.

We record the deflections $w(r_i)$, $i = 1, 2, ..., q$ at q plate points employing DSPI and determine the load P using the least square method:

$$P = \frac{\sum_{i=1}^{q} \overline{w}^2(r_i)}{\sum_{i=1}^{q} \overline{w}(r_i) w^{\exp}(r_i)} \tag{4}$$

where $w^{\exp}(r_i)$ are the experimentally found deflections.

STAGE 2. The circular plate is loaded by the already found load P which is kept constant. At this stage plate viscous deflections - $w = w^c(r,t)$, develop in time and a deflection $\Delta w_1^c = w^c(r,t_1) - P\overline{w}$ takes place for a time interval $\Delta t_1 = t_1 - t_0$ due to the material viscosity. t_0 denotes the initial moment, at which instantaneous plate deflection $w(r,t_0) = P\overline{w}$ took place according to STAGE 1- relations (2) and (3). Assume that to an arbutrary small time interval $\Delta t_j = t_j - t_{j-1}, j = 1,2,...m, t_j \in (t_0,t_1)$ correspond a viscous deflection Δw_j^c and radial and peripheral strains - $\Delta\varepsilon_r(j)$ and $\Delta\varepsilon_\theta(j)$. Our aim is to outline an adequate stress – strain relation, so that material rheology parameters be identified.

Various constitutive equations have been proposed for the description of rubber visco-elastic properties – [3], [4]. We use here the power law of nonlinear viscous deformation – [2]:

$$\frac{d\underline{\zeta}}{dt} = \frac{1}{2G}\frac{d\underline{S}}{dt} + \frac{\tau_0^{n-1}}{\eta}\underline{S} \tag{5}$$

where $\underline{\zeta}$ is the deviator of the strain tensor $\underline{\epsilon}$, $\underline{S}$ – the deviator of the stress tensor $\underline{\sigma}$, G – elastic shear modulus, $\tau_0 = [\frac{1}{2}\underline{S}^T\underline{S}]^{0.5}$ – shear stress intensity, n and η – material constants which are to be identified. The additional strains are viscous only. They depend, according to relation (5), on stresses and on the material parameters n and η. If the latter are constant, then, at a moment t_j, we get

$$\underline{\varepsilon}_j(t_j - t_0) = \frac{\tau_0^{n-1}(t_0)}{\eta}\underline{S}(t_0)(t_j - t_0) \tag{6}$$

If, however, n and η are not constant in time, then the constitutive law is much more complex. In such a case the parameters n and η depend on the creep history through, for example, the viscous strain intensity $\underline{\xi}_0^c = \left[\frac{1}{2}\underline{\zeta}^T\underline{\zeta}\right]^{0.5}$. For the plane case considered here, relation (5) takes the form:

$$\begin{aligned} \Delta\varepsilon_{r_j} &= \frac{1}{\eta}\frac{1}{3^{\frac{n-1}{2}}}(\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2)^{\frac{n-1}{2}} S_{r_j}\Delta t_j \\ \Delta\varepsilon_{\theta_j} &= \frac{1}{\eta}\frac{1}{3^{\frac{n-1}{2}}}(\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2)^{\frac{n-1}{2}} S_{\theta_j}\Delta t_j \end{aligned} \tag{7}$$

where $S_{r_j} = \frac{1}{3}(2\sigma_r^2 - \sigma_\theta)_j$, $S_{\theta_j} = \frac{1}{3}(2\sigma_\theta - \sigma_r)_j$. Considering the geometry relations $\Delta\varepsilon_{r_j} = -z\frac{d^2(\Delta w_i)}{dr^2}\Delta t_j, \Delta\varepsilon_{\theta_j} = -z(\frac{d(\Delta w_i)}{dr})\Delta t_j$, where z is the coordinate normal to the plate middle surface, we can determine Δw_j after a discretization with respect to r_i, $i = 1, 2, \ldots, q$. The deflections $\Delta w_j^{\exp}(r_i)$ are experimentally found using the DSPI – [5], [6], at points r_i, $i = 1, 2, \ldots, q$ and for each time inteval Δt_j, $j = 1, 2, \ldots, q$. The parameters n and η are calculated using the least square method. We obtain from the extremum conditions of the functional

$$(8) \qquad \Phi_j = \sum_i [\Delta w_j(r_i) - \Delta w_j^{\exp}(r_i)]^2$$

a system of nonlinear equations for n and η. The system can be solved by using the intersection method, i.e. by giving a finite number of values n_k, $k = 1, 2, \ldots, s$ in a physically admissible interval and by a subsequent calculation of η_k. The process stops when a previously given accuracy of η is attained.

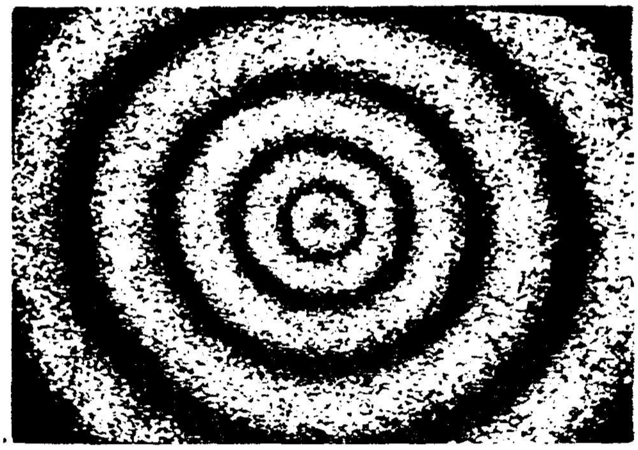

Fig. 3.

3. Experimental

As stated, the deflections are experimentally found by means of the DPSI. It must be noted, however, that an original technique is used here, which registers out-of-plane (along z) deflections of the plate – [6].

In brief, the plate surface is illuminated by a laser beam before applying the load and a TV-picture is taken. Then, after loading the plate, a set of pictures is also taken. Subtracting the post-loading picture from the pre-loading ones, we get an interferrogramme which consists of light and dark bands – Fig. 3. We obtain the geo-

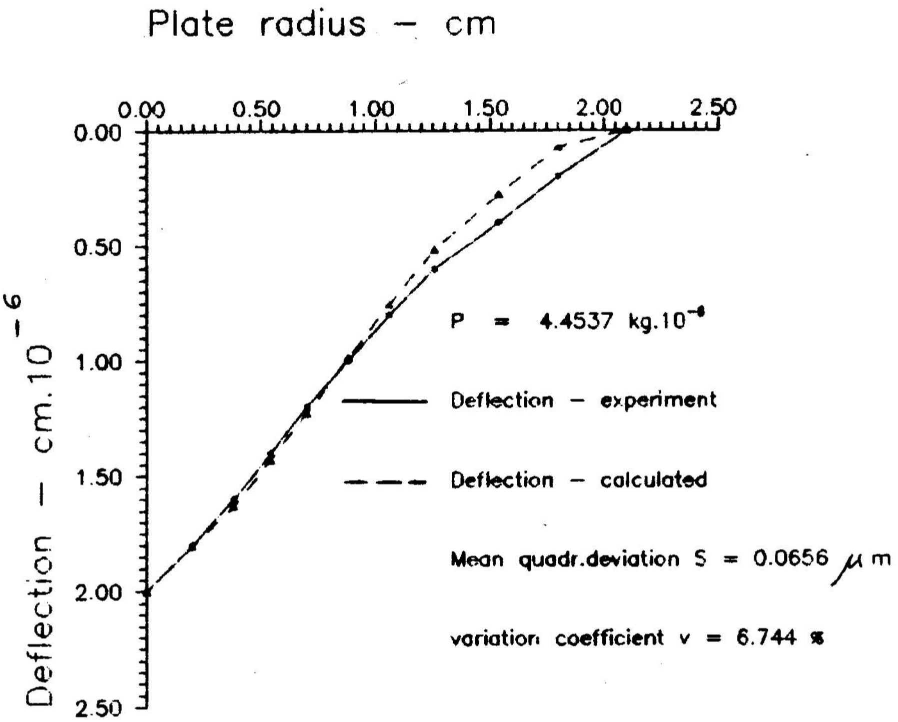

Fig. 4.

metry of the deformed surface and the viscous plate deflections on the basis of the localized maxima of the light intensity. Then, after applying the method outlined, we calculate the values of n and η through rel.(8).

The following values of the identified parameters were determined: $P = 4.452.10^{-6}$ kg, $n = 1.413$, $\eta = 175.614$ cm^2/kg. The experimentally found and the calculated instantaneous and visco-elastic deflections of the plate are shown in Fig. 4 and Fig. 5. Satisfactory agreement between theory and experiment is obtained.

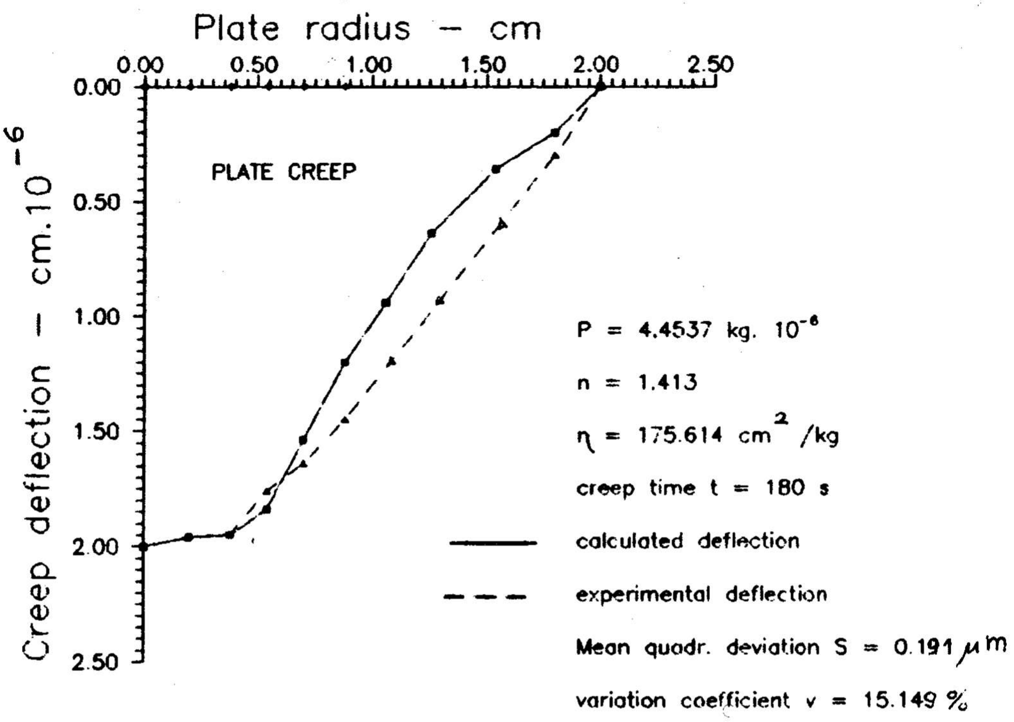

Fig. 5.

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Presented at the 7th Congress on Theoretical and Applied Mechanics, Sofia, August 30 – September 1, 1993
section: Solid Mechanics