

LINEAR ANALYSIS AND SIMULATION OF INTERFACIAL SLIP BEHAVIOUR FOR COMPOSITE BOX GIRDERS*

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ABSTRACT. The slip at the steel-concrete interface in steel-concrete composite beams was studied analytically and numerically. A theoretical description for steel-concrete composite box beams with partial shear interaction based on the partial interaction theory was derived, and equilibrium of the rotation angle w' was introduced to allow convenient computation of deformation of composite box beams. Numerical simulations of steel-concrete composite box beams subjected to concentrated load and/or uniformly distributed load were conducted. The analytical solutions show excellent agreement with the numerical results. For typical composite box beams used in practice, shear slip in partial composite box beams makes a significant contribution to beam deformation. Even for full composite box beams, slip effects may result in stiffness reduction. However, slip effects are ignored in many design specifications which use

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transformed section methods; an exception is the American Institute of Steel Construction [1] specifications, which recommend a calculation procedure in the commentary. Finally, the proposed method was extended to analyze the interface slip for shear connectors of different pitch and, to some extent, confirm the accuracy of the predictions.

KEY WORDS: Composite box beams, partial interaction, interfacial slip, analysis, simulation.

1. Introduction

Composite steel construction has gained in popularity during the past decades, taking advantage of the high tensile strength of steel materials and high compressive strength of concrete materials. The partial shear interactions between steel and concrete have been studied extensively. One of the earliest papers dealing with this problem was by Newmark et al. [2], and the model proposed at that time is usually referred to as the Newmark model. This model couples two Euler–Bernoulli beams, i.e. one for the reinforced concrete slab and one for the steel beam, by means of a deformable shear connection distributed at their interface. The shear connection enables relative longitudinal displacements to occur between the two components (partial shear interaction) while preventing their vertical separation. Since then, many researchers have extended the Newmark model to include vertical separation, material nonlinearities, and time dependent behaviour of concrete, shear-lag effects, geometric nonlinearities, and out-of-plane bending as well as torsion. It is beyond the scope of this paper to provide a review of the current state of the art; useful reviews were presented in recent works such as [3, 4].

The fundamental equations for one-dimensional, linear elastic, partial composite action for beams and columns subjected to static loads were first developed independently by Stussi [5] in Sweden, Newmark et al. in the USA, and Pleshkov [6]. This “Newmark model” has been applied many times for analysis of composite beams with interlayer slip, e.g. by Goodman [7]. Earlier analysis of composite beam conducted by Girhammar and Gopu [8] dealt with only one particular axial loading case and regarded discretely-located interlayer connectors as continuous. Recently, Ranzi et al. [9] presented a general method of analysis of composite beams with partial interaction. Dall’Asta [10] developed a three-dimensional theory for composite beams with shear weak connection. The theory included combinations of bending in the symmetry plane and torsion and transverse bending in the plane parallel to the shear-connector interface.

Numerical techniques provide important tools for structural analysis, and the finite element technique has been applied to analyze steel-concrete

composite beams for a number of years. Ansourian [11] presented an ambitious finite element technique that could be applied to continuous steel-concrete composite beams, and included realistic material properties for the steel and concrete and the non-linear load-slip characteristics of the stud shear connectors. More general displacement-type approaches have also been used, such as those described by Arizumi et al. [12]. Daniels et al. [13] presented an “exact” finite element model based on an “exact” analytical expression for the stiffness matrix. Ranzi and Bradford [14] presented a numerical model for time-dependent behaviour of partially composite beams. Different numerical and finite element formulations for the analysis of composite beams with interlayer slip have been suggested [15, 16]. Gara et al. [17] developed a finite element model for composite beams with both longitudinal slip and vertical uplift of the interface connection. Other models include those of Luigino Dezi et. al [18], Weichen Xue et. al [19] and XiangGuo Wu et. al [20].

The classical approach to the solution of structural mechanics problems, like the one mentioned above, is to formulate the governing differential equation and obtain the analytical solution. In addition, the finite element analysis of composite beams makes it possible to obtain useful information about the structural behaviour; however, the use of such an approach introduces a curvature-locking problem [21], which causes numerical instabilities for high and low values of the stiffness of the shear connection.

This paper presents an exact theoretical analysis and numerical simulation procedure for interlayer slip for composite box beams. The proposed practical analysis method is general in nature and can be applied to arbitrary boundary and loading conditions. Thus, it is well suited for designers, and should be considered for inclusion in codes. The purpose of this paper is to derive, by the introduction w' of the angle displacement function about the axial deformation of composite box beams, the ordinary differential equations for the interface slip and internal actions and all the different shear connectors and pertaining admissible boundary conditions for partially composite box beam. In particular, the aim is to derive the exact closed form characteristic equations within the validity of the Euler-Bernoulli beam theory for partially composite box beams and to compare the theory with the finite element solution for different shear connectors and different load types.

2. Theoretical analysis of interface slip for steel-concrete composite box beams

This partial composite action theory for Euler–Bernoulli beams with inter-layer slip is based on the Newmark model, for further details, see [22, 23].

The mechanical fasteners are evenly spaced and assumed to produce uniformly-distributed slip forces or interlayer shear stresses with a constant slip modulus ku [N/m²]. The secant slip modulus is frequently used for members subjected to static loads. Full composite action (infinite slip stiffness, $ku \rightarrow \infty$) and non-composite action (zero slip stiffness, $ku \rightarrow 0$) represent the upper and lower bounds for the partial composite action. The geometric parameters defining a typical composite box beam with two sub-elements of different materials are shown in Fig. 1. The z-axis of the coordinate system is located at the centroid of the fully-composite section.

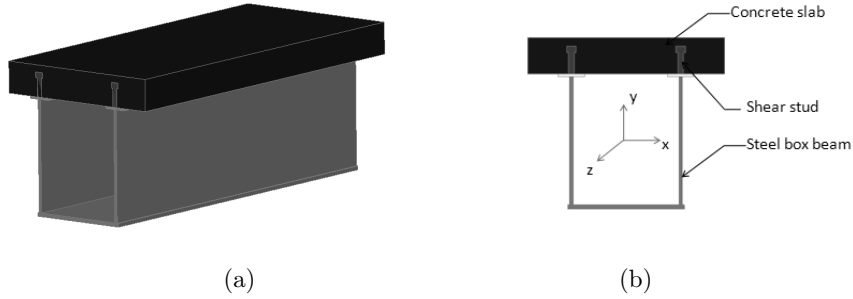


Fig. 1. Calculation model for simply-supported composite box beam:
 (a) Simply-supported beam, (b) Typical composite steel-concrete box girder

The composite beam boundary conditions are arbitrary and the beam is subjected to a distributed transverse loading q_y , and an axial load (P) acting at the centroid of the fully composite section. In the figure, the following forces are defined: moments (M, M_c, M_s), shear forces (Q, Q_c, Q_s), and normal forces (N_c, N_s). The total elastic bending stiffness (both steel and reinforced concrete slab without considering interaction), $EI_x = E_c I_{cx} + E_s I_{sx}$, is for convenience denoted simply as EI_x . The same kind of notation will be used for the bending stiffness, $(EI_x) = EI$. The interlayer slip is ΔS . The length of the composite box beam is L .

3. Governing equations

In this section we recall the field equations for a two-layer composite beam with partial shear interaction for small displacements. We assume that plane sections remain plane (Bernoulli's assumption). All variables subscripted with "c" belong to concrete slab and those with "s" belong to steel box beam.

The main equilibrium equations are derived by considering a differential element dz located at an arbitrary position z :

$$\begin{aligned}
 (1) \quad & M_{cx} = -E_c I_{cx} y'', \\
 (2) \quad & M_{sx} = -E_s I_{sx} y'', \\
 (3) \quad & N_c = E_c A_c w_c', \\
 (4) \quad & N_s = E_s A_s w_s',
 \end{aligned}$$

where:

A_s = section area of steel box for composite beams,

A_c = section area of reinforced concrete slab for composite beams.

The equations cited below of the composite beam in terms of the strength of materials can be easily derived by from the equations given above:

$$\begin{aligned}
 (5) \quad & -E_c I_{cx} y''' = Q_{cy} - h_c q_z, \\
 (6) \quad & -E_s I_{sx} y''' = Q_{sy} - h_s q_z, \\
 (7) \quad & E_c A_c w_c'' = -q_u, \\
 (8) \quad & E_s A_s w_s'' = q_u.
 \end{aligned}$$

The constitutive relationships for the sections are obtained simply by integrating the appropriate uni-axial constitutive model over each cross-section. For a linearly elastic material, these relationships are as follows:

$$\begin{aligned}
 (9) \quad & M_x = M_{cx} + M_{sx}, \quad M_y = M_{cy} + M_{sy}, \\
 (10) \quad & Q_x = Q_{cx} + Q_{sx}, \quad Q_y = Q_{cy} + Q_{sy}, \quad N = N_c = -N_s, \\
 (11) \quad & MN_x = -N_h, \\
 (12) \quad & m_x = q_u h, \\
 (13) \quad & EI_x = E_c I_{cx} + E_s I_{sx}, \\
 (14) \quad & EI_0 = h^2 / \left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s} \right), \\
 (15) \quad & \frac{GA}{\mu} = \frac{G_c}{\mu_c} + \frac{G_s}{\mu_s}, \\
 (16) \quad & h = h_c + h_s,
 \end{aligned}$$

where

$m_x = M'_{N_x}$;

M_{N_x} = the bending moment carried by the shear force;

EI_0 = bending stiffness of centroid deformation;

w_c = centroid deformation of concrete;

w_s = centroid deformation of steel;

h_s = 1/2 depth of steel;

h_c = 1/2 depth of concrete.

The partial interaction between slab and girder results in interface slip; based on the hypothesis that the slab and the girder have the same deflection along their axes, the shear force q_u may be expressed as follows [24]:

$$(17) \quad q_u = k_u S = k_u (w_s - w_c + (h_c + h_s) y'),$$

$$(18) \quad k_u = K/u,$$

where

K = slip shear stiffness of each shear stud;

k_u = equivalent shear stiffness per unit length;

u = spacing of stud connectors;

S = slip between steel and concrete.

Equilibrium of the rotation angle w' for axis deformation gives:

$$(19) \quad w' = (w_c - w_s)/h.$$

Combining Eqs. (17) and (19) yields:

$$(20) \quad m_x = C^*(y' - w'),$$

$$(21) \quad C^* = k_u h^2.$$

Combining Eqs. (5) and (6) yields:

$$(22) \quad -EI_x y''' = Q_y - h q_z.$$

Combining Eqs. (7) and (8) yields:

$$(23) \quad w''' = -m_x/EI_0.$$

Considering Eq. (23), equation (20) was differentiated twice to give:

$$(24) \quad m_x'' = C^* y''' + C^* \frac{m_x}{EI_0}.$$

Combining Eqs. (22) and (24) yields:

$$(25) \quad \frac{d^6 y}{dt_*^6} - \beta^2 \frac{d^4 y}{dt_*^4} = -\frac{\beta(\alpha - 1)L^4}{EI_x \alpha} q_y,$$

$$(26) \quad \beta = L\sqrt{(C^* \alpha / (EI_x))},$$

$$(27) \quad \alpha = 1 + EI_x / EI_0,$$

where:

- β_z = shear connection degree coefficient;
- α_z = steel and concrete composite degree coefficient.

4. Slips at interface subjected to a uniformly distributed load

Uniform load q_y was assumed within the current calculation model for a simply supported box beam. Solving Eq. (25):

$$(28) \quad y = C1 + C2t_* + C3t_*^2 + C4t_*^3 + Ash\beta t_* + Bch\beta t_* + \frac{(\alpha - 1)L^4}{24EI_x\alpha} q_y t_*^4,$$

where:

- $t_* = z/L$;
- $C1, C2, C3, C4, A,$ and B are constants for general solution of the equation.

Boundary conditions of the simply supported composite box beams were recommended according to structural characteristics as:

$$(29) \quad \left\{ \begin{array}{l} y = 0, \quad \text{when } t_* = 0 \\ -EI_x y'' = 0, \quad \text{when } t_* = 0 \\ -EI_o w'' = 0, \quad \text{when } t_* = 0 \\ y = 0, \quad \text{when } t_* = 1 \\ -EI_x y'' = 0, \quad \text{when } t_* = 1 \\ -EI_o w'' = 0, \quad \text{when } t_* = 1 \end{array} \right. .$$

The above constants can be written in terms of the boundary conditions as

$$(30) \quad C_1 = -\frac{L^4 q_y}{\beta^4 EI_x \alpha},$$

$$(31) \quad C_2 = \frac{L^4 q_y}{24 EI_x \alpha} (\alpha + \frac{12}{\beta^2} - 1),$$

$$(32) \quad C_3 = -\frac{L^4 q_y}{2 EI_x \alpha \beta^2},$$

$$(33) \quad C_4 = -\frac{L^4 (\alpha - 1) q_y}{12 EI_x \alpha},$$

$$(34) \quad A = \frac{(1 - \text{ch } \beta) L^4 q_y}{\beta^4 EI_x \alpha \text{ sh } \beta},$$

$$(35) \quad B = \frac{L^4 q_y}{\beta^4 EI_x \alpha}.$$

The relationship between deflection and interface-slip gives:

$$(36) \quad w' = \frac{EI_o}{C^*} \left(y''' - \frac{EI_x}{C^*} y^{(5)} - \frac{q'_y}{C^*} \right) + y'.$$

By combining Eqs. (28), (30–35), and (37), the slip may be determined as:

$$(37) \quad S = \frac{L^3 h q_y}{\beta^3 EI - x \operatorname{sh} \beta} \left[\operatorname{ch} \beta - \operatorname{ch} \beta (1 - t_*) - \frac{\beta}{2} (2t_* - 1) \operatorname{sh} \beta \right],$$

where q_y = uniform load at the composite box beam.

5. Slips at interface subjected to concentrated load

The composite box beam was divided into two parts (with lengths L_1 and L_2) to facilitate structure analysis, under concentrated load, according to the location of the concentrated force. The physical equation for the composite box girder under a concentrated load based on the physical equation (25) for composite box girders under uniform load, was proposed:

$$(38) \quad \frac{d^6 y}{dt_*^6} - \beta^2 \frac{d^4 y}{dt_*^4} = 0.$$

The calculation for a composite box girder subjected to concentrated load was divided into two parts, one for each part of the beam using the above equation in the current calculation model for a simply supported box beam. When $0 \leq z \leq L_1$, the deflections can be obtained as a general solution to the above equation:

$$(39) \quad y_1(t) = C_1 + C_2 t_* + C_3 t_*^2 + C_4 t_*^3 + C_5 \operatorname{sh}(\beta t_*) + C_6 \operatorname{ch}(\beta t_*),$$

where:

C_1 to C_6 are constants for the general solution of the equation and $t_* = z/L_1$.

Also, Eq. (25) may be solved routinely to produce the deflections for the situation where $L_1 \leq z \leq L_1 + L_2$:

$$(40) \quad y_2(t_*) = d_1 + d_2(1 - t_*) + d_3(1 - t_*)^2 \\ + d_4(1 - t_*)^3 + d_5 \operatorname{sh}(\beta(1 - t_*)) + d_6 \operatorname{ch}(\beta(1 - t_*)),$$

where:

d_1 to d_6 are constants for general solution of the equation and $t_* = z/(L_1 + L_2)$.

Therefore, boundary conditions for the simply supported composite box beams (when $0 \leq z \leq L_1$) were chosen based on the structural characteristics:

$$(41) \quad \begin{cases} y_1 = 0, & \text{when } t_* = 0 \\ -EI_x y_1'' = 0, & \text{when } t_* = 0 \\ -EI_o w_1'' = 0, & \text{when } t_* = 0 \\ y_1' = w_1', & \text{when } t_* = t_o \\ -EI_x y_1'' = 0, & \text{when } t_* = t_o \end{cases} .$$

From these boundary conditions, the constants in eq. (41) were calculated:

$$(42) \quad C_1 = C_3 = C_6 = 0,$$

$$(43) \quad C_4 = -\frac{PL^3(1-t_o)}{6EI_o\alpha},$$

$$(44) \quad C_5 = \frac{PL^3(1-t_o)}{\beta^3 EI_o \alpha (1-\alpha) \text{ch}(\beta t_o)}.$$

The relationship between deflection and interface-slip gives:

$$(45) \quad w_1'(t) = \frac{EI_o}{L^3 C^*} [6C_4 + (1-\alpha)\beta^3 (C_5 \text{ch}(\beta t_*) + C_6 \text{sh}(\beta t_*))] + y_1'.$$

Combining Eqs. (39), (42-44), and (45) and solving gives the slip as:

$$(46) \quad S_1 = \frac{hP(1-t_o)}{\alpha C^*} \left(1 - \frac{\text{ch} \beta t_*}{\text{ch} \beta t_o} \right).$$

Also, the boundary conditions for the simply supported composite box beams when $L_1 \leq z \leq L_1 + L_2$ were:

$$(47) \quad \begin{cases} y_2 = 0, & \text{when } t_* = 1 \\ -EI_x y_2'' = 0, & \text{when } t_* = 1 \\ -EI_o w_2'' = 0, & \text{when } t_* = 1 \\ y_2' = w_2', & \text{when } t_* = t_o \end{cases} .$$

Based on these boundary conditions, the constants of eq. (40) were calculated as:

$$(48) \quad d_1 = d_3 = d_6 = 0,$$

$$(49) \quad d_4 = \frac{PL^3 t_o}{6EI_o \alpha},$$

$$(50) \quad d_5 = -\frac{PL^3 t_o}{\beta^3 EI_o \alpha (1-\alpha) \text{ch}(\beta(1-t_o))}$$

The relation between deflection and interface-slip gives:

$$(51) \quad w'_2(t) = -\frac{EI_o}{L^3 C^*} [6d_4 + (1 - \alpha)\beta^3(d_5 \operatorname{ch}(\beta(1 - t_*))) + d_6 \operatorname{sh}(\beta(1 - t_*))] + y'_2.$$

Combining Eqs. (47), (48–50), and (51) may be solved routinely to produce the slip as:

$$(52) \quad S_2 = \frac{hPt_o}{\alpha C^*} \left[1 - \frac{\operatorname{ch} \beta(1 - t_*)}{\operatorname{ch} \beta(1 - t_o)} \right],$$

where:

P = total load at the composite box beam;

t_o = acting position of total load;

y_1 = deflection for left side of the simply supported composite box beams;

y_2 = deflection for right side of the simply supported composite box beams.

6. Numerical simulations

In this paper, the explicit FE code ANSYS (version 10) was used to predict the response of the composite box beams subjected to different loading situations. The material constants used in the simulation are given in Table 1. The program offers a wide range of options regarding element types,

Table 1. Material constants

Material	Property	Value
Concrete	Young's modulus (MPa)	36 600
	Poisson's ratio	0.2
	Compressive strength (MPa)	68
	Tensile strength (MPa)	5
Steel beam	Young's modulus (MPa)	210 000
	Poisson's ratio	0.3
	Yield stress (MPa)	355
	Ultimate strength (MPa)	560
Reinforced steel bar	Young's modulus (MPa)	210 000
	Poisson's ratio	0.3
	Yield stress (MPa)	495
	Ultimate strength (MPa)	580

material behaviour and numerical solution controls, as well as graphical user interfaces, auto-meshers, and sophisticated postprocessors and graphics to speed the analyses. The finite element types considered in the model are as follows: elastic-plastic shell (SHELL181) and solid (SOLID65) elements for the steel section and the concrete slab, respectively, and nonlinear springs (COMBIN39) to represent the shear connectors. The element SHELL181 is defined by four nodes having six degrees of freedom at each node. The element SOLID65 is used for three-dimensional modelling of solids with or without reinforcing bars. The element has eight nodes and three degrees of freedom (translations) at each node. Spring elements (COMBIN39) were used in the ANSYS model to connect the deck to the top flange of the girder. The element COMBIN39 is defined by two node points and a generalized force–deflection curve. In the finite element model, there were 11360 elements in the solid, with each unit size at most $10 \times 20 \times 75$ mm; there were 1000 plate shell units, each floor unit size is 50×75 mm, and web unit size is 45×75 mm; there were 80 spring units, and the relationship between spring tension and slip, as given by equation (58), was input into ANSYS. The load conditions included uniformly distributed load $q = 100$ kN/m and concentrated load $P = 300$ kN, and the loaded area was 100×100 mm. Then, static analysis of the composite box girders was carried out by applying the model with the appropriate loading, and boundary conditions and solving. The results were then subjected to post-processing and analysis steps. Numerical simulation methods especially finite element method, play an important role in the analysis of the mechanical behaviour of steel and concrete composite structures. To obtain more information about the mechanical performance of the composite box beams, the materials used in the simulations possessed nonlinear and inelastic characteristics, although the proposed analytical solution is based on a linear elastic analysis. In order to compare with the linear elastic analytic results, in the current analysis, only the elastic analysis results are put forward as evidence to support the proposed analytical solution. The nonlinear and inelastic characteristics will be discussed in future work.

7. Material properties

The stress–strain relationship in compression is not of primary interest since the bending behaviour of RC beams subjected to monotonic loading is much more affected by the tensile rather than by the compressive behaviour of concrete. Among the numerous mathematical models currently used in the analysis of RC structures, the monotonic envelope curve introduced by Kent and Park [25] and later extended by Scott et al. [26] is adopted in this paper

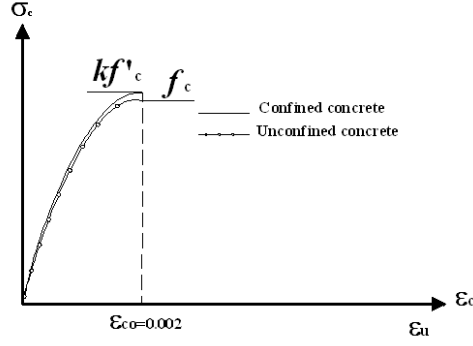


Fig. 2. Stress-strain relationship for concrete

because of its simplicity and computational efficiency. In this model, as shown in Fig. 2, the monotonic concrete stress-strain relationship in compression is described by three regions:

$$(53) \quad \sigma_c = kf'_c \left[2 \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right) - \left(\frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right], \varepsilon_c \leq \varepsilon_{c0},$$

$$(54) \quad \sigma_c = kf'_c [1 - Z_i(\varepsilon_c - \varepsilon_{c0})], \varepsilon_{c0} \leq \varepsilon_c \leq \varepsilon_u,$$

$$(55) \quad \sigma_c = 0.2kf'_c, \varepsilon_c \geq \varepsilon_u,$$

where:

$$(56) \quad \varepsilon_{c0} = 0.002k, \quad k = 1 + \frac{\rho_s f_{yh}}{f'_c},$$

$$(57) \quad Z_i = \frac{0.5}{\frac{3 + 0.0284f'_c}{14.21f'_c - 1000} + 0.75\rho_s \sqrt{\frac{h'}{s_h}} - 0.002k} \quad (i = 1, 2),$$

where ε_{c0} is the concrete strain at maximum stress, k is a factor which accounts for the strength increase due to confinement, Z_i is the strain softening slope, f'_c is the concrete compressive strength in kg/cm^2 ($1 \text{ kg}/\text{cm}^2 = 0.098 \text{ MPa}$), f_{yh} is the yield strength of the stirrups in kg/cm^2 , ρ_s is the ratio of the volume of hoop reinforcement to the volume of the concrete core measured to the outside of the stirrups, h' is the width of the concrete core measured to the outside of the hoops or ties, and s_h is the center to center spacing of tie or hoop sets.

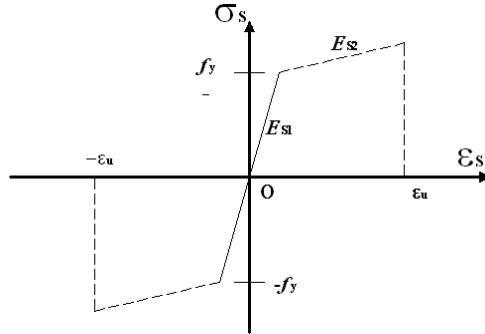


Fig. 3. Stress-strain relationship for steel

In the case of RC beams whose behaviour is greatly dominated by bending. The value of ϵ_o is derived from the fracture mechanics concept by equating the crack energy release with the fracture toughness of concrete G_f . The experimental study by Welch and Haismen [27] indicates that for normal strength concrete, the value of G_f/f'_t is in the range of 0.005–0.01 mm. ϵ_o can be determined if G_f and f'_t are known from the measurements. In addition, reinforcing steel is modelled as a linear elastic, linear strain hardening material with yield stress f_y , as shown in Fig. 3. Only the linear elastic constitutive relation of materials in Figs 2–3 are given in order to compare with the linear elastic analytic results, in the current analysis.

8. Load-slip relationship

The flexural and slip behaviour of these composite beams is greatly influenced by the shear connectors, which are characterized by their ductility and stiffness since composite beams are equipped with shear connectors between the concrete slab and girder to unify the behaviour of the total structure. Usually, the static behaviour of the shear connectors, which governs the slip behaviour at the interface, can be explained through the shear stiffness in the elastic region, and the ultimate shear strength and the corresponding ultimate slip are measured by a push-out test. Many tests have been performed to obtain and predict the slip behaviour of shear connectors. A typical load-slip curve was introduced [28]. The expression applied here for the force-slip curve is:

$$(58) \quad F = (1 - e^{-0.7s})^{0.4}[F],$$

where $[F]$ = ultimate horizontal shear; F = horizontal shear; and S = slip

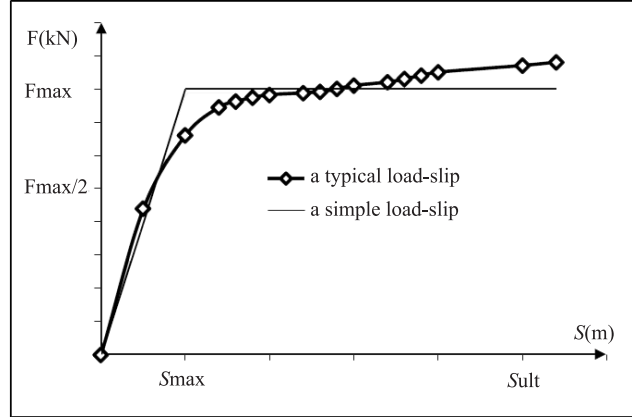


Fig. 4. Simple load-slip curve of a shear connector

between steel and concrete. In addition, the equivalent ultimate horizontal shear $[F]$ can be calculated from the following equation:

$$(59) \quad [F] = 0.373d^2 \sqrt{E_c f_c} \leq 1.005A_d f_d,$$

where d = stud diameter; A_d = cross-sectional area of stud; E_c = modulus of the concrete; f_d = ultimate tensile strength of stud; and f_c = axial compressive strength of concrete. A typical load-slip curve determined from Eq. (58) is shown in Fig. 4.

9. FE formulation

A composite box beam, 3000 mm long, of steel-concrete cross-section with partial interaction, and 150 mm and 450 mm pitches of shear connectors (D1 = 150 mm, D3 = 450 mm) was considered in this analysis. The FE model of the composite box beam is shown in Fig. 5(a, b). A shear connector was introduced between the steel and concrete. Theoretical solutions and numerical results for the slip and deflection are shown in Figs 6–7.

As the theoretical values showed good agreement with the numerical results, the proposed formulas can be reliably applied to the analysis and design of steel-concrete composite box beams, except that for a uniform load, the theoretical solution is a little higher than the numerical prediction. The main reason for this is because the distributions of shear-force for a simply supported beam subjected to concentrated load are not well understood, and thus this requires further study. A test plan will be conducted to modify the

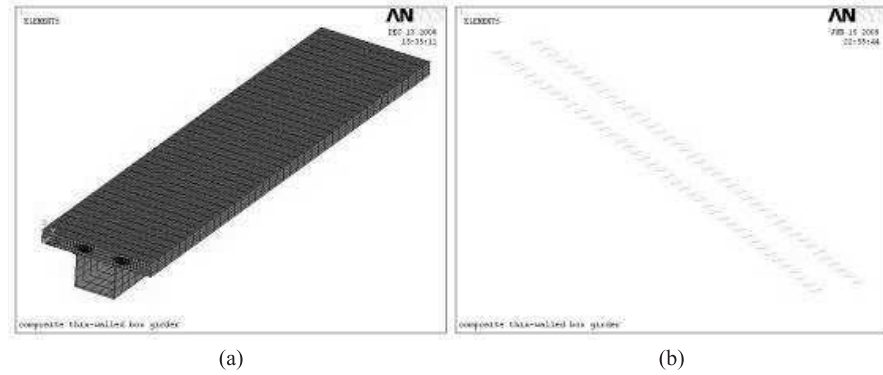


Fig. 5. Finite element model of simply supported beams:
 (a) Whole finite element model; and (b) Slip model

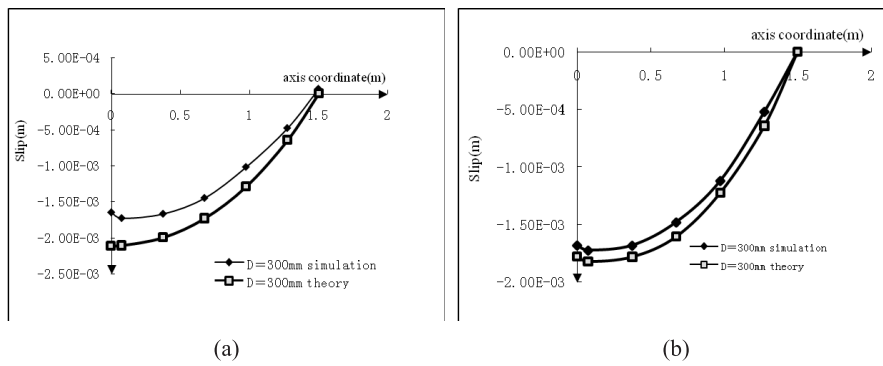


Fig. 6. Interface slip distribution curves ($D_2 = 300$ mm)
 (a) Subjected to concentrated load; (b) Subjected to uniform load

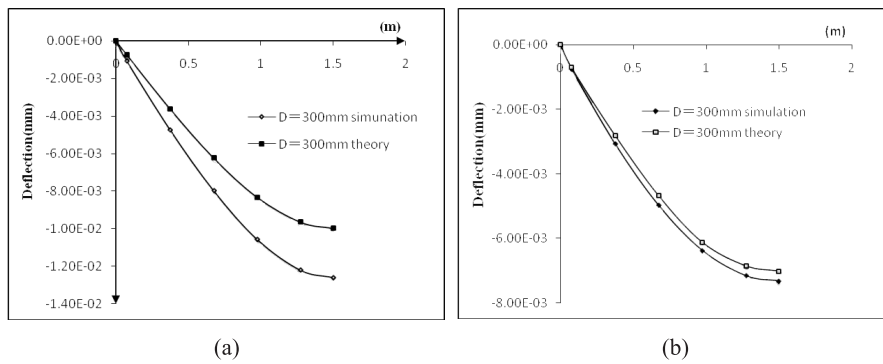


Fig. 7. Interface deflection distribution curves ($D_2 = 300$ mm):
 (a) Subjected to concentrated load; (b) Subjected to uniform load

theoretical solution mentioned above, and the results of these modifications will be introduced in another paper.

10. Conclusions

The equations to be used in this new approach are (17), (19), and (20). They are derived from the solution for a simply-supported beam with concentrated load or uniformly distributed load. In addition, the commercial finite element program ANSYS can simulate the interfacial interaction behaviour under concentrated load or uniformly distributed load. According to the ANSYS combin39 element, it was possible to simulate the interfacial interaction behaviour. The analysis and simulation model can be used effectively in determining the structural response of composite beam structures when it represents partial bond-slip with an increase in loading because ignoring the bond-slip effect may cause overestimation of the resisting capacity of structures. The validity of the introduced analysis model was verified through correlation studies between analysis and simulation results and its importance can be found from the efficiency in all kinds of structure forms and a variety of complex forms of the supporting and load, and we can also take the homogeneous solution of the analytical equation as a displacement function of composite beam finite element model. At the same time, the proposed method was also extended to analyze the deflection, internal force and dynamic characteristics etc of steel-concrete composite box beam subjected to different load.

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