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STRUCTURAL OPTIMIZATION OF LINEAR VIBRATION ISOLATION SYSTEMS

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ABSTRACT: This article addresses the problem of structural optimization of linear vibration isolation systems in various actions. A generalized criterion is formulated. The criterion reduces the problem of finding the conditional extremum of the functionality to the finding of the absolute extreme. Equations for the generalized criterion are obtained by the transfer function for deterministic, arbitrary kinematic and force actions. The optimal transfer function of the vibration isolation system is determined in the process of optimal synthesis. The solution to the problems of optimal synthesis for kinematic and random effects is given in the form of “white noise” and an exponentially correlated process, as well as by determined harmonic kinematic impact.

KEY WORDS: vibration protection, optimization, functionality extremum, transfer function, dynamic system.

1 INTRODUCTION

The development of high-performance machines and high-speed vehicles forced by power, loads and other performance characteristics, inevitably leads to increasing the intensity, expansion of the spectrum of vibration, and vibration acoustic fields. Harmful vibration violates the laws of movement of machines, mechanisms and control systems planned by the designer, causes instability of working processes and can cause a failure and complete detuning of the entire system. Developing effective tools of protection against vibration and shock is one of the important problems of the present day technology.

The development of the theory of vibration isolation began with solving the problems of vibration isolation of equipment in linear formulation. The main assumptions of this theory can be found in work [1]. Vibration isolation systems are often exposed to separate or repetitive shock pulses. Short duration of shock processes and non-stationary of oscillations necessitates the direct integration of the equation of movement with considering the initial conditions. Extensive material on the results of calculating linear vibration isolation systems for shock loads is given in works [2, 3].

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In a lot of practically important cases, disturbing effects are random. These are the loads on vehicles: cars, ships, airplanes, and others. This led to the need of using statistical methods for solving the problems of vibration isolation [4]. Entire departments of institutes work on the problems of vibration isolation of equipment. The results of the joint work of Soviet and Czechoslovak scholars have been published [5].

Despite the completeness of theory of linear vibration isolation, research in this field continues, as it is evident by [6, 7] and recent articles [8–13].

Recently, the need of developing methods of parametric optimization and optimal synthesis of vibration isolation systems has become increasingly obvious. This is due to the interrelatedness of the parameters and the contradictory requirements for the design of vibration isolation systems. A number of monographs are dealing with these questions, for example [14, 15], as well as a number of articles [16, 17]. The need of solving optimization problems of equipment for vibration isolation in turn contributed to automation of calculations and design of vibration isolation systems. Recent works in this field are [18] and [19]. They consider mathematical models, methods, algorithms, and tools for solving the problems of conceptual design of vibration isolation systems.

Many researchers attribute the problem of improving the quality and efficiency of vibration isolation to solving optimization problems [20–24]. Most of these works are dealing with selecting the optimal parameters of the vibration isolation system according to one of the work criteria. At the same time, the tasks of designing vibration insulation systems are always multi-criteria, and some of the criteria are the opposite of changes compared to system parameters. Therefore, for vibration isolation systems, the issues of parametric optimization with the selected structure and optimal synthesis are relevant.

The formulation of optimal synthesis problems is associated with the controversial pattern of the requirements imposed on vibration isolation systems. The limitations on these systems are imposed on the absolute accelerations of the points of the vibration-isolating object (kinematic action) or on the power transmitted from the object to the base (power action):

$$A \le \bar{A}\,,$$

where A is the functionality from the rated parameter of vibration r.

With insufficient rigidity of vibration isolators, dynamic effects can cause their large relative displacements, which leads to increasing the overall dimensions of vibration isolators. Therefore, vibration isolation can become practically unfeasible. In the presence of rigid stops, these movements lead to impacts on the supports ("breakdown" of the suspension) and the effect of vibration isolation will be lost. Therefore, restrictions are also imposed on the relative movement of the object and the base:

$$B \leq B_0,$$

where B is the functionality from the relative movement δ.

The restrictions mentioned have an opposite nature: the decreasing of the A functionality leads to the increase of the B functionality. The optimal synthesis problem is reduced to determining the conditional extremum:

$$A = A_{\min}\,, \quad B \leq B_0\,.$$

The problem of determining the conditional extremum can be reduced to finding the unconditional extremum by means of introducing a generalized criterion of the form:

$$C = A + \rho B\,, \tag{1}$$

where ρ has the meaning of the Lagrange multiplier.

The solution of the problem consists in determining the minimum of the C functionality and building the structure of the vibration isolation system that implements the resulting functionality. These issues receive little attention in the studies.

The purpose of this work is an analytical solution of the problem of optimal synthesis of linear vibration isolation systems according to criterion (1) with deterministic and random effects.

2 Methodology of Solving the Problem

The following values can be selected as functionalities A and B:

- under impact effects in the form of separate pulses:

$$A = \max|r(t)|\,, \quad B = \max|\delta(t)|\,;$$

- under deterministic oscillations:

$$A = \int_0^\infty r^2(t)dt\,, \quad B = \int_0^\infty \delta^2(t)dt\,; \tag{2}$$

- under random oscillations:

$$A = \sigma_r^2\,, \quad B = \sigma_\delta^2\,, \tag{3}$$

where σ^2 is dispersion of the corresponding process.

The solution of the optimal synthesis problem is as follows. A set of values ρ is given, for each value generalized criterion (1) is minimized. For this purpose, special methods are used to search for the extremum of functionalities. As a result of solving the problem of the unconditional extremum, a pair of optimal values A_{opt}, B_{opt} is found, and there is built the dependence of these values on the ρ parameter. Accepting $B_{\text{opt}} = B_0$ according to the charts, A_{opt} and the corresponding ρ are found. In the general case, this problem is solved numerically.

For linear vibration isolation systems, the optimal synthesis problem can be solved analytically.

Let us consider the problem of optimal synthesis of one-dimensional linear vibration isolation systems according to criterion (1) with deterministic and random vibrations. For the integral quadratic functionality of the function is valid the Parseval formula:

$$\int_0^\infty x^2(t)dt = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} x(p)x(-p)dp\,, \tag{4}$$

where $p = i\omega$, and $x(p)$ is the image of the function $x(t)$ according Fourier.

For the random process dispersion $x(t)$ is valid a private case of the Wiener-Khinchin formula:

$$\sigma_x^2 = \frac{1}{i}\int_{-i\infty}^{i\infty} S_x(p)dp\,, \tag{5}$$

where $S_x(p)$ is the spectral density of the $x(t)$ process.

In the course of optimal synthesis, the optimal transfer function $W(p)$ of the vibration isolation system is found, providing the best quality in the class of linear systems according to the selected quadratic criterion. This function is found by solving the Wiener-Hopf equation [25]. This equation can be obtained from the condition that the variation of functionality (1) on $W(p)$ is equal to zero.

Functionalities included in criterion (1) are linearly associated with the input effects. For example, for kinematic vibration isolation when transmitting vibration from an input point movement $U(p)$ to an output point movement$X(p)$, we obtain the following expression for the relative movement:

$$\delta(p) = U(p) - X(p) = [1 - W(p)]U(p)\,. \tag{6}$$

The expression for accelerating the output point will take the form:

$$p^2X(p) = p^2W(p)U(p)\,. \tag{7}$$

Using formulas (4) and (7), we obtain the explicit expressions for criterion (1). For deterministic action, when the functionalities are calculated as (2), we have:

$$C = \frac{1}{2\pi i} \int\limits_{-i\infty}^{i\infty} \{p^4 W(-p)W(p) + \rho[1 - W(-p)][1 - W(p)]\} U(-p)U(p)dp\,. \tag{8}$$

For random action, when functionalities are calculated as (3), we obtain:

$$C = \frac{1}{i} \int\limits_{-i\infty}^{i\infty} \{p^4 W(-p)W(p) + \rho[1 - W(-p)][1 - W(p)]\} S_U(p)dp\,. \tag{9}$$

In case of power vibration isolation, the transfer function is found as the ratio of the power in the vibration isolator $R(t)$ to the power acting on the object (input) $F(t)$, that is:

$$W(p) = R(p)/F(p)\,.$$

Then the equation of the movement of the object with the mass "m" in the image space has the form:

$$mp^2 X(p) + W(p)F(p) = F(p)\,.$$

From here we have:

$$X(p) = [1 - W(p)]F(p)/mp^2\,.$$

Using these expressions, we can write expressions for the generalized criterion:
(10)

$$C = \frac{1}{2\pi i} \int\limits_{-i\infty}^{i\infty} \{W(p)W(-p) + (\rho/m^2p^4)[1 - W(-p)][1 - W(p)]\} F(p)F(-p)dp\,.$$

Expression (10) is for deterministic action.

$$C = \frac{1}{i} \int\limits_{-i\infty}^{i\infty} \{W(p)W(-p) + (\rho/m^2p^4)[1 - W(-p)][1 - W(p)]\} S_F(p)dp\,. \tag{11}$$

Expression (11) is for random action.
Let's introduce:

$$C = \frac{1}{i} \int\limits_{-i\infty}^{i\infty} I(p)dp\,,$$

where $I(p)$ is an expression under integral sign.

Let's derive a derivative from the expression under the integral sign (9):

$$\frac{\partial I(p)}{\partial W(-p)} = [(p^4 + \rho)W(p) - \rho]S_U(p)\,.$$

Let's denote:

(12) $$\Delta(p) = (p^4 + \rho)S_u(p)\,, \quad \Gamma(p) = \rho S_U(p)\,.$$

Then:

(13) $$\frac{\partial I(p)}{\partial W(-p)} = \Delta(p)W(p) - \Gamma(p)\,.$$

Similarly, for the power effect we will obtain (13), where:

(14) $$\Delta(p) = (1 + \rho/m^2p^4)S_F(p)\,, \quad \Gamma(p) = \rho S_F(p)/m^2p^4\,.$$

The fractional rational function $\Delta(p)$ is presented in the form of two multipliers:

$$\Delta(p) = \Delta^+(p)\Delta^-(p)\,,$$

where $\Delta^+(p)$ contains zeros and poles only in the left semi-plane, and $\Delta^-(p)$ – only in the right semi-plane. This operation is called spectrum factoring.

Let us equate (13) to zero and write down its solution in the form:

(15) $$W(p) = \frac{1}{\Delta^+(p)}\left[\frac{\Gamma(p)}{\Delta^-(p)}\right]_+\,,$$

where $\left[\frac{\Gamma(p)}{\Delta^-(p)}\right]_+$ is a part of expression $\frac{\Gamma(p)}{\Delta^-(p)}$, containing poles only in the left semi-plane.

Let's present the external effect in the factorized form:

$$S(p) = \phi(p)\phi(-p)\,.$$

Then the presentation $\Delta(p)$ will take the form:

$$\Delta^+ = (p^2 + \sqrt{2}\omega_c p + \omega_c^2)\phi(p)\,; \quad \Delta^- = (p^2 - \sqrt{2}\omega_c p + \omega_c^2)\phi(-p)\,,$$

(16) $$\omega_c = \sqrt[4]{\rho}\,,$$

and the optimal transfer function:

$$W(p) = \frac{1}{\Delta^+(p)} \left[\frac{\omega_c^4 \phi(p)}{p^2 - \sqrt{2}\omega_c p + \omega_c^2} \right]_+ . \tag{17}$$

This expression is also valid for criterion (8), if $\phi(p)$ is substituted by $U(p)$.

Let us consider the solution of the power vibration isolation problem. In this case, taking into account (14):

$$\Delta^+ = \frac{1}{p^2}(p^2 + \sqrt{2}\omega_c p + \omega_c^2)\phi(p), \quad \omega_c = \sqrt[4]{\rho/m^2}\,, \tag{18}$$

$$\frac{\Gamma(p)}{\Delta^-(p)} = \frac{\omega_c^4 \phi(p)}{p^2(p^2 - \sqrt{2}\omega_c p + \omega_c^2)} . \tag{19}$$

For deterministic criterion (10) $\phi(p)$ in these expression must be substituted by $F(p)$.

Then, by the form of the obtained transfer function a block diagram of the dynamic system of vibration isolation is constructed, consisting of standard links connected in an appropriate way.

The following standard links are distinguished:

- aperiodic link:
$$W_{(p)} = \frac{a_0}{p + q_0}\,;$$
- oscillatory link:
$$W_{(p)} = \frac{a_0}{p^2 + q_1 p + q_0}\,; \quad \frac{q_1}{2\sqrt{q_0}} < 1\,;$$
- integrating link:
$$W_{(p)} = \frac{a_0}{p}\,;$$
- differentiating link:
$$W_{(p)} = a_1 p\,;$$
- delay link:
$$W_{(p)} = a_0 e^{-\tau p}\,.$$

There is serial, parallel connection of links and systems with feedback (closed systems). When the links are connected in series, the transfer function of the system is equal to the product of the transfer functions of the links.

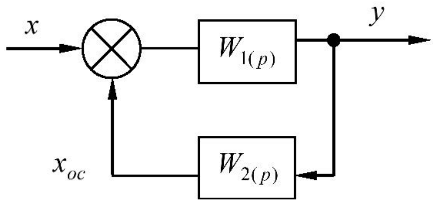

Fig. 1. Connection of the links with feedback.

When connecting the links in parallel, the transfer function of the system is equal to the sum of the transfer functions of the links.

In the systems with feedback (Fig. 1) described by the equations:

$$y = W_{1(p)}\left(x \pm x_{oc}\right) ,$$

$$x_{oc} = W_{2(p)} y ,$$

the transfer function is determined from the operator solution of the equation system and is equal:

$$W_{(p)} = \frac{W_{1\,(p)}}{1 - W_{1\,(p)} W_{2\,(p)}} .$$

By appropriate selection of standard links and links between them, it is possible to simulate any complex dynamic system. The idea of calculating vibration isolation systems by moving from dynamic models to dynamically equivalent structural schemes in order to apply the methods of the theory of automatic control, including simulation, is described in works [26, 27].

In physical implementation of the dynamic model, real standard elements are used. Taken are elastic, dissipative inertial elements, as well as throttle regulators and motion transducers. Their spectral densities give random external effects and for deterministic effects their images are found by the Laplace transformation.

3 Applications to the Vibration Isolation Systems

Let us consider the examples of determining the optimal transfer function. Let the object is affected by a random power of the "white noise" type $S_F(p) = s_0/2\pi$. The factorized values of the input spectrum are recorded as follows:

$$\phi(p) = 1, \phi(-p) = s_0/2\pi .$$

Let us expand (19) to fractions:

$$\frac{\omega_c^4}{p^2(p^2 - \sqrt{2}\omega_c p + \omega_c^2)} = \frac{A}{p^2} + \frac{B}{p} + \frac{Cp + D}{p^2 - \sqrt{2}\omega_c p + \omega_c^2} .$$

In this expression in the left semi-plane there are poles relating only to the first two members. From the previous equality we have:

$$A = \omega_c^2 , \quad B = \sqrt{2}\omega_c .$$

Then:

$$\left[\frac{\Gamma(p)}{\Delta^-(p)}\right]_+ = \frac{\sqrt{2}\omega_c p + \omega_c^2}{p^2} .$$

Finally we obtain:

$$W(p) = \frac{\sqrt{2}\omega_c p + \omega_c^2}{p^2 + \sqrt{2}\omega_c p + \omega_c^2} .$$

This optimal transfer function describes a linear vibration isolation system containing a parallel spring and a damper. In this case:

$$W(p) = \frac{bp + c}{m^2 p + bp + c} = \frac{2\varepsilon p + \omega_0^2}{p^2 + 2\varepsilon p + \omega_0^2} ,$$

where c, b are characteristics of the spring and damper; $\varepsilon = b/m$ is the coefficient of oscillation damping; $\omega_0 = \sqrt{c/m}$ is the frequency of own oscillations.

From these expressions we obtain:

$$\omega_0 = \omega_c \quad \text{or} \quad c = \sqrt{\rho}; \quad b = \sqrt{2cm} .$$

The optimal value of the relative damping is:

$$n = \varepsilon/\omega_0 = 1/\sqrt{2} .$$

If we consider kinematic vibration isolation with the input effect of the “white noise” type on acceleration, then $S_U(p) = s_0/2\pi p^4$. Assuming that:

$$\phi(p) = 1/p^2 , \quad \phi(-p) = s_0/2\pi(-p)^2 ,$$

using formulas (16) and (17), we will obtain the same optimal transfer function.

Let us consider vibration isolation when the base is moving according to the harmonic law

$$u = \frac{a}{\omega^2} \sin \omega t ,$$

where is the amplitude of the base acceleration.

In this case:

$$U(p) = \frac{a}{\omega(p^2 + \omega^2)} ; \quad \frac{\Gamma(p)}{\Delta^-(p)} = \frac{\omega_c^4 a}{\omega} \left[\frac{Ap + B}{p^2 + \omega^2} + \frac{Cp + D}{p^2 - \sqrt{2}\omega_c p + \omega_c^2}\right] .$$

In the second expression in the left semi-plane there are poles relating only to the first summand. From this expression we find:

$$A = \frac{\sqrt{2}\omega_c}{\omega_c^4 + \omega^4}\,, \quad B = \frac{\omega_c^2 - \omega^2}{\omega^4 + \omega_c^4}\,.$$

Then the optimal transfer function will take the form:

$$W(p) = \frac{\omega_c^4}{\omega_c^4 + \omega^4} \cdot \frac{\sqrt{2}\omega_c p + \omega_c^2 - \omega^2}{p^2 + \sqrt{2}\omega_c p + \omega_c^2}\,.$$

Physical implementation of this vibration isolation system is shown in Fig. 2. Here the system parameters are determined as follows:

$$\frac{\omega_c^4}{\omega_c^4 + \omega^4} = \frac{e}{d} = \bar{e} < 1 \quad \text{or} \quad \omega_c^4 = \rho = \omega^4 \frac{\bar{e}}{1 - \bar{e}}\,;$$

$$\omega_0 = \sqrt{c/m}/\bar{e} = \omega_c\,; \quad b/m\bar{e}^2 = \sqrt{2}\omega_c\,.$$

If we set $\bar{e}$, then

$$c = m\omega^2\bar{e}^2\sqrt{\frac{\bar{e}}{1 - \bar{e}}}\,; \quad b = \sqrt{2}m\omega\bar{e}^2\sqrt{\frac{\bar{e}}{1 - \bar{e}}}\,.$$

Let us consider a random power effect with the spectral density:

$$S_F(p) = \sigma^2\alpha/[\pi(\omega^2 + \alpha^2)]\,.$$

That is presented in the factorized form:

$$\phi(p) = \frac{1}{\alpha + ip}\,, \quad \phi(-p) = \frac{\sigma^2\alpha}{\pi(\alpha - ip)}\,.$$

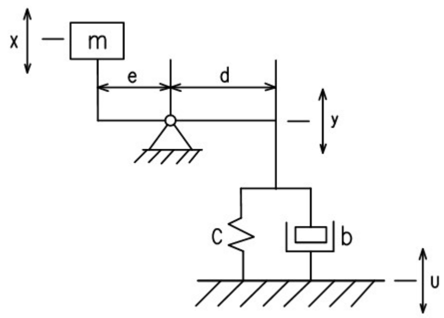

Fig. 2. Vibration isolation system with the optimal transfer function under harmonic action.

According to expression (19):

$$\frac{\Gamma(p)}{\Delta^{-}(p)} = \frac{\omega_c^4}{(\alpha+ip)p^2(p^2-\sqrt{2}\omega_c p+\omega_c^2)} == \omega_c^4(\frac{A}{p^2}+\frac{B}{p}+\frac{E}{\alpha+ip}+\frac{Cp+D}{p^2-\sqrt{2}\omega_c p+\omega_c^2}).$$

Here in the left semi-plane there are radicals relating to the first three summands. From here we will determine the corresponding constants:

$$A = \frac{1}{\alpha\omega_c^2,}, \quad B = \frac{\sqrt{2}\alpha - i\omega_c}{\alpha^2\omega_c^3}, \quad E = -\frac{\omega_c^2-\alpha^2+i\sqrt{2}\alpha\omega_c}{\alpha^2(\omega_c^4+\alpha^4)}.$$

Then from expression (15), taking into account (18) and the above-mentioned, we obtain the following optimal transfer function:

$$W(p) = \frac{\lambda p^2+\sqrt{2}\omega_c p+\omega_c^2}{p^2+\sqrt{2}\omega_c p+\omega_c^2}, \tag{20}$$

where $\lambda = \omega_c^2\frac{\omega_c^2-\alpha^2}{\omega_c^4+\alpha^4}$.

Physical implementation of this vibration isolation system contains an additional inertial element with a movement conversion mechanism. The structural diagram of such a system is shown in Fig. 3.

Let us determine the physical parameters of the model. From the equation of the mass movement it follows that:

$$X(p) = \frac{F}{m}\cdot\frac{1}{p^2+2\varepsilon p+\omega_0^2},$$

where $\varepsilon = b/2m$; $\omega_0^2 = c/m$.

The image of the power transferred to the base is equal to:

$$R(p) = (bp+c)X(p) + m_1(d/e)p^2X(p).$$

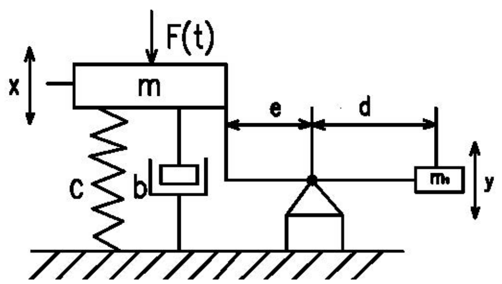

Fig. 3. Vibration isolation system with the optimal transfer function under the exponential-correlated action.

From here the transfer function is:

$$W(p) = \frac{(m_1 d/me)p^2 + 2\varepsilon p + \omega_0^2}{p^2 + 2\varepsilon p + \omega_0^2}\,.$$

Comparing it with (20), we can write down:

$$\omega_c = \omega_0\,, \quad 2\varepsilon = \sqrt{2}\omega_c\,, \quad \lambda = m_1 d/me\,.$$

From here we have:

$$b = \sqrt{2cm} \quad \text{or} \quad n = \varepsilon/\omega_0 = 1/\sqrt{2}\,;$$

$$m_1 = \frac{me}{d}\frac{1-\gamma^2}{1+\gamma^4}\,, \quad \gamma = \alpha/\omega_0\,.$$

4 Conclusion

The obtained theoretical results allow finding the optimal transfer function of a linear vibration isolation system that best satisfies the requirements for the type of actions under consideration. This transfer function is used to construct a block diagram of a vibration isolation system, consisting of standard links connected in an appropriate way. After this, an optimal dynamic model of the vibration isolation system is constructed with indication of standard elements and their interrelations. In this case, external effects can be deterministic or random, of power or kinematic type. The paper presents examples of the optimal synthesis of vibration isolation systems under various effects.

It is possible to generalize the results obtained to a combination of different types of effects. To do this, based on the nature of the effects, it is necessary to formulate a new kind of the generalized criterion and to repeat the procedure for determining the optimal transfer function.

The structural approach to the synthesis of a dynamic system provides great opportunities for expanding the scope of application of the results. The advantages of this approach consist in the possibility of widespread use of simulation modeling using advanced computer technologies, as well as in the possibility of replacing the parameters of the links themselves or entire blocks without changing the calculation procedure. This allows carrying out optimization and design calculations, changing the parameters of the links according to a specific algorithm, or performing structural design, changing the types of links. In this respect, the most effective are interactive CAD systems.

The solution of the optimal synthesis problem for nonlinear vibration isolation systems is performed numerically using special methods of determining the extremum of functionalities, as noted above. Another prospect of the work is extending the analytical solution to nonlinear vibration isolation systems. A possible trend of the studies is seen in the combination of this approach with the linearization of the vibration isolator characteristics. The application of nonlinear standard structural elements also gives certain prospects.

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