Journal of Theoretical and Applied Mechanics, Sofia, Vol.50 (2020) pp. 142-157
INTERDISCIPLINARY TOPICS

HUMAN-INDUCED VIBRATIONS ON FOOTBRIDGES. CURRENT CODES OF PRACTICE – OVERVIEW

KRUMKA KASAPOVA*, DOBROMIR DINEV

University of Architecture, Civil Engineering and Geodesy;
1 Smirnenski Blvd., 1046, Sofia, Bulgaria

[Received: 25 September 2019. Accepted: 03 February 2020]

ABSTRACT: Considering the trend in the last decades of using lightweight materials and, in some sense extraordinary structural configurations, structures have become more and more slender and therefore more susceptible to dynamic excitation. This holds especially for long-span and slender footbridges, since such structures commonly have natural frequencies that fall within the respective frequency range of human excitation.

The present paper regards in details the procedures of the current codes of practice. It may serve as a background for subsequent assessment of the dynamic behaviour of footbridges.

KEY WORDS: Human-induced vibrations, footbridges.

1 INTRODUCTION

The pedestrian structures appear to be quite vulnerable when subjected to human-induced loads. Pedestrians exert different dynamic loads with regard to their type, magnitude and frequency. The current codes of practice which are thoroughly reviewed in this article, adopt simplified procedures for the vibration serviceability assessment of footbridges [1, 2].

Their procedures are reduced to assuming a resonant load condition. A deficiency of the aforementioned regulations is that they disregard the human-human interaction, which may have a substantial effect on the maximum acceleration levels of the structure under consideration [3].

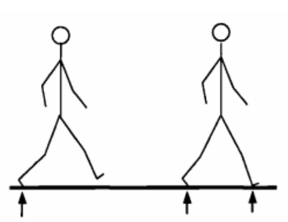
2 HUMAN-INDUCED VIBRATIONS OF SLENDER STRUCTURES

Human motion is a complex combination between translational and rotational motion of various body segments and as such it is hard to describe it without the usage of specialized software tools [4]. Factors influencing human gait and dynamic effects are: pacing frequency, stride length, purpose, surface type, gender, weight etc. but the most important one is the pacing frequency, as it has major influence on the

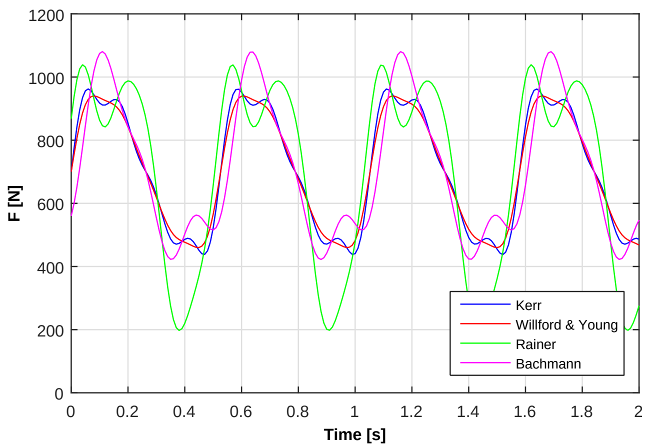
*Corresponding author e-mail: krumka_kasapova@abv.bg

induced forces. Values of the step frequency, pertaining to a wide range, can be found in [5,6]. Depending on circumstances, the environment, human body specificity etc., step frequency can vary from 1.5 Hz to 2.5 Hz or even more.

When walking a person is always in contact with the surface. The two phases of the walking cycle – single support phase and double support phase – are depicted in Fig. 1a. As regards the vertical forces, the motion can be described starting with 'heel strike' (when the heel touches the ground), followed by flat position of the foot and finishing with 'toe-off' (when the toes apply pressure on the floor and the other foot makes contact with the surface). When both legs are in contact with the surface the 'heel strike' of the current step and the 'toe-off' of the previous one result in a strongly pronounced dynamic effect as their impacts are superimposed (Fig. 1b).



(a) Single support phase and double support phase of the gait cycle [5]



(b) Critical frequency ranges in horizontal direction

Fig. 1: Phases of gait cycle and continuous foot force function for $f_s = 1.9$ Hz according to different authors

2.1 Generated forces

The load exerted by a pedestrian on a structure is assumed as a sequence of point forces applied along pedestrian's trajectory. Therefore, the loading forces are represented as a function of both time and current position: $P(x,t) = F(t)\delta(x - vt)$, wherein $F(t)$ – time component of the force, represented as a periodic function (Fig. 1b), $\delta(x - vt)$ – space component, fully-defined by the pedestrian's position with regard to the centerline of the footbridge, the particular time instant t and the velocity of the pedestrian v, assumed to be constant, δ – Dirac function.

The periodic forcing function $F(t)$ is developed as Fourier series up to certain limited harmonic number, since beyond a certain number the significance of the harmonics becomes negligible: $F(t) = G + \sum_{i=1}^{n} G\alpha_i \sin(2\pi i f_s t - \phi_i)$, where G – static component of the force, namely the weight of the pedestrian; α_i – dynamic load factor of the i-th component; f_s – step frequency; ϕ_i – phase shift of the i-th component; n – number of harmonic components accounted for.

Different authors give different formulations and different values for the dynamic load factors. This fact can be easily explained by the huge variability of the determining parameters (step frequency, weight etc.) from one pedestrian to another (inter-subject variability) as well as from one step to another for one and the same human (intra-subject variability).

3 Guidelines and Current Codes of Practice

The currently available codes of practice [7–11], which are relevant for the vibration serviceability analysis of the footbridges provide different simplified procedures for quantifying vibration levels in structures and assessing their impact on the comfort of the pedestrians. A vibration analysis, based on simplified load models (accounting for the effects induced by a crowd or a group) and on assumed comfort levels (which are strictly individual for every pedestrian), may be quite subjective.

3.1 Eurocode 5: Part 2, Appendix B

Although mentioned in a code specialized for design of timber structures, this procedure states a valuable and concise principle for evaluating the maximum accelerations of footbridges in direct dependence on their mass and natural frequency [7]. In vertical and lateral direction, natural frequencies are considered as potentially critical if $f_{\text{vert}} \le 5$ Hz and $f_{\text{lat}} \le 2.5$ Hz, respectively. If not, no further analysis is required. The application range of the proposed procedure is limited to structures with a calculation model corresponding to a simply supported beam.

The vertical accelerations resulting from a single pedestrian passing the bridge can be calculated as follows:

(1) $$a_{\text{vert},1} = \begin{cases} \dfrac{200}{M\xi}, & \text{for } f_{\text{vert}} \le 2.5\,\text{Hz} \\ \dfrac{100}{M\xi}, & \text{for } 2.5\,\text{Hz} < f_{\text{vert}} \le 5\,\text{Hz} \end{cases},$$

where $M = m\ell$ – total mass of the bridge [kg]; m – mass per unit length [kg/m]; ℓ – span of the bridge [m]; ξ – damping ratio; f_{vert} – natural frequency of the first vertical mode of vibrations.

The obtained vertical acceleration due to a single pedestrian is further used as a

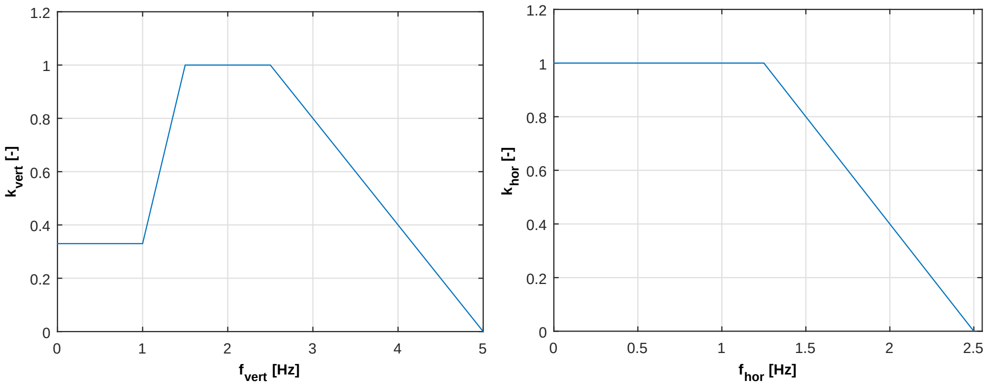


Fig. 2: Relationships between the natural frequency of the first mode of vertical vibrations and the coefficient k_{vert} (left), and the natural frequency of the first mode of lateral vibrations and the coefficient k_{lat} (right)

base for the calculation of the acceleration caused by a group or a crowd

(2) $$a_{\text{vert},n} = 0.23\, a_{\text{vert},1} n k_{\text{vert}}\,,$$

where n – number of pedestrians; k_{vert} – coefficient depending on the natural frequency under consideration (Fig. 2). Depending on the specific situation n can be adopted as $n = 13$ – for a distinct group, or $n = 0.6\,A$ – for a continuous stream of pedestrians, where A is the area of the bridge deck, [m^2].

The exposed procedure can be used for some preliminary prediction of the acceleration levels, but it may be appropriate to mention two issues

- The calculation considers only two loading cases - load by a group of pedestrians ($n = 13$) and load by a continuous flow ($n = 0.6A$). Nothing is mentioned about the synchronization of pedestrians which may result in a considerable reduction in the acceleration levels.
- The acceleration levels which should be satisfied are 0.7 m/s^2 for vertical vibrations. These limits are stated in EN 1990 A1 A.2.4.3.2 Comfort criterion for pedestrians (serviceability limit state) [12].

3.2 BS 5400 - Part 2: Specification for loads

In compliance with Eurocode 5 [7], the British Standard [8] states that pedestrian structures with fundamental frequency $f_0 \leq 5$ Hz should be studied for excessive vibration levels. This methodology does not mention a special treatment for the different loading intensities. It regards only single-pedestrian loading. BS 5400 offers two calculation procedures:

- Simplified procedure

Here, only simple structures, as the ones presented in Table 1b, are regarded. The maximum vertical accelerations due to a single-pedestrian loading are to be calculated using the following formula:

$$a = 4\pi^2 {f_0}^2 y_s K \psi \,, \tag{3}$$

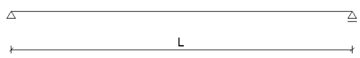
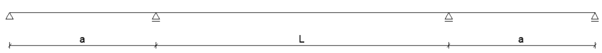
wherein f_0 – the fundamental natural frequency [Hz]; y_s – static deflection at the midpoint of the span resulting from concentrated force $F = 0.7$ [kN] applied at this point; K – configuration factor depending on the static scheme of the structure (Table 1b); ψ – dynamic response factor given as a function of the bridge span and logarithmic decrement of decay (Table 1a).

Table 1: Logaritmic decrement of decay and configuration factors

(a) Logaritmic decrement of decay

Structure type	δ
Steel with asphalt or epoxy surfacing	0.03
Composite steel/RC	0.04
Prestressed and reinforced concrete	0.05

(b) Configuration factors [8]

Configuration of the structure	a/L	K
	–	1.0
	1.0 0.8 < 0.6	0.7 0.9 1.0
	1.0 0.8 < 0.6	0.6 0.8 0.9

- Complex procedure

In the cases when the structure does not belong to the simple structure types mentioned in Table 1b, a simplified load function should be used for deriving the critical acceleration levels, namely:

$$F = 180\sin(2\pi f_0 t), \quad V_t = 0.9 f_0\,, \tag{4}$$

where the factor 180 can be regarded as dynamic load factor for the first harmonic and F represents moving point load, where the pedestrian moves with a constant speed of V_t [m/s].

The comfort criteria sought-after here states that calculated maximum acceleration a should be smaller than $0.5\sqrt{f_0}$.

3.3 Human-induced vibrations of steel structures

The Human-Induced Vibrations of Steel Structures (HIVOSS) design guide [9] is designated for steel structures, although modal damping ratios for other types of structures are given, as well. It gives two methodologies for assessing the maximum vibration levels, which are far more comprehensive than the above-mentioned, but both based on the natural modes of vibration. The HIVOSS procedure is explained in the following.

3.3.1 Critical frequency ranges

As opposed to Eurocode 5 [7] and BS 5400 [8], HIVOSS considers the possibility not only for adverse vertical vibrations, but for horizontal ones, as well. The critical ranges adopted here can be seen in Table 2a.

Since the human loading is regarded as periodic one, the guide attaches importance to the second harmonic of the vertical impact, too. Two deficiencies should be borne in mind:

- A gap between the frequency values 2.3 and 2.5 Hz for vertical vibrations is observed, which makes these values irrelevant regarding dynamic analysis (see Fig. 4).
- The guide regards the second harmonic of the human-induced loads as minor basing that on the lack of structure cases with excessive vibrations. Nevertheless, Bachmann [13] reported a case of a structure essentially excited not only by the first and the second harmonic, but by the intermediate harmonic in between these two, as well.

3.3.2 Traffic classes

The intensity of usage of a footbridge depends predominantly on its particular location and on the time interval of the day. Since there is a certain amplification of the structural response with the increase of the pedestrians, the crowd density is used as a governing parameter (Table 2b).

Table 2: Critical frequency ranges and traffic classes

(a) Critical frequency ranges

Direction	Harmonic	Frequency range, Hz
Vertical	1	1.25–2.3
Vertical	2	2.5–4.6
Lateral	1	0.5–1.2

(b) Traffic classes

Traffic class	Density d, Ped/m^2	Motion type
TC1	$d = 15/A$	Group of 15 pedestrians
TC2	$d = 0.2$	Unhampered motion
TC3	$d = 0.5$	Dense traffic
TC4	$d = 1.0$	Hampered motion
TC5	$d = 1.5$	Extraordinarily dense crowd

3.3.3 Comfort classes

Assessment of comfort classes is a quite subjective process as the perception of vibrations is different for every person and may depend on various factors (position, duration of the loading, structure outlook, etc.). The maximum expected accelerations of the structure are taken as benchmark of the pedestrians' comfort (Table 3)

Table 3: Comfort classes

Comfort class	Degree of comfort	Vertical acceleration level	Lateral acceleration level
CL1	Maximum	$< 0.5\,\mathrm{m/s^2}$	$< 0.1\,\mathrm{m/s^2}$
CL2	Medium	$0.5 - 1.0\,\mathrm{m/s^2}$	$0.1 - 0.3\,\mathrm{m/s^2}$
CL3	Minimum	$1.0 - 2.5\,\mathrm{m/s^2}$	$0.3 - 0.8\,\mathrm{m/s^2}$
CL4	Intolerable	$> 2.5\,\mathrm{m/s^2}$	$> 0.8\,\mathrm{m/s^2}$

3.3.4 Mass modification

Depending on the particular structure, the static mass of the pedestrians can affect, more or less, the modal parameters of the coupled system. Therefore, HIVOSS suggests the static mass of the crowd to be incorporated in the modal analysis, thus resulting in several overall modal characteristics for every crowd density considered:

$$\omega_i' = 2\pi f_i' = \sqrt{\frac{k_i}{m_i}}\ , \tag{5}$$

where k_i – modal stiffness, $m_i = \int_{L_d} \mu_d \rho \phi(x)^2 dx$ – modal mass, $\rho = \frac{\mu_d + \mu_p}{\mu_d}$ – influence factor, μ_d – distributed mass of the bridge deck, μ_p – distributed mass from pedestrians.

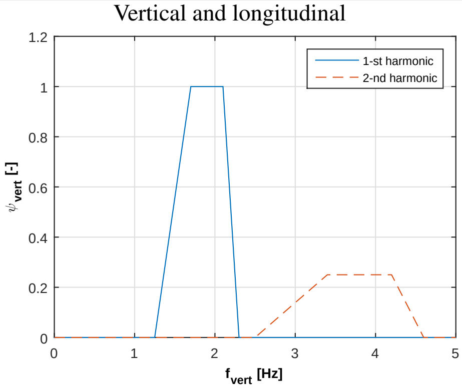
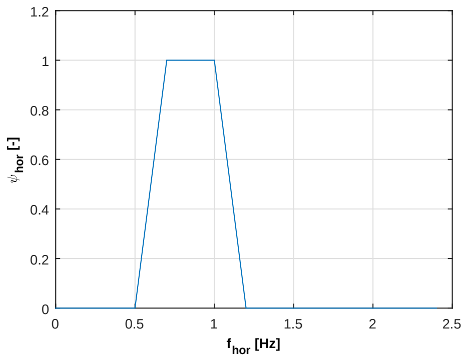
3.3.5 PREDICTION AND EVALUATION OF THE MAXIMUM ACCELERATION – SINGLE-DEGREE OF FREEDOM METHOD

The evaluation of the maximum acceleration levels is based on the acquired natural frequencies and mode shapes after performing modal analysis. Next, the harmonic load due to a certain number of pedestrians is defined. Key element is that the number of randomly walking pedestrians is replaced by equivalent number of perfectly synchronized pedestrians who result in the same maximum acceleration levels. The equivalent number of pedestrians are assumed to walk with a pacing rate or its 2-nd harmonic coinciding with the natural frequency of the considered mode falling within the critical frequency range, thus imitating resonant conditions for the structure. The harmonic load is then calculated as follows:

$$p(t) = P\cos(2\pi f_s t)n'\psi\,, \tag{6}$$

wherein P – harmonic component of the load due to a single pedestrian moving with pacing rate f_s [N]; f_s – natural frequency under consideration, which happens to

Table 4: Characteristics of the harmonic load model

<table>
<tr><td colspan="3">Harmonic load component P, N</td></tr>
<tr><td>Vertical P = 280</td><td>Longitudinal P = 140</td><td>Lateral P = 35</td></tr>
<tr><td colspan="3">Reduction factor ψ</td></tr>
<tr><td colspan="2">Vertical and longitudinal
</td><td>Lateral
</td></tr>
<tr><td colspan="3">Equivalent number of pedestrians - n’</td></tr>
<tr><td colspan="2">TC1 to TC3 ($d < 1.0\,\mathrm{ped./m^2}$)
$n' = 10.8\frac{\sqrt{\xi n}}{S}$</td><td>TC4 and TC5 ($d \geq 1.0\,\mathrm{ped./m^2}$)
$n' = 1.85\frac{\sqrt{n}}{S}$</td></tr>
</table>

coincide with the step frequency of the pedestrians or its 2-nd harmonic [Hz]; n' – equivalent number of pedestrians evaluated depending on the assumed traffic class (Table 2b), ψ – reduction factor considering the probability of coincidence between the natural frequency under consideration and the pedestrians' step frequency (Table 4).

Subsequently, harmonic load is to be applied to the structure, where the sign of the load must correspond to the mode shape sign (Fig. 3) and thereby leading to the extreme response of the structure. Employing the basic principles of structural dynamics, the maximum acceleration response at resonance in a particular mode n can be evaluated in direct relationship with the damping ratio, namely

$$a_{\max,n} = \frac{p_n}{m_n}\frac{1}{2\xi_n} , \tag{7}$$

wherein p_n – generalised load; m_n – generalised mass; ξ_n – modal damping ratio.

3.4 Sétra guide

The French Sétra guide [10] gives a procedure for the assessment of the dynamic behaviour of the structures which coincides with almost all aspects with the one used in HIVOSS. Similarly, the dynamic assessment of the structure starts with modal analysis, passes through assessment of the proneness to human-induced loads, assessment of the traffic circumstances, evaluation of the maximum accelerations based on harmonic load models and ends with comparison between the adopted comfort levels and the evaluated maximum accelerations.

3.4.1 Load representation

Eqs. (8) and (9) for the equivalent number of pedestrians are derived from numerical simulations. The pedestrians in the crowd correspond to a certain crowd density, where every pedestrian moves at random frequency (according to the Gaussian distribution – $\mu = 2$ Hz, $\sigma = 0.175$ Hz) and with random shift ϕ_i. The maximum response in terms of accelerations is registered and equated to the response induced by equivalent number of fully synchronized pedestrians applying loads with a sign coinciding with the sign of the respective modal ordinates (Fig. 3).

The formulae used for determining the equivalent number of pedestrians are different for sparse/dense and very dense crowd (Table 7):

- Sparse and dense crowd

$$n' = 10.8\sqrt{n\xi} ; \tag{8}$$

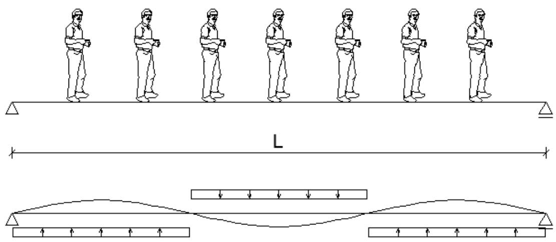


Fig. 3: Procedure exhibiting the way of applying the load by the equivalent synchronized pedestrians.

- Very dense crowd

$$n' = 1.85\sqrt{n}\ . \tag{9}$$

The main feature characterizing the crowd flow is the assumed pedestrian density. Based on it the respective number of the pedestrians is obtained. Afterwards, depending on the particular value of the pedestrian density an equivalent number of pedestrians is calculated with the premise the equivalent number of pedestrians move fully synchronized and cause the same structural response (as the actual number of pedestrians) in terms of the maximum acceleration experienced by the structure.

3.4.2 Critical frequency ranges

The guide differentiates four frequency ranges, in both vertical and horizontal direction, which can be critical for a structure to a different degree. Tables 5 present the critical frequency ranges in vertical and lateral direction.

Table 5: Critical frequency ranges in vertical and lateral directions

(a) Critical frequency ranges in vertical direction

No	Risk	Frequency range, Hz
1	Maximum	1.7-2.1
2	Medium	1-1.7 $\cup$ 2.1-2.6
3	Low	2.6-5
4	Negligible	0-1 $\cup > 5$

(b) Critical frequency ranges in lateral direction

No	Risk	Frequency range, Hz
1	Maximum	0.5-1.1
2	Medium	0.3-0.5 $\cup$ 1.1-1.3
3	Low	1.3-2.5
4	Negligible	0-0.3 $\cup > 2.5$

The adopted frequency limits are in compliance with many other regulations and articles ([7,13,14], etc.), where values of the frequency in the vicinity of 2.00 Hz are recognized as the most important regarding human-induced vibrations.

3.4.3 Critical accelerations and comfort levels

The comfort levels, assumed as acceleration ranges can be seen in Tables 6.

Table 6: Comfort levels and corresponding limits in vertical and horizontal directions

(a) Comfort levels and corresponding limits in vertical direction

No	Comfort level	Acceleration, $\mathrm{m/s^2}$
1	Maximum	< 0.5
2	Mean	0.5-1
3	Minimum	1-2.5
4	Incomfortable	> 2.5

(b) Comfort levels and corresponding limits in horizontal direction

No	Comfort level	Acceleration, $\mathrm{m/s^2}$
1	Maximum	< 0.15
2	Mean	0.15-0.3
3	Minimum	0.3-0.8
4	Incomfortable	> 0.8

3.4.4 Dynamic load cases

Depending on the significance of the footbridge on the intensity of use Sétra guide regards two load cases which are further bound to three values of the crowd density.

Case 1: sparse and dense crowd.

For the case of sparse and dense crowd the equivalent number of pedestrians is defined by eq. (8). Analogously, the harmonic load per unit area is defined using the equivalent number of pedestrians and the reduction factor which defined using Fig. 4.

Slight difference between HIVOSS and Sétra guide can be recognized in the frequency ranges. Nevertheless, all assumptions made here result in approximately the

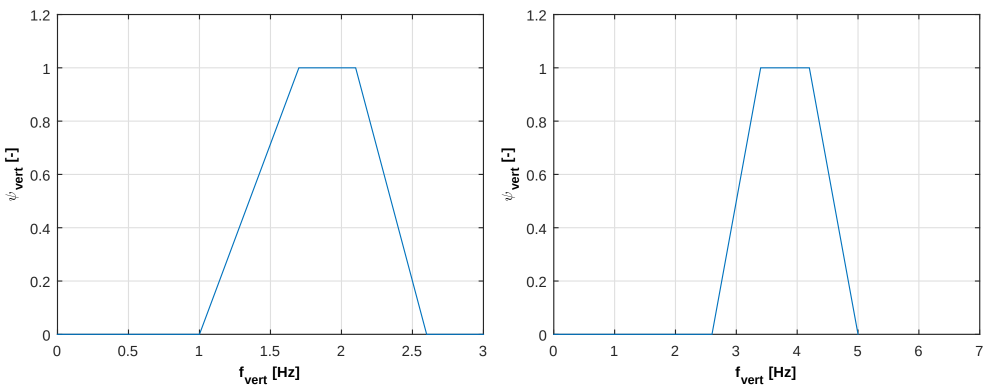


Fig. 4: Reduction factor in vertical direction the first (left) and second (right) harmonic component.

same harmonic load as the one defined above

$$p(t) = dP\cos(2\pi f_s t)10.8\sqrt{\frac{\xi}{n}}\psi\,, \tag{10}$$

wherein P – harmonic component of the load due to a single pedestrian; f_s – natural frequency under consideration, which happens to coincide with the step frequency of the pedestrians; ψ – reduction factor considering the probability of coincidence between the natural frequency under consideration and the pedestrians' step frequency (Figures 4). Values respectively for the first and the second harmonic component of the load are as follows: $P_{\text{vert},1} = 280$ N and $P_{\text{vert},2} = 70$ N, $P_{\text{long},1} = 140$ N and $P_{\text{long},2} = 35$ N, $P_{\text{lat},1} = 35$ N and $P_{\text{lat},2} = 7$ N.

Case 2: very dense crowd.

The assumed crowd density is $d = 1\,\text{ped}/\text{m}^2$ (Table 7). This case should be considered for structures of high importance which are frequently used by dense crowds in highly populated areas.

Table 7: Crowd density

Footbridge class	Description	Crowd density, $\text{ped.}/\text{m}^2$
III	Standard use	0.5
II	Heavy traffic	0.8
I	High pedestrian densities	1.0

The harmonic load model is calculated as follows:

$$p(t) = dP\cos(2\pi f_s t)1.85\sqrt{\frac{1}{n}}\psi\,, \tag{11}$$

3.5 UK National annex to Eurocode 1

The UK National annex presents a procedure to estimate of both vertical and lateral vibrations [11]. Regarding the vertical vibrations, it gives the opportunity of calculating the response due to single pedestrian, groups of pedestrians and crowd with a specific density.

3.5.1 Bridge classes

Depending on the intensity of the traffic and the situation of the pedestrian facility, UK National annex gives the following bridge classes:

It should be pointed out that the values for the crowd density are recommendatory and can be modified depending on the particular structure.

3.5.2 DYNAMIC LOADS

The UK National Annex differentiates two load models – due to a group and due to a crowd. The dynamic loads are calculated as sine functions which are fully defined by a set of constants accounting for different aspects of the of the walking load.

Vertical response due to single pedestrian or group of pedestrians.

The load is considered to move across the structure at a constant speed for a signle pedestrian or a group of pedestrians. The loading function is defined as follows:

$$p(t) = Pk(f_n)\sqrt{1+\gamma(N-1)}\sin(2\pi f_n t)\,, \tag{12}$$

where N – is the number of pedestrians in the group (Table 8); $P = 280$ N – amplitude of the dynamic load; $k(f_n)$ – coefficient taking into account the effects of a more realistic pedestrian population, harmonic responses, the relative weighting of pedestrian sensitivity to response (Fig. 5a); t – time [s]; γ – a reduction factor depending on the damping ratio and effective span.

Vertical response due to crowd loading.

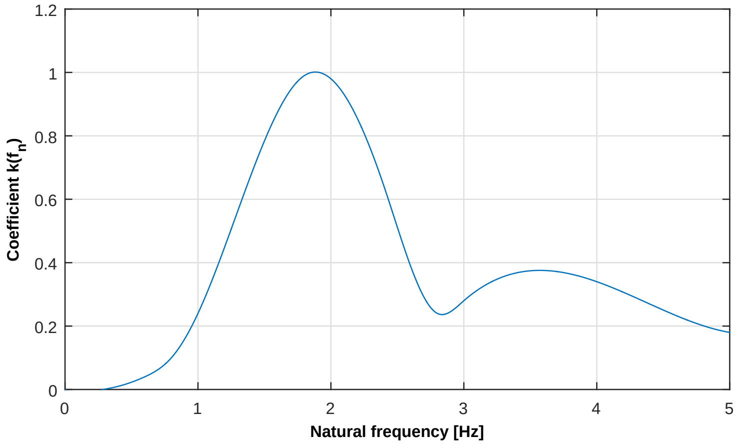
The loading due to a crowd is represented by a distributed harmonic load over the bridge deck as follows:

$$p(t) = 1.8\frac{P}{A}k(f_n)\sqrt{\frac{\gamma N}{\lambda}}\sin(2\pi f_n t)\,, \tag{13}$$

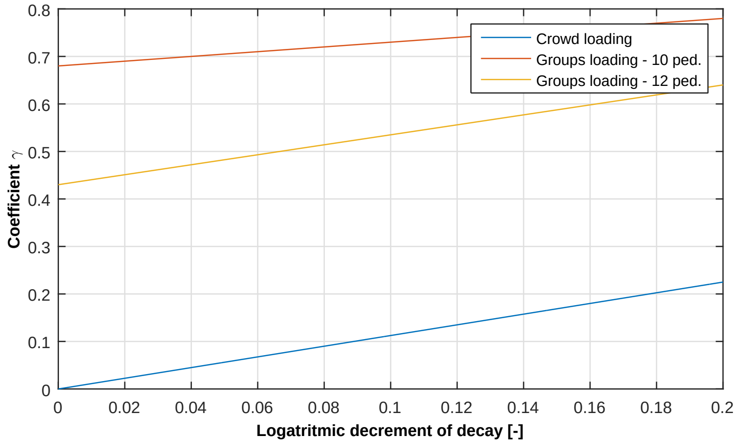
where $N = \rho A = \rho SW$ – total number of pedestrians distributed over area of the bridge, ρ – crowd density [ped./m^2] (Table 8), W – bridge width [m], λ – reduction factor. The distributed load is applied over the bridge area with sign corresponding to the sign of the mode shape (Fig. 3). The structural response can then be calculated using eq. (7).

Table 8: Bridge classes according to [11]

Bridge class	Bridge usage	Group size (walking)	Crowd density $\mathrm{ped./m^2}$ (walking)
A	Seldom used – sparsely populated areas	N = 2	0
B	Suburban location – likely to experience slight variations in pedestrian loading	N = 4	0.4
C	Urban routes – significant variation in daily usage	N = 8	0.8
D	Primary access to major public assembly facilities	N = 16	1.5



(a) Complex factor $k(f_n)$ [11]



(b) Reduction factor to allow for the unsynchonized combination of pedestrian actions among groups and crowds [11]

Fig. 5: Coefficients according to [11].

3.5.3 Comfort criteria

The comfrot limit is to be calculated with the formula

$$a_{\text{limit}} = 1.00 k_1 k_2 k_3 k_4 \,, \tag{14}$$

where $0.5 \leq a_{\text{limit}} \leq 2.0$ [m/s^2]; k_1 – site usage factor; k_2 – route redundancy factor; k_3 – height of structure factor; $k_4 = 1$ – exposure factor.

4 Conclusuions

A comprehensive exposure of the most used current codes of practice accounting for either single pedestrian loading and crowd loading, and assessing the dynamic behaviour of pedestrian structures is presented.

The detailed procedures for analyzing crowd loading provided by Sétra [10] and HIVOSS [9] overlap in most assumptions. Nevertheless, due to slight differences in the reduction factor definition, the results for common crowd densities may not be in a good conformity. This explicitly affirms the stochastic nature of the human-induced vibration and the necessity of more elaborated studies stemming from it.

References

[1] V. Rizov, A. Doicheva, A. Mladensky, G. Keradjisky (2015) Analysis of a pedestrian bridge with respect to the comfort using the EC. *Annual of UACEG* **XLVIII** 79-89.

[2] E. Dulevski, S. Ivanov (2004) Assessment of the condition of the suspension bridges for pedestrian in the rodopi mountain. In: *Proceeding of the 3rd National Conference on Design and Construction of Buildings and Transport Facilities*, Plovdiv, Bulgaria.

[3] Kr. Kasapova (2018) Human-induced vibrations on footbridges and the impact of human-human interaction. PhD thesis, UACEG, So

a, Bulgaria.

[4] K. Van Nimmen, G. Lombaert, I. Jonkers, G. De Roeck, P. Van den Broeck (2014) Characterisation of walking loads by 3d inertial motion tracking. *Journal of Sound and Vibration* **333** 5212-5226.

[5] S.C. Kerr (1998) Human induced loading on staircases. PhD Thesis, University of London.

[6] C. Heinemeyer, M. Feldmann (2008) European design guide for footbridge vibration. In: *Proceedings of Third International Conference on Footbridges "Footbridge 2008 – Footbridges for Urban Renewal"*, Porto, Portugal, 2-4 July 2008.

[7] European committee for standardization (2004) Eurocode 5: Design of timber structures - Part 2: Bridges.

[8] British Standard (1978) BS 5400: Steel, concrete and composite bridges (Part 2: Specification for loads).

[9] HIVOSS (2008) "Design of Footbridges". Research Fund for Coal and Steel.

[10] Association Française de Génie Civil, Sétra/AFGC (2006) Evaluation du comportement vibratoire des passerelles piétonnes sous l'action des piétons. Sétra, Paris.

[11] British Standard (2003) BS NA EN 1991-2: UK National Annex to Eurocode 1. Actions on structures. Traffic loads on bridges.

[12] EUROPEAN COMMITTEE FOR STANDARDIZATION (2005) Eurocode 0: Basis of structural design.

[13] H. BACHMANN, W. AMMANN (1987) “Vibrations in structures: induced by man and machines”. IABSE, Zurich, Switzerland.

[14] Y. MATSUMOTO, H SHIOJIRI, T. NISHIOKA (1978) Dynamic design of footbridges. In: *IABSE proceedings* No. P-17/78, pp. 1-15.