Journal of Theoretical and Applied Mechanics, Sofia, Vol.50 (2020) pp. 222-237
SOLID MECHANICS

FRACTIONAL ORDER THEORY OF THERMAL STRESSES IN A TWO DIMENSIONAL TRANSVERSELY ISOTROPIC MAGNETO THERMOELASTIC MATERIAL

PARVEEN LATA, IQBAL KAUR*

Department of Basic and Applied Sciences, Punjabi University, Patiala, Punjab, India

[Received: 29 July 2019. Accepted: 03 February 2020]

ABSTRACT: A mathematical model for homogeneous transversely isotropic magneto-thermoelastic (HTIMT) material and fractional order theory (FOT) of thermal stresses due to ramp type heat and with hall current effect is formulated. To solve the field equations, Fourier and Laplace transforms are used. The mathematical expressions of components of displacement, temperature, current density and stress are solved in the transformed domain and then obtained in physical domain using transform inversion techniques. The fractional order parameter by considering various cases of weak, normal and strong conductivity are represented graphically.

KEY WORDS: Fractional-order Theory, Hall current, Magneto Thermoelastic, Transversely Isotropic, Ramp Type Heat.

NOMENCLATURE

δ_{ij} : Kronecker delta,
$\vec{H}_0$: magnetic field intensity vector,
τ_v : temperature gradient phase lag,
T : temperature,
c_{ijkl} : elastic parameters,
ρ : medium density,
ε_0 : electric permeability,
τ_t : heat flux phase lag,
K^*_{ij} : materialistic constant,
C_E : specific heat,
e_{ij} : strain tensors,
F_i : components of Lorentz force,
T_0 : reference temperature,
$\vec{j}$: current density vector,
t_{ij} : stress tensors,
μ_0 : magnetic permeability,
$\vec{u}$: displacement vector,
τ_0 : relaxation time,
u_i : components of displacement,
φ : conductive temperature,
ω : frequency,
β_{ij} : thermal elastic coupling tensor,
K_{ij} : thermal conductivity,
τ_q : phase lag of thermal displacement,
α_{ij} : linear thermal expansion coefficient,
a_{ij} : two temperature parameters,
$\delta(t)$: Dirac's delta function,
α : fractional parameter
t_0 : rise time of heat
T_1 : constant temperature
T_{ij} : Maxwell's stress component

*Corresponding author e-mail: bawahanda@gmail.com

1 INTRODUCTION

In last decade, the attraction of ample number of researchers for magneto-thermoelastic (MT) material (composite material) structures is continuously increasing since magneto-thermoelastic materials exhibits the prerequisite coupling effect among electric and magnetic fields. Due to these electric and magnetic field, Hall voltage is produced in the medium. Similarly, studying the coupling effect in a thermoelastic medium among magnetic fields and stress is known as Magneto-thermoelasticity and has a numerous applications in the domain of science and technology mainly in nuclear reactors, geophysics, and related domains.

Sherief *et al.* [1] proposed a model of thermoelasticity by using fractional calculus and Youseff [2, 3] offered an alternative theory of thermoelasticity by using the fractional calculus with extensive range $(0 < \alpha \leq 2)$. Ezzat *et al.* [4–6] established a additional theory on FOT using Taylor series. Bachher *et al.* [7] examined the Caputo time-FOT for MT response of a 2D isotropic solid including rotating. Kumar *et al.* [8] examined the FOT proposed by Sherief [1] and Ezzat & Youssef [2] in a micropolar solid and ramp heat source. Also, Kumar *et al.* [9] studied the thermomechanical exchanges in a rotating transversely isotropic thermoelastic (TIT) solid with two temperatures (2T). Sheoran *et al.* [10] analysed the future predictions of FOT. Abbas [11] discussed 2-D GN-III model with FOT. Additionally, other researcher discussed another theories of thermoelasticity as Marin [12,13], Marin *et al.* [14,15], Lata and Kaur [16,17], Marin [18,19], Kumar *et al.* [20] Lata and Kaur [21–23], Lata & Kaur [24–26], Marin and Craciun [27], Kaur and Lata [28]. Numerous research is done by researchers to solve different problem using fraction order generalized thermoelasticity. Regardless of these, no considerable research to study fractional order three phase lag (TPL) effect due to Hall current has been taken. In this research, a mathematical model is purposed to study the effect of FOT with hall current and ramp type heat in HTIMT solid.

2 BASIC EQUATIONS

The Maxwell's stress components and simplified linear equations for slow moving conducting elastic solid following [29] are:

$$\text{curl}\,\vec{h} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\,, \tag{1}$$

$$\text{curl}\,\vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t}\,, \tag{2}$$

$$\vec{E} = -\mu_0 \left(\frac{\partial \vec{u}}{\partial t} + \vec{H}_0 \right), \tag{3}$$

$$\operatorname{div} \vec{h} = 0 \,. \tag{4}$$

$$T_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}) \,. \tag{5}$$

The equation of motion [30] for a TITM with the Lorentz force $F_i = \mu_0 \left(\vec{j} \times \vec{H}_0 \right)$ is given as under:

$$t_{ij} = c_{ijkl} e_{kl} - \beta_{ij} T \,, \tag{6}$$

$$t_{ij,j} + F_i = \rho \ddot{u}_i \,. \tag{7}$$

The equations (6) and (7) using the universal Ohm's law with Hall current and with constant conductivity is given by

$$J = \frac{\sigma_0}{1+m^2} \left(E + \mu_0 \left(\dot{u} \times H - \frac{1}{e n_e} J \times H_0 \right) \right) \,. \tag{8}$$

The TPL heat conduction equation with FOT is

$$\begin{aligned} K_{ij} \left(1 + \frac{(\tau_t)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) \dot{T}_{,ji} + K^*_{ij} \left(1 + \frac{(\tau_v)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} \right) T_{,ji} \\ = \left(1 + \frac{(\tau_q)^\alpha}{\alpha!} \frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} \right) \left[\rho C_E \ddot{T} + \beta_{ij} T_0 \ddot{e}_{ij} \right] , \end{aligned} \tag{9}$$

where

$$\beta_{ij} = c_{ijkl} \alpha_{ij} \,, \quad e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \,, \quad \beta_{ij} = \beta_i \delta_{ij} \,, \quad K_{ij} = K_i \delta_{ij} \,, \tag{10}$$

where $i, j = 1, 2, 3$ and i, j is not summed. Here $c_{ijkl} = c_{klij} = c_{jikl} = c_{ijlk}$.

3 Formulation and Solution of the Problem

We consider a HTIMT material with TPL fractional order heat transfer at a constant temperature T_0, and with a magnetic field $\vec{H}_0 = (0,\, H_0,\, 0)$ along y-axis. Furthermore, the Cartesian coordinates x, y, z are considered with origin at $z = 0$ towards z-axis with ramp heat directing vertically downwards.

For a 2-D model in the xz-plane, we take

$$\mathbf{u} = (u, 0, w)$$

Also, we consider that

$$\mathbf{E} = 0.$$

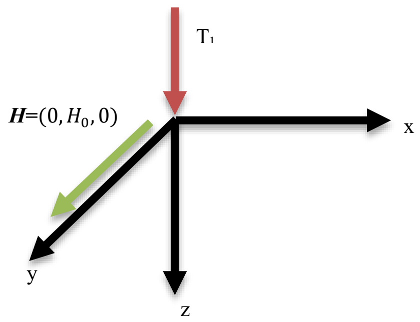

Fig. 1: Geometry of the problem.

And thus from the universal Ohm's law, we have

$$J_2 = 0\,, \tag{11}$$

J_1 and J_3 using (8) are given as

$$J_1 = \frac{\sigma_0 \mu_0 H_0}{1+m^2}\left(m\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right), \tag{12}$$

$$J_3 = \frac{\sigma_0 \mu_0 H_0}{1+m^2}\left(\frac{\partial u}{\partial t} + m\frac{\partial w}{\partial t}\right). \tag{13}$$

Following Slaughter [31] apply transformation on Eqs. (7 – (9) yeilds

$$c_{11}\frac{\partial^2 u}{\partial x^2} + c_{13}\frac{\partial^2 w}{\partial x \partial z} + c_{44}\Big(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\Big) - \beta_1\frac{\partial}{\partial x}T - \mu_0 J_3 H_0 = \rho\Big(\frac{\partial^2 u}{\partial t^2}\Big), \tag{14}$$

$$(c_{13}+c_{44})\frac{\partial^2 u}{\partial x \partial z} + c_{44}\frac{\partial^2 w}{\partial x^2} + c_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}T - \mu_0 J_1 H_0 = \rho\Big(\frac{\partial^2 w}{\partial t^2}\Big), \tag{15}$$

$$\begin{aligned} & K_1\Big(1+\frac{(\tau_t)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\Big)\frac{\partial^2 \dot{T}}{\partial x^2} + K_3\Big(1+\frac{(\tau_t)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\Big)\frac{\partial^2 \dot{T}}{\partial z^2} \\ & \quad + K_1^*\Big(1+\frac{(\tau_v)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\Big)\frac{\partial^2 T}{\partial x^2} + K_3^*\Big(1+\frac{(\tau_v)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\Big)\frac{\partial^2 T}{\partial z^2} \\ & \quad = \Big(1+\frac{(\tau_q)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\Big)\Big[\rho C_E \ddot{T} + T_0\Big\{\beta_1\frac{\partial \ddot{u}}{\partial x} + \beta_1\frac{\partial \ddot{w}}{\partial z}\Big\}\Big] \end{aligned} \tag{16}$$

and

$$t_{xx} = c_{11}e_{xx} + c_{13}e_{xz} - \beta_1 T\,, \tag{17}$$

$$t_{zz} = c_{13}e_{xx} + c_{33}e_{zz} - \beta_3 T\,, \tag{18}$$

$$t_{xz} = 2c_{44}e_{xz}\,, \tag{19}$$

where

$$\beta_1 = (c_{11} + c_{12})\,\alpha_1 + c_{13}\alpha_3\,, \qquad \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3\,.$$

Following dimensionless (DL) quantities are used to solve the problem:

$$\begin{aligned} &x' = \frac{x}{L}\,, \quad u' = \frac{\rho c_1^2}{L\beta_1 T_0}u\,, \quad t' = \frac{C_1}{L}t\,, \quad w' = \frac{\rho c_1^2}{L\beta_1 T_0}w\,, \\ &T' = \frac{T}{T_0}\,, \quad t'_{xx} = \frac{t_{xx}}{\beta_1 T_0}\,, \quad t'_{zz} = \frac{t_{zz}}{\beta_1 T_0}\,, \quad t'_{xz} = \frac{t_{xz}}{\beta_1 T_0}\,, \\ &z' = \frac{z}{L}\,, \quad \tau'_T = \frac{C_1}{L}\tau_T\,, \quad \tau'_v = \frac{C_1}{L}\tau_v\,, \quad \tau'_q = \frac{C_1}{L}\tau_q\,. \end{aligned} \tag{20}$$

By using these DL quantities defined in (20) in the Eqs. (14 – (16), and thereafter suppressing primes, yields

$$\frac{\partial^2 u}{\partial x^2} + \delta_1\frac{\partial^2 w}{\partial x \partial z} + \delta_2\frac{\partial^2 u}{\partial z^2} - \frac{\partial T}{\partial x} = \frac{M}{1+m^2}\left[\frac{\partial u}{\partial t} + m\frac{\partial w}{\partial t}\right] + \frac{\partial^2 u}{\partial t^2}\,, \tag{21}$$

$$\delta_1\frac{\partial^2 u}{\partial x \partial z} + \delta_2\frac{\partial^2 w}{\partial x^2} + \delta_3\frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1}\frac{\partial T}{\partial z} = -\frac{M}{1+m^2}\left[m\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right] + \frac{\partial^2 w}{\partial t^2}\,, \tag{22}$$

$$\begin{aligned} &\left(1 + \frac{C_1(\tau_t)^\alpha}{\alpha! L}\frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)\left(K_1\frac{\partial^2 T}{\partial x^2} \right.\\ &\left. + K_3\frac{\partial^2 T}{\partial z^2}\right) + \left(1 + \frac{(\tau_v)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(K_1^*\frac{\partial^2 T}{\partial x^2} + K_3^*\frac{\partial^2 T}{\partial z^2}\right) \\ &= \left(1 + \frac{(\tau_q)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left[\rho C_E \ddot{T} + \frac{\beta_1}{\rho}T_{{}_0}\left\{\beta_1\frac{\partial \ddot{u}}{\partial x} + \beta_1\frac{\partial \ddot{w}}{\partial z}\right\}\right], \end{aligned} \tag{23}$$

where

$$\delta_1 = \frac{c_{13}+c_{44}}{c_{11}}\,, \quad \delta_2 = \frac{c_{44}}{c_{11}}\,, \quad \delta_3 = \frac{c_{33}}{c_{11}}\,, \quad M = \left(\frac{L\sigma_0\mu_0^2 H_0^2}{\rho C_1}\right).$$

According to initial and symmetry conditions

$$\begin{aligned} \mathbf{u}\,(x,z,0) &= \dot{u}\,(x,z,0) = 0\,, \\ \mathbf{w}\,(x,z,0) &= \dot{w}\,(x,z,0) = 0\,, \\ \mathbf{T}\,(x,z,0) &= \dot{T}\,(x,z,0) = 0\,. \end{aligned}$$

For $z \geq 0$ and $-\infty \leq x \leq \infty$ $\mathbf{w}(x,z,t) = u(x,z,t) = \mathbf{T}(x,z,t) = 0$, when $t > 0$ while $z \to \infty$.

The Laplace and Fourier transforms are defined as

$$\tilde{f}\left(x,z,s\right) = \int_{0}^{\infty} f\left(x,z,t\right) e^{-st} dt\,, \tag{24}$$

$$\hat{f}\left(\xi,z,s\right) = \int_{-\infty}^{\infty} \tilde{f}\left(x,z,s\right) e^{i\xi x} dx\,. \tag{25}$$

On Eqs. (17) – (19) we get a set of equations

$$\begin{aligned}\left[-\xi^2 - s^2 + \delta_2 D^2 - \frac{Ms}{1+m^2}\right] \hat{u}\left(\xi,z,s\right) \\ + \left[\delta_1 D i\xi - \frac{mMs}{1+m^2}\right] \hat{w}\left(\xi,z,s\right) + \left(-\mathrm{i}\xi\right) \hat{T}\left(\xi,z,s\right) = 0\,,\end{aligned} \tag{26}$$

$$\begin{aligned}\left[\delta_1 D i\xi + \frac{mMs}{1+m^2}\right] \hat{u}\left(\xi,z,s\right) \\ + \left[-\delta_2\xi^2 + \delta_3 D^2 - s^2 - \frac{Ms}{1+m^2}\right] \hat{w}\left(\xi,z,s\right) - \frac{\beta_3}{\beta_1} D\hat{T}\left(\xi,z,s\right) = 0\,,\end{aligned} \tag{27}$$

$$\begin{aligned}&\frac{\beta_1^2 T_0 s^2 i\xi}{\rho}\left[1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!}\right]\hat{u}(\xi,z,s) \\ &+ \frac{\beta_1\beta_3 T_0 s^2}{\rho}\left[1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!}\right] D\hat{w}(\xi,z,s) \\ &+ \left\{\rho C_E C_1^2 s^2 \left(1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!}\right) + K_1\xi^2\left(1 + \frac{C_1\tau_T^\alpha s^{\alpha+1}}{\alpha! L}\right)\right. \\ &+ K_1^*\xi^2\left(1 + \frac{\tau_v^\alpha s^\alpha}{\alpha! L}\right) - \left[K_3\left(1 + \frac{C_1\tau_T^\alpha s^{\alpha+1}}{\alpha! L}\right)\right. \\ &\left.\left. + K_3^*\left(1 + \frac{\tau_v^\alpha s^\alpha}{\alpha! L}\right) D^2\right]\right\}\hat{T}(\xi,z,s) = 0\,.\end{aligned} \tag{28}$$

The non-trivial solution of (26) – (28) yields

$$AD^6 + BD^4 + CD^2 + E = 0\,, \tag{29}$$

where

$$
\begin{aligned}
&D = \frac{d}{dz}\,, \qquad A = -\delta_2\delta_3 A_4\,,\\
&B = -A_4A_9\delta_2 + \delta_2\delta_3A_3 - A_5A_4\delta_3 - A_2A_{10}\delta_2 + \delta_1^2A_7^2A_4\,,\\
&C = A_3A_9\delta_2 - A_9A_4A_5 + A_5A_3\delta_3 - A_2A_{10}A_5 + A_8A_4A_6 - \delta_1^2A_7^2A_3\\
&\qquad + A_7A_1\delta_1A_{10} - \delta_1A_7^2A_2 + A_1A_7\delta_3\,,\\
&E = A_5A_9A_3 - A_6A_8A_3 + A_1A_9A_7\,,\\
&A_1 = \frac{\beta_1^2T_0s^2i\xi}{\rho}\left[1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha}s^{2\alpha}}{2\alpha!}\right],\\
&A_2 = \frac{\beta_1\beta_3T_0s^2}{\rho}\left[1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha}s^{2\alpha}}{2\alpha!}\right],\\
&A_3 = \rho C_E C_1^2 s^2\left[1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha}s^{2\alpha}}{2\alpha!}\right] + K_1\xi^2\left[1 + \frac{C_1\tau_T^\alpha s^{\alpha+1}}{\alpha!L}\right]\\
&\qquad + K_1^*\xi^2\left[1 + \frac{\tau_v^\alpha s^\alpha}{\alpha!L}\right],\\
&A_4 = K_3\left[1 + \frac{C_1\tau_T^\alpha s^{\alpha+1}}{\alpha!L}\right] + K_3^*\left[1 + \frac{\tau_v^\alpha s^\alpha}{\alpha!L}\right],\\
&A_5 = -\xi^2 - s^2 - \frac{Ms}{1+m^2}, \qquad A_6 = -\frac{mMs}{1+m^2}\,,\\
&A_7 = i\xi\,, \quad A_8 = \frac{mMs}{1+m^2}\,, \quad A_9 = -\xi^2\delta_2 - s^2 - \frac{Ms}{1+m^2}\,, \quad A_{10} = -\frac{\beta_3}{\beta_1}\,.
\end{aligned}
$$

The $\pm\lambda_j$, $(j = 1, 2, 3)$ are the roots of Eq. (29). The result of Eqs. (26) – (28) satisfying the radiation condition, i.e. $\tilde{u}$, $\tilde{v}$, $\tilde{w} \to 0$ as $z \to \infty$ yields

(30) $$\hat{u}\,(\xi, z, s) = \sum_{j=1}^{3} B_j e^{-\lambda_j z}\,,$$

(31) $$\hat{w}\,(\xi, z, s) = \sum_{j=1}^{3} d_j B_j e^{-\lambda_j z}\,,$$

(32) $$\hat{T}\,(\xi, z, s) = \sum_{j=1}^{3} l_j B_j e^{-\lambda_j z}\,,$$

where B_j, $j = 1, 2, 3$ being unknown constants and

$$d_j = \frac{\delta_2 A_4 \lambda_j^4 + (-A_5 A_4 + \delta_2 A_3)\lambda_j^2 + A_1 A_7 + A_5 A_3}{-\delta_3 \zeta_4 \lambda_j^4 + (\delta_3 A_3 - A_2 A_{10} - A_4 A_9)\lambda_j^2 + A_3 A_6},$$

$$l_j = \frac{\delta_2 \delta_3 \lambda_j^4 + \left(\delta_2 A_9 + A_5 \delta_3 - \delta_1^2 A_7^2\right)\lambda_j^2 + A_5 A_9 - A_6 A_8}{-\delta_3 \zeta_4 \lambda_j^4 + (\delta_3 A_3 - A_2 A_{10} - A_4 A_9)\lambda_j^2 + A_3 A_6}.$$

4 Boundary Conditions

The half-space surface $(z = 0)$ is subjected to ramp type heat, therefore, we have

(33) $$t_{zz}(x, z, t) = 0,$$

(34) $$t_{xz}(x, z, t) = 0,$$

(35) $$T(x, z, t) = G(t)\delta(x),$$

where $G(t)$ is a function defined as

$$G(t) = \begin{cases} 0; & t \le 0 \\ T_1 \dfrac{t}{t_0}; & 0 \le t \le t_0 \\ T_1; & t > t_0 \end{cases}.$$

Applying the transforms as defined by equations (24) – (25) to (35)

$$\hat{T}(\xi, 0, s) = \bar{G}(s),$$

where $\bar{G}(s) = T_1\left(1 - e^{-st_0}\right)/t_0 s^2$.

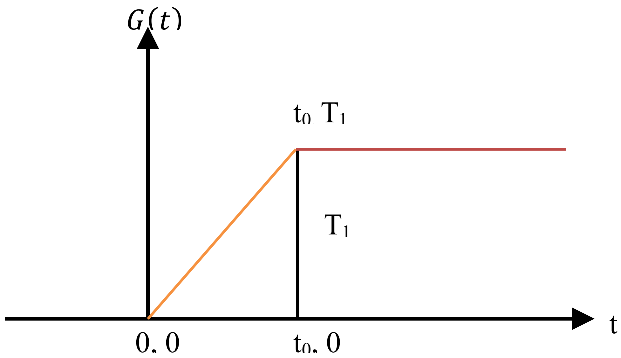

Fig. 2: Geometry of Ramp type heat.

After using (24) – (25) respectively on (33) – (35) and (14) – (16) by using (30) – (32), we obtained the components of displacement, current density and stress, as well as conductive temperature

$$(36)\quad [\hat{u}, \hat{w}, \hat{T}, \hat{t}_{xz}, \hat{t}_{zz}, \hat{J}_1, \hat{J}_3](\xi, z, s) = \frac{\bar{G}(s)}{\chi} \sum_{j=1}^{3} [1, d_j, l_j, \chi_{2j}, \chi_{1j}, \psi_j, \varrho_j] \varsigma_j e^{-\lambda_j z} ,$$

where

$$\chi_{1j} = C_{13} i\xi - C_{33} d_j \lambda_j - \beta_3 l_j , \quad \chi_{2j} = (\lambda_j + i\xi d_j) , \quad \chi_{3j} = l_j , \quad j = 1, 2, 3$$

and

$$\chi = \varsigma_1 \chi_{11} - \varsigma_2 \chi_{12} + \varsigma_3 \chi_{13} , \qquad \varsigma_1 = \bar{G}(s)(\chi_{12}\chi_{23} - \chi_{13}\chi_{22}) ,$$
$$\varsigma_2 = \bar{G}(s)(\chi_{11}\chi_{23} - \chi_{21}\chi_{13}) , \qquad \varsigma_3 = \bar{G}(s)(\chi_{11}\chi_{23} - \chi_{21}\chi_{12}) ,$$
$$\psi_j = \frac{\sigma_0 H_0 \mu_0}{(1+m^2)} \mathrm{s}(m - d_j) , \qquad \varrho_j = \frac{\sigma_0 H_0 \mu_0}{(1+m^2)} \mathrm{s}(1 + m d_j) .$$

5 Inversion of the Transformation

To obtain the results in physical domain, Fourier transform in Eq. (36) need be inverted by using

$$(37)\quad \tilde{f}(x, z, s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi, z, s) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} | \cos(\xi x) f_e - i \sin(\xi x) f_0 | d\xi .$$

The Laplace transform function $\tilde{f}(x, z, s)$ following Honig and Hirdes [32], inverted to $f(x,\, z,\, t)$ by

$$(38)\qquad f(x, z, t) = \frac{1}{2\pi i} \int_{\nu - i\infty}^{\nu + i\infty} \tilde{f}(x, z, s) e^{-st} ds .$$

Romberg's integration [33] is used for inversion integral in Eq. (37).

6 Numerical Results and Discussion

The physical data for cobalt material [34] is used to validate the theoretic outcomes and influence of fractional order parameter and Hall current, is given as

$$c_{11} = 3.07 \times 10^{11}\ \mathrm{Nm}^{-2}, \quad \mathrm{T}_0 = 298\ \mathrm{K}, \quad c_{44} = 1.510 \times 10^{11}\ \mathrm{Nm}^{-2},$$

$$\varepsilon_0 = 8.838\times10^{-12}\ \mathrm{Fm}^{-1}, \quad \beta_1 = 7.04\times10^{6}\ \mathrm{Nm}^{-2}\mathrm{deg}^{-1},$$
$$\beta_3 = 6.90\times10^{6}\ \mathrm{Nm}^{-2}\mathrm{deg}^{-1}, c_{33} = 3.581\times10^{11}\ \mathrm{Nm}^{-2}, \quad \rho = 8.836\times10^{3}\ \mathrm{kgm}^{-3},$$
$$C_E = 4.27\times10^{2}\ \mathrm{Jkg}^{-1}\mathrm{deg}^{-1}, \quad \mathrm{H}_0 = 1\ \mathrm{Jm}^{-1}\mathrm{nb}^{-1}, \quad K_1 = 0.690\times10^{2}\ \mathrm{Wm}^{-1}\mathrm{K}^{-1},$$
$$c_{13} = 1.027\times10^{10}\ \mathrm{Nm}^{-2}, \quad K_3 = 0.690\times10^{2}\ \mathrm{Wm}^{-1}\mathrm{K}^{-1}, \quad L = 1.$$

Dimensionless field variables are compared with distinct values of fractional order factor and constant Hall current factor $m = 0.5$ is shown graphically. Figure 3 and Fig. 4 exhibit the variations in u and w w.r.t x respectively. For different values of α the displacement variables u and w, first decreases when $0 \le x \le 2$, and thereafter exhibits oscillatory pattern.

Figure 5 shows the variations in t_{zx} w.r.t. x and Fig. 6 illustrates the t_{zz} w.r.t. x. For $\alpha = 1.0$ the tangential stress t_{zx} shows an oscillatory behaviour. However, when $\alpha = 0.5$, t_{zx} and t_{zz} shows an oscillatory behaviour with declining amplitude with rise in x and the normal stress t_{zz} shows oscillatory behaviour with larger amplitude with increase in x. For $\alpha = 1.5$, t_{zx} abruptly increases when $0 \le x \le 2$ and then become oscillatory while normal stress t_{zz} abruptly rises when $0 \le x \le 2$ and then shows oscillatory behaviour with declining amplitude with rise in x. Figure 7 shows the variations in T with x. Initially, it sharply decrease and then become oscillatory for rest of the length. Figure 8 illustrates the J_1 and Fig. 9 demonstrates the J_3 w.r.t.

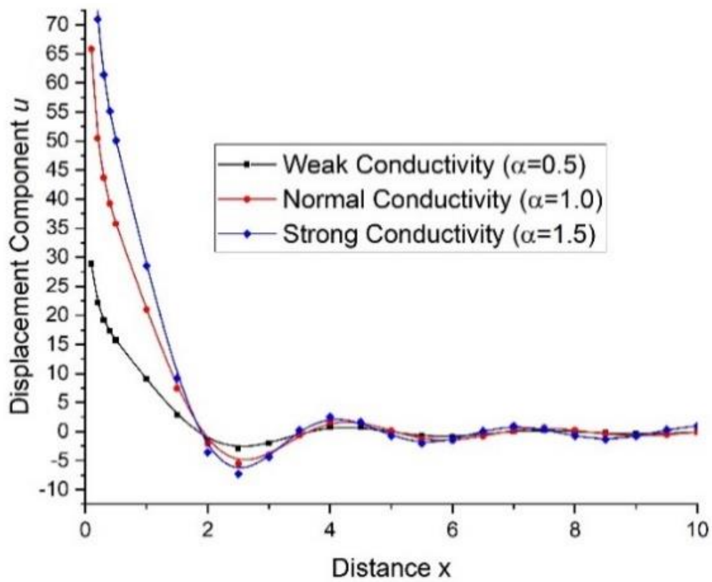

Fig. 3: Deviations of u w.r.t. x.

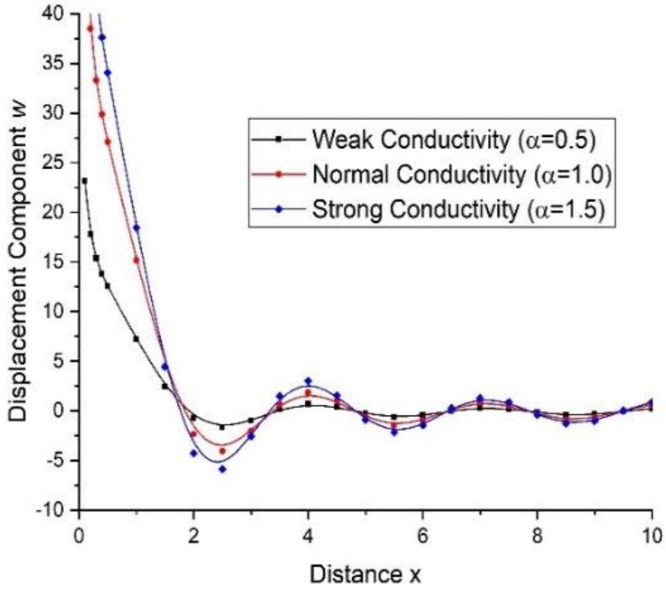

Fig. 4: Variations of w w.r.t. x.

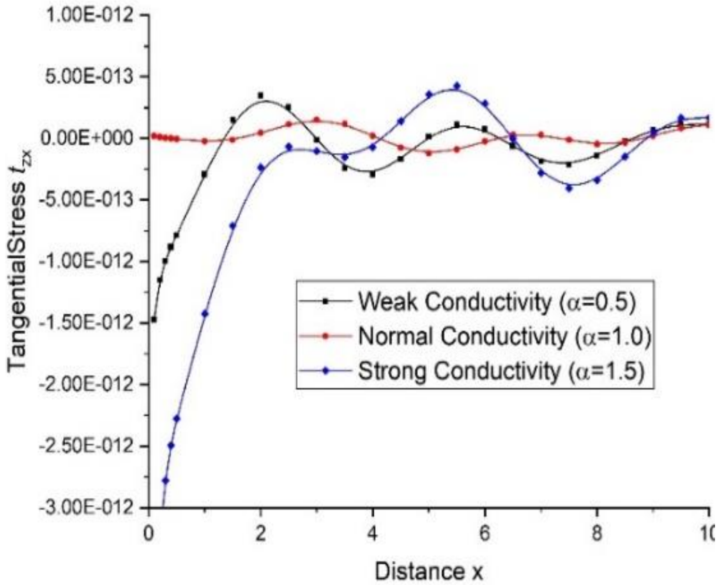

Fig. 5: Deviations of t_{zx} w.r.t. x.

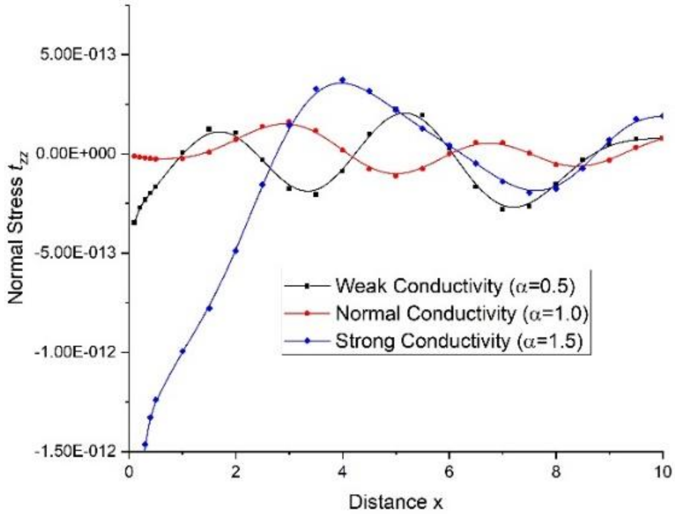

Fig. 6: Deviations of t_{zz} w.r.t. x.

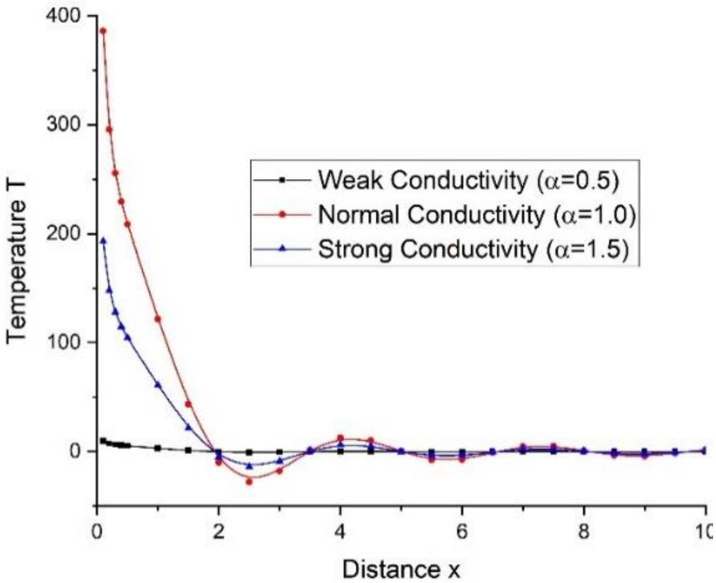

Fig. 7: Deviations of T with x.

x. for $\alpha = 0.5$ and $\alpha = 1.0$ the J_1 is remain similar, while J_3 for $\alpha = 0.5$ the value remains similar and for $\alpha = 1.0$ the value of of J_3 first increases when $0 \le x \le 2$ and then remains identical as for $\alpha = 0.5$.

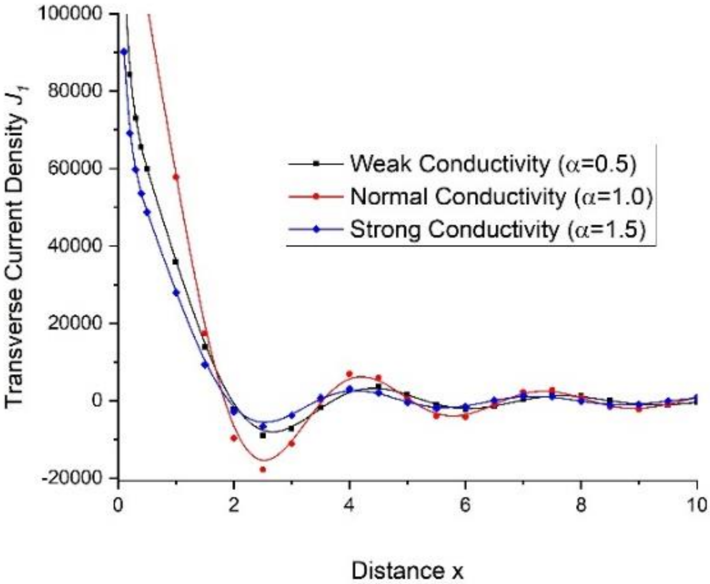

Fig. 8: Deviations of J_1 w.r.t. x.

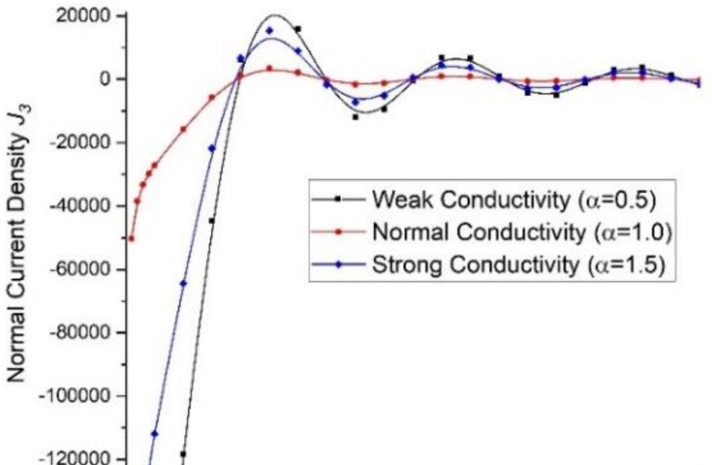

Fig. 9: Deviations of J_3w.r.t. x.

7 Conclusion

- A novel mathematical model is purposed for the study of HTIMT material with FOT of thermal stresses due to ramp type heat and Hall current effect

- Fractional order parameter α has a substantial impact on the various parameters of HTIMT medium. As the value of α increases, it shows the major effect on the various components.

- The effect of ramp heat in HTIMT medium with Hall Effect and fractional order factor plays a significant role in the analysis of the deformed media.

- The graphs shows the effect of α with constant value of m on the media and also fulfil research motive.

References

[1] H.H. Sherief, A.M.A. El-Sayed, A. El-Latief (2010) Fractional Order Theory of Thermoelasticity. *International Journal of Solids and Structures* **47**(2) 269-275.

[2] H. Youssef (2006) Two-Dimensional Generalized Thermoelasticity Problem For A Half Space Subjected To Ramp - Type Heating. *European Journal of Mechanics A/Solid* **25** 745-763.

[3] H. Youssef (2010) Theory of Fractional Order Generalized Thermoelasticity. *ASME J. Heat Transfer* **132** 1-7.

[4] M.A. Ezzat, A.S. Karamany, M.A. Fayik (2011) Fractional Order Theory in Thermoelastic Solid with Three-Phase Lag Heat Transfer. *Archive of Applied Mechanics* **82** 557-572.

[5] M.A. Ezzat, A.S. El-Karamany, S.M. Ezzat (2012) Two-Temperature Theory in Magneto-Thermoelasticity with Fractional Order Dual-Phase-Lag Heat Transfer. *Nuclear Engineering and Design (Elsevier)* **252** 267-277.

[6] M.A. Ezzat, A.S. El-Karamany, A.A. El-Bary (2017) Two-Temperature Theory in Green–Naghdi Thermoelasticity with Fractional Phase-Lag Heat Transfer. *Microsystem Technologies – Springer Nature* **24**(2) 951-961.

[7] M. Bachher, N. Sarkar (2016) Fractional Order Magneto-Thermoelasticity in a Rotating Media with One Relaxation Time. *Mathematical Models In Engineering* **2**(1) 56-68.

[8] R. Kumar, K. Singh (2015) Interaction of Fractional Order Theories in Micropolar Thermoelastic Solid Due to Ramp Type Heating. *Proceedings of The World Congress on Engineering* I. London, U.K.

[9] R. Kumar, N. Sharma, P. Lata (2016) Thermomechanical Interactions Due to Hall Current in Transversely Isotropic Thermoelastic with and without Energy Dissipation with Two Temperatures and Rotation. *Journal of Solid Mechanics* **8**(4) 840-858.

[10] S.S. SHEORAN, P. KUNDU (2016) Fractional Order Generalized Thermoelasticity Theories: A Review. *International Journal of Advances in Applied Mathematics and Mechanics* **3**(4) 76-81.

[11] I. ABBAS (2018) A Study on Fractional Order Theory in Thermoelastic Half-Space under Thermal Loading. *Physical Mesomechanics* **21**(2) 150-156.

[12] M. MARIN (1997) Cesaro Means in Thermoelasticity of Dipolar Bodies. *Acta Mechanica* **122**(1-4) 155-168.

[13] M. MARIN (1994) The Lagrange Identity Method in Thermoelasticity of Bodies with Microstructure. *International Journal of Engineering Science* **32**(8) 1229-1240. DOI: 10.1016/0020-7225(94)90034-5.

[14] M. MARIN, R. AGARWAL, L. CODARCEA (2017) A Mathematical Model For Three-Phase-Lag Dipolar Thermo-Elastic Bodies. *Journal of Iinequalities and Application* **2017**(1) 1-16. DOI: 10.1186/S13660-017-1380-5.

[15] M. MARIN, E. CRACIUN, N. POP (2016) Considerations On Mixed Initial-Boundary Value Problems for Micropolar Porous Bodies. *Dynamic Systems and Applications* **25**(1-2) 175-196.

[16] P. LATA, I. KAUR (2018) Effect of Hall Current in Transversely Isotropic Magnetothermoelastic Rotating Medium with Fractional Order Heat Transfer Due to Normal Force. *Advances in Materials Research* **7**(3) 203-220.

[17] P. LATA, I. KAUR (2019) Transversely Isotropic Thermoelastic Thin Circular Plate with Time Harmonic Sources. *Geomechanics and Engineering* **19**(1) 29-36. DOI: 10.12989/gae.2019.19.1.029.

[18] M. MARIN (2009) On the Minimum Principle for Dipolar Materials with Stretch. *Nonlinear Analysis Real World Applications* **10**(3) 1572-1578.

[19] M. MARIN (2010) Lagrange Identity Method for Microstretch Thermoelastic Materials. *J. of Mathematical Analysis and Applications* **363**(1) 275-286.

[20] R. KUMAR, P. KAUSHAL, R. SHARMA (2018) Transversely Isotropic Magneto-Visco Thermoelastic Medium with Vacuum and Without Energy Dissipation. *Journal of Solid Mechanics* **10**(2) 416-434.

[21] P. LATA, I. KAUR (2019) Axisymmetric Thermomechanical Analysis of Transversely Isotropic Magneto Thermoelastic Solid Due to Time-Harmonic Sources. *Coupled Systems Mechanics* **8**(5) 415-437. DOI: 10.12989/csm.2019.8.5.415.

[22] P. LATA, I. KAUR (2019) Plane Wave Propagation in Transversely Isotropic Magnetothermoelastic Rotating Medium with Fractional Order Generalized Heat Transfer. *Structural Monitoring and Maintenance* **6**(3) 191-218. DOI: 10.12989/smm.2019.6.3.191.

[23] P. LATA, I. KAUR (2019) Effect of Rotation and Inclined Load on Transversely Isotropic Magneto Thermoelastic Solid. *Structural Engineering And Mechanics* **70**(2) 245-255.

[24] P. LATA, I. KAUR (2019) Thermomechanical Interactions in Transversely Isotropic Magneto Thermoelastic Solid with Two Temperatures and without Energy Dissipation. *Steel and Composite Structures* **32**(6) 779-793. DOI: 10.12989/scs.2019.32.6.779.

[25] P. LATA, I. KAUR (2019) Transversely Isotropic Magneto Thermoelastic Solid With Two Temperature and Without Energy Dissipation in Generalized Thermoelasticity Due to Inclined Load. *Sn Applied Sciences* **1** 426. DOI: 10.1007/s42452-019-0438-z.

[26] P. LATA, I. KAUR (2019) Thermomechanical Interactions Due to Time Harmonic Sources in a Transversely Isotropic Magneto Thermoelastic Solids with Rotation. *International Journal of Microstructure and Materials Properties* **14**(6) 549-577.

[27] M. MARIN, E.CRACIUN (2017) Uniqueness Results For A Boundary Value Problem In Dipolar Thermoelasticity To Model Composite Materials. *Composites Part B: Engineering* **126** 27-37.

[28] I. KAUR, P. LATA (2019) Rayleigh Wave Propagation in Transversely Isotropic Magneto Thermoelastic Medium with Three Phase Lag Heat Transfer and Diffusion. *International Journal of Mechanical and Materials Engineering* **14**(12) 1-11. DOI: 10.1186/s40712-019-0108-3.

[29] R. KUMAR, N. SHARMA, P. LATA (2017) Effects of Hall Current and Two Temperatures Intransversely Isotropic Magnetothermoelastic with and without Energy Dissipation Due to Ramp-Type Heat. *Mechanics of Advanced Materials and Structures* **24**(8) 625-635.

[30] M. SCHOENBERG, D. CENSOR (1973) Elastic Waves in Rotating Media. *Quarterly of Applied Mathematics* **31** 115–125.

[31] W. SLAUGHTER (2002) “The Linearised Theory of Elasticity”. Birkhausar.

[32] G. HONIG, U. HIRDES (1984) A Method for the Numerical Inversion of Laplace Transform. *Journal of Computational and Applied Mathematics* **10** 113-132.

[33] W. PRESS, S.A. TEUKOLSHY W.T. VELLERLING, B. FLANNERY (1986) “Numerical Recipes in Fortran”. Cambridge University Press, Cambridge.

[34] R. DHALIWAL, A. SINGH (1980) “Dynamic Coupled Thermoelasticity”. New Delhi-India: Hindustan Publication Corporation.