Journal of Theoretical and Applied Mechanics, Sofia, Vol.51 (2021) pp. 3-21
FLUID MECHANICS

EFFECT OF TEMPERATURE DEPENDENT VISCOSITY ON THERMAL CONVECTION IN FERROFLUID LAYER

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[Received: 18 November 2019. Accepted: 05 October 2020]

ABSTRACT: A linear stability analysis of onset of thermal convection in ferromagnetic fluid layer heated from below with temperature-dependent viscosity is investigated for general bounding surfaces. The eigenvalue problem is solved using a single term Galerkin technique and the values of critical Rayleigh numbers for different combinations of stress free and no-slip boundary conditions are computed for linear and exponential temperature dependent viscosity variations. The effects of temperature dependent viscosity parameter, magnetic Rayleigh number and the magnetization parameter on the onset of stationary convection are investigated numerically. It is found that increase in temperature dependent viscosity and decrease in magnetic number delay the onset of ferroconvection, while the nonlinearity of fluid magnetization has marginal influence on the stability of the system.

KEY WORDS: thermal convection, temperature dependent viscosity, ferrofluids, Galerkin method, Rayleigh number, magnetic Rayleigh number.

1 INTRODUCTION

Due to some physical properties of ferrofluids and its important applications in different fields of industry and medicine, the study of thermal convection in ferrofluids has gained much attention in the recent past. Ferrohydrodynamics, in contrast to the magnetohydrodynamics of ordinary fluids, is of special interest, since the flow here occurs without the imposition of electrical currents and thus in the absence of corresponding *Lorentz* forces. Ferrofluids are very important and useful tool in variety of applications in the fields of technology and biomedical sciences. Rosensweig [1] provided a detailed introduction to the subject of ferrofluids and its properties in his monograph and described the ferrofluids as stable colloidal suspensions consisting of single-domain magnetic particles coated with surfactant and immersed in a carrier fluid. The particles are commonly magnetite and of order 10 nm. in diameter, while the carrier liquid is typically an oil or water base. He observed that magnetization is a function of magnetic field, temperature and density of the fluid in general, and

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any variation in these properties impart change in the body force distribution in the fluid, which eventually give rise to convection in ferrofluids in the presence of a gradient of magnetic field and the phenomenon is known as ferroconvection, analogous to classical thermal convection (Chandrasekhar [2]).

Finlayson [3] studied the convective instability of a ferromagnetic fluid layer heated from below in the presence of a uniform vertical magnetic field and concluded from the characteristic equation, under the approximation namely; magnetic parameter M_2 very small ($M_2 \approx 0$), that the oscillatory motions of growing amplitude are not allowed and hence the stationary convection is the only mode of instability. Further, he derived an expression for the Rayleigh number for the case of stationary convection from the exact solution of the problem for the case of both free boundaries. He also obtained the numerical values of the critical magnetic Rayleigh number from an approximate solution for the case of both rigid boundaries using Galerkin method. Since then many authors including: Curtis [4], Lalas and Carmi [5], Shliomis [6] and Schwab *et al.* [7] have studied the convective instability in ferrofluids under varying assumptions of ferrohydrodynamics.

One of the well-known phenomena generated by the influence of magnetic field on the ferrofluids is the change of their viscosity. Rosensweig *et al.* [8] in the study of concentrated magnetite ferrofluids revealed that the increasing magnetic field strength increases the viscosity of ferrofluids. Shliomis [6] investigated the effect of a homogeneous magnetic field on the viscosity of a fluid with solid particles possessing intrinsic magnetic moments. The effect of magnetic field dependent (MFD) viscosity on the onset of ferroconvection in a rotating medium has been investigated by Vaidyanathan *et al.* [9]. Nanjundappa *et al.* [10] have also studied the effect of magnetic field dependent (MFD) viscosity on the onset of ferroconvection.

Review of the literature has revealed that for most of the fluids, viscosity is temperature dependent and shows a significant variation with respect to temperature. Torrance and Turcotte [11] found that the viscosity of liquids decreases with increasing temperature, while reverse trend is observed in gases. Early studies related to convection in fluids are by Palm [12] and Stengel *et al.* [13], who respectively considered the *linear* and *exponential* variations of temperature dependent viscosity. Jenkins [14] compared the *linear* and *exponential* viscosity variations with temperature and revealed that the linear dependence is realistic for fluids with small values of viscosity, while exponential dependence is more realistic for fluids with high viscosity. Dhiman and Kumar [15] and Dhiman and Sharma [16, 17] investigated the effect of temperature-dependent viscosity on the Rayleigh-Bénard convection problem in classical fluid, nanofluid and ferrofluid layers, respectively for different cases of boundary conditions.

A critical review of the above studies reveals that the ferrofluid motions have shown strong dependence upon some physical parameters (temperature, magnetic field and density), therefore, it seems that the temperature-dependent viscosity may be one of the rheological properties that influence the convective motions of the ferrofluids. The present study is motivated by the above point and thus deals with the study of the effect of temperature dependent viscosity on the onset of convection in a ferromagnetic fluid layer in the presence of a uniform vertical magnetic field for different combinations of boundary conditions. The eigenvalue problem is solved using the Galerkin technique for different possible cases of combination of *dynamically free* and *rigid* boundary conditions. The effects of temperature dependent viscosity parameter, magnetic Rayleigh number and the nonlinear fluid magnetization parameter on the onset of stationary convection are discussed.

2 Physical Configuration and Eigenvalue Problem

Let an infinite horizontal layer of ferromagnetic fluid of thickness 'd' is confined between two horizontal boundaries $z = 0$ and $z = d$ in the force field of gravity $\vec{g}(0, 0, -g)$. The lower and upper boundaries are respectively maintained at uniform temperature T_0 and T_1 $(T_0 > T_1)$, thus maintaining a uniform adverse temperature gradient (β) having tendency to increase the density of the fluid in vertical direction z. The fluid is taken to be electrically non-conducting and is also permeated with a uniform vertical magnetic field $\vec{H}(0, 0, H_0)$. Here, it is also assumed that magnetic field $\vec{H}$ and magnetic induction $\vec{B}$ act parallel to each other. The fluid is assumed to be incompressible (Boussinesq) and the viscosity is temperature dependent given by; $\mu = \mu_0 f(T)$.

Following the usual steps of linear stability theory, the non-dimensional linearized perturbation equations and boundary conditions governing the above physical problem are derived in Appendix A; equations (A.22)–(A.26).

System of equations (A.22)–(A.24) and boundary conditions (A.25)–(A.26) constitutes an *eigenvalue problem* for complex growth rate $(p = p_r + ip_i)$ for the given values of other parameters. If $p_r \geq 0$ implies $p_i = 0$ (or *equivalently*, $p_i \neq 0$ implies $p_r < 0$) for all wave number a^2, then for neutral instability $(p_r = 0)$, we have $p = 0$. This situation in hydrodynamic stability theory is termed as *principle of exchange of stabilities* (PES), which means that instability sets in as stationary convection. Proving the validity of PES for any problem results in the elimination of the unsteady terms form the governing equations of the stability problems and thus the transition from stability to instability occurs via a marginal stationary state characterized by $p = 0$. Thus, putting $p = 0$ in governing perturbations, we obtain an eigenvalue problem for Rayleigh number.

The problem of thermal convection in ferrofluids [3] with temperature dependent viscosity was recently studied by Dhiman and Sharma [17] for general cases boundary conditions and proved the validity of the *principle of exchange of stabili-*

ties (PES) using Pellew and Southwell method of conjugate eigenfunctions for this problem governed by equations (A.22)–(A.26), when the magnetic parameter M_2 is very small ($M_2 \approx 0$). This yields that convective instability in ferromagnetic fluids sets in as stationary convection, which also validates the claim of Finlayson [3] regarding the nonexistence of oscillatory motions (for constant viscosity) for the cases of both free and both rigid boundary conditions. Since the typical value of M_2 for magnetic fluids is of the order of 10^{-6}, hence its effect can be neglected as compared to unity in the eigenvalue problem governed by these equations.

In view of the above discussion, the eigenvalue problem for Rayleigh number can be obtained by taking $p = 0$. Hence, equations (A.22)–(A.24) together with the combinations of the *rigid* and *dynamically free* boundary conditions (A.25)–(A.26) for stationary convection with $p = 0$ at $M_2 = 0$, assume the following form:

$$f(D^2 - a^2)^2 w + 2(Df)D(D^2 - a^2)w + D^2 f(D^2 + a^2)w = Ra^2[(1 + M_1)\theta - M_1 D\varphi]\,, \tag{1}$$

$$(D^2 - a^2)\theta = -w\,, \tag{2}$$

$$(D^2 - a^2 M_3)\varphi = D\theta\,, \tag{3}$$

subject to either of the following cases of boundary conditions:

Case 1: *Both boundaries dynamically free*

$$w = 0 = \theta = D\varphi = D^2 w \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1\,. \tag{4}$$

Case 2: *Both boundaries rigid*

$$w = 0 = \theta = D\varphi = Dw \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1\,. \tag{5}$$

Case 3: *One rigid and other free boundary*

$$\begin{cases} w = 0 = \theta = D\varphi = Dw & \text{at} \quad z = 0 \quad (\text{or} \quad z = 1) \\ w = 0 = \theta = D\varphi = D^2 w & \text{at} \quad z = 1 \quad (\text{or} \quad z = 0)\,. \end{cases} \tag{6}$$

3 Stability Analysis

We shall now use the Galerkin method to solve the above eigenvalue problem. For this, we shall use a single term Galerkin method (*cf.* [3, 15] and [16]) to find the values of Rayleigh numbers for each case of boundary conditions (4)–(6). Let us take a single-term in the expansions of w, θ and φ as

$$w = Aw_1(z)\,, \quad \theta = B\theta_1(z)\,, \quad \varphi = C\varphi_1(z)\,, \tag{7}$$

where w_1, θ_1 and φ_1 are suitably chosen trial functions which satisfy the respective boundary conditions (4)–(6) and A, B and C are constants.

Substituting the above values of w, θ and φ in equations (1)–(3), multiplying each of the resulting equations (the residuals) by w_1, θ_1 and φ_1, integrating these equations by parts suitable number of times using the relevant boundary conditions (4)–(6), we obtain three homogeneous algebraic equations in A, B and C having its coefficients as definite integrals. For the existence of the nontrivial solution, the determinant of the coefficients must vanish, which yields the following eigenvalue equation involving the relation between Rayleigh number R and wave number a (*the cell size*), as a first order

$$R = \frac{(I_1 + a^2 I_2) I_3}{a^2 I_4 \left\{ (1 + M_1) I_4 + \dfrac{I_5 (M_1 I_6)}{I_7} \right\}} . \tag{8}$$

Here

$$I_1 = \int_0^1 f\left((D^2 w_1)^2 + 2a^2 (D w_1)^2 + a^4 (w_1)^2\right) dz , \quad I_2 = a^2 \int_0^1 (D^2 f)(w_1)^2 dz ,$$
$$I_3 = \int_0^1 \left((D\theta_1)^2 + a^2 (\theta_1)^2\right) dz , \quad I_4 = \int_0^1 \theta_1 w_1 dz , \quad I_5 = \int_0^1 D\theta_1 \varphi_1 dz ,$$
$$I_6 = \int_0^1 D\varphi_1 w_1 dz , \quad I_7 = \int_0^1 \left((D\varphi_1)^2 + a^2 M_3 (\varphi_1)^2\right) dz .$$

From the above expression (8), we shall compute the minimum of the eigenvalues; *i.e.* the critical Rayleigh number R_c, at which the onset of stationary instability occurs for each case of boundary conditions for both *exponential* and *linear* temperature dependent viscosity variations. For this, we consider following non-dimensional *exponential* and *linear* viscosity variation laws (*cf.* [15] and [16]):

$$f = e^{\delta z} \quad \text{and} \quad f = (1 + \delta z) , \tag{9}$$

where $\delta = \gamma \beta d$ is the viscosity variation parameter.

We shall now calculate the values of *critical* Rayleigh numbers for both *exponential* and *linear* variation of viscosity numerically, for each case of boundary conditions, separately.

Case 1: Both boundaries dynamically free. For the present case, let us choose the following suitable polynomial trial functions satisfying the boundary conditions (4)

$$w = z^4 - 2z^3 + z , \quad \theta = z(z-1) \quad \text{and} \quad \varphi = \frac{z^2}{2} - \frac{z^3}{3} . \tag{10}$$

Using the trial functions (10) and *linear* viscosity variation law (9) in the integrals I_1 to I_7 above, computing each of it numerically and finally using the values in expression (8), we obtain the following Rayleigh number for *linear* viscosity variation:

$$(11)\quad R_{\text{ff}}^{\text{lin}} = \frac{14(42+13a^2M_3)}{867a^2(42+13a^2(1+M_1)M_3)} \times (10+a^2)(31a^4+612a^2+3024)(2+\delta)\,.$$

Similarly using the numerical values of integrals I_1 to I_7 calculated using the *exponential* viscosity law (9) in expression (8), we obtain the following expression for Rayleigh number for the case of *exponential* viscosity variation:

$$(12)\quad R_{\text{ff}}^{\text{exp}} = \frac{(42+13a^2M_3)}{a^2(42+13a^2(1+M_1)M_3)}\,\frac{11760}{289\delta^9}\Big[(10+a^2)\Big\{144\delta^4\big(-12-6\delta - \delta^2+e^{\delta}(12-6\delta+\delta^2)\big)+a^4\big(-20160-1080\delta-1440\delta^2+120\delta^3 + 48\delta^4-\delta^6+e^{\delta}(20160-10080\delta+1440\delta^2+120\delta^3-48\delta^4+\delta^6)\big) + 2a^2\delta^2\big(-15840-7290\delta-1152\delta^2+84\delta^3+36\delta^4-\delta^6 + e^{\delta}(15840-7290\delta+1152\delta^2+84\delta^3-36\delta^4+\delta^6)\big)\Big\}\Big]\,.$$

Now, from expression (11) for the case of *linear* viscosity variation, the critical value of the wave number (the *critical cell size*) is determined from the condition, $d(R_{\text{ff}}^{\text{lin}})/da = 0$, which yields the following algebraic equation, whose positive root gives the critical wave number (a_c) depending upon M_1 and M_3:

$$(13)\quad C_1a^{10}+C_2a^8+C_3a^6+C_4a^4+C_5a^2+C_6 = 0\,.$$

Here

$$\begin{aligned}
C_1 &= \left(5239\,(1+M_1)\,M_3^2\right)\,,\\
C_2 &= (77909\,(1+M_1)\,M_3^2+(8463M_1+33852)M_3)\,,\\
C_3 &= (503412M_3+54684)\,,\\
C_4 &= \left(-2555280\,(1+M_1)\,M_3^2+2496312M_1M_3+813204\right)\,,\\
C_5 &= (-16511040\,(1+M_1)\,M_3)\,,\\
C_6 &= -26671680\,.
\end{aligned}$$

Note that equation (13) is a 5th degree algebraic equation in a^2 with coefficients involving M_1 and M_3 implicitly and it is cumbersome to solve analytically equation (13) for any positive root. Hence, in the present investigation, equation (13) is solved numerically (using *Mathematica*® software) and the values of the critical wave numbers a_c for particular set of values of δ, M_1 and M_3 are obtained and consequently using these values of a_c, the values of *critical* Rayleigh numbers $(R_{ff}^{lin})_c$ are obtained. The results so obtained are also presented in Tables 1 and 2 for the case of *linear viscosity* variation.

Now, for the case of *ordinary fluid*, *i.e.* in the absence of magnetic parameters when $M_1 = 0 = M_3$, equation (13) reduces to the following equation:

$$31a^6 + 461a^4 - 15120 = 0\,, \tag{14}$$

and the only positive root of this equation is given by $a_c = 2.22$.

Thus, the minimum (*critical*) value of Rayleigh number $(R_{\text{ff}}^{\text{lin}})$ given by equation (11), using $a_c = 2.22$, is given by

$$(R_{\text{ff}}^{\text{lin}})_c \approx 332.267(2+\delta) \quad \text{at} \quad a_c = 2.22\,, \tag{15}$$

which is the same value as obtained by Dhiman and Kumar [15] for the thermal convection problem for ordinary fluid (*Newtonian fluid*) with temperature dependent viscosity.

Further, for the case of *constant viscosity* $(\delta \to 0)$, the expression (15) yields

$$R_c = 664.526 \quad \text{at} \quad a_c = 2.22\,, \tag{16}$$

which is about 6.5% higher than the value $R_c = 657.51$ as obtained by Chandrasekhar [2] using full numerical solution, in comparison to the present first approximation solution.

Also, for the case of *constant viscosity*, the expressions (11) and (12) in view the values of the integrals I_1 to I_7, yield the following same expression for Rayleigh number using *exponential* and *linear* variations of viscosity laws:

$$R_{\text{ff}} = \frac{28(10+a^2)(3024+612a^2+31a^4)(42+13M_3a^2)}{867a^2(42+13a^2(1+M_1)M_3)}\,. \tag{17}$$

The minimum value of the R_{ff} given in (17) can be calculated numerically using the critical values of wave number a_c computed from equation (13) for any positive root.

Finlayson [3] in his analysis of ferroconvection (with *constant* viscosity) obtained the following expression for Rayleigh number for the case of both *free boundaries*:

$$R = \frac{(\pi^2+a^2)^3(\pi^2+M_3a^2)}{a^2(\pi^2+a^2(1+M_1)M_3)}\,, \tag{18}$$

and the *critical* value of this Rayleigh number (R) denoted as R_c can computed using critical wave number a_c, which can be obtained from the following equation obtained analogous to equation (13) above:

$$D_1 a^{10} + D_2 a^8 + D_3 a^6 + D_4 a^4 + D_5 a^2 + D_6 = 0\,, \tag{19}$$

where

$$D_1 = 2(1 + M_1)M_3^2\,, \qquad D_2 = ((4 + 3M_1)\,M_3 + M_1 + 4))M_3\pi^4\,,$$
$$D_3 = 2((1 + 3M_1)\,M_3^2 + 3M_3 + 1)\pi^4\,, \quad D_4 = (-\,(3 + M_3)\,M_1 M_3 + 3)\,\pi^6\,,$$
$$D_5 = (-2\,(1 + M_1)\,M_3)\pi^8\,, \qquad D_6 = -\pi^{10}\,.$$

Now, the values of critical wave numbers (a_c^2) computed from expressions (13) and (19) respectively, we obtain the following values of critical Rayleigh numbers $(R_{\rm ff})_c$ and R_c from expressions (17) and (18): $(R_{\rm ff})_c = 342.52$ and $(R)_c = 357.235$, for $a_c^2 = 5.174$ (with $M_1 = 1$ and $M_3 = 10$); $(R_{\rm ff})_c = 65.973$ and $(R)_c = 75.139$, for $a_c^2 = 5.352$ (with $M_1 = 10$ and $M_3 = 6$).

It is clear from the above set of values that the critical Rayleigh numbers computed using (present) *Galerkin first approximation* solution method for the case of *constant viscosity* are in close agreement with the respective values obtained by Finlayson [3] in his analysis for this problem.

Further, using Finlayson [3] analysis, we have following expressions for N (the *magnetic Rayleigh number*) for very large values of M_1, obtained from expressions (11) and (12) respectively for *linear* and *exponential* variations of viscosity

$$(N_{\rm ff}^{\rm lin}) = (R_{\rm ff}^{\rm lin} M_1) = \frac{14(2+\delta)(30240+9144a^2+922a^4+31a^6)(42+13a^2M_3)}{11271a^4M_3} \tag{20}$$

$$\begin{aligned}(N_{\rm ff}^{\rm exp}) = (R_{\rm ff}^{\rm exp} M_1) = {} & \frac{11760(10 + a^2)(42 + 13a^2M_3)}{3757\delta^9 a^4 M_3} \\ & \times \Big[\Big\{144\delta^4(-12 - 6\delta - \delta^2 + e^\delta(12 - 6\delta + \delta^2)) \\ & + a^4\big(-20160 - 10080\delta - 1440\delta^2 + 120\delta^3 + 48\delta^4 - \delta^6 \\ & + e^\delta(20160 - 10080\delta + 1440\delta^2 + 120\delta^3 - 48\delta^4 + \delta^6)\big) \\ & + 2a^2\delta^2\big(-15840 - 7290\delta - 1152\delta^2 + 84\delta^3 + 36\delta^4 - \delta^6 \\ & + e^\delta(15840 - 7290\delta + 1152\delta^2 + 84\delta^3 - 36\delta^4 + \delta^6)\big)\Big\}\Big]\,,\end{aligned} \tag{21}$$

which represent the magnetic mechanism operating in the absence of buoyancy effects.

Following the above analysis, the numerical values of critical wave numbers a_c at which the minimum (*critical*) values $(N_{\text{ff}}^{\text{exp}})_c$ and $(N_{\text{ff}}^{\text{lin}})_c$ of N_{ff} (for both *exponential* and *linear* cases) exist can be computed analogously. The values so obtained for *linear* case of viscosity variations are presented in Table 3.

Now, proceeding as in Case 1 of boundary conditions above, we can obtain the analogous expressions for Rayleigh numbers for other two cases of boundary conditions (Case 2 and Case 3), also.

Case 2: Both boundaries rigid. The suitable polynomial trial functions satisfying the boundary conditions (5) chosen are

$$w = z^4 - 2z^3 + z^2\,, \quad \theta = z(z-1) \quad \text{and} \quad \varphi = \frac{z^2}{2} - \frac{z^3}{3}\,.$$

Using these trial functions and the viscosity variation laws (9) in expression (8) and proceeding as in Case 1 above, we obtain following expressions for Rayleigh numbers respectively for *linear* and *exponential* variation of viscosity:

$$(22) \qquad (R_{\text{rr}}^{\text{lin}}) = \frac{14(2520(4+744\delta)+(744a^2+34a^4+a^6)(2+\delta)(42+13a^2M_3)}{27a^2(42+13a^2(1+M_1)M_3)}\,,$$

$$\begin{aligned}(23) \quad (R_{\text{rr}}^{\text{exp}}) = \frac{7840(10+a^2)[42+13a^2M_3]}{3a^2\delta^9[42+13a^2M_3+13a^2M_1M_3]}\Big[\Big\{6a^4\big(-1680-840\delta \\ -180\delta^2-20\delta^3-\delta^4+e^\delta(1680-840\delta-180\delta^2-20\delta^3-\delta^4)\big) \\ +\delta^4\big(-864-432\delta-96\delta^2-12\delta^3-\delta^4+e^\delta(864-432\delta+96\delta^2 \\ -12\delta^3+\delta^4)\big)+2a^2\delta^2\big(-7920-3960\delta-852\delta^2-96\delta^3-5\delta^4 \\ +e^\delta(7920-3960\delta+852\delta^2-96\delta^3+5\delta^4)\big)\Big\}\Big].\end{aligned}$$

Case 3: Lower rigid and upper free boundaries. Let us choose following suitable polynomial trial functions satisfying the either of boundary conditions (6)

$$w = 2z^4 - 5z^3 + 3z^2\,, \quad \theta = z(z-1) \quad \text{and} \quad \varphi = \frac{z^2}{2} - \frac{z^3}{3}\,,$$

and proceeding analogously as in other two cases of boundary conditions above, upon using these trial functions and the viscosity variation laws given in (9) in expression (8), we obtain following expressions for Rayleigh numbers for *linear* and *exponential* variation of viscosity, respectively:

(24)

$$(R_{\text{rf}}^{\text{lin}}) = \frac{7(42+13a^2M_3)(10+a^2)(1512(12+5\delta)+216a^2(8+5\delta)+a^4(76+43\delta))}{507a^2(42+13a^2(1+M_1)M_3)}$$

$$(25) \qquad (R_{\text{rf}}^{\text{exp}}) = \frac{11760(10+a^2)[42+13a^2M_3]}{169a^2\delta^9[42+13a^2M_3+13a^2M_1M_3]}\Big[\Big\{18\delta^4\big(-384 - 240\delta - 66\delta^2 - 10\delta^3 - \delta^4 - 6e^{\delta}(64-24\delta+3\delta^2)\big) + a^4\big(-36(2240+1400\delta+370\delta^2+50\delta^3+3\delta^4) + e^{\delta}(80640-30240\delta+3240\delta^2+240\delta^3-72\delta^4+\delta^6)\big) + 2a^2\delta^2\big(-18(3520+2200\delta+584\delta^2+80\delta^3+5\delta^4 + e^{\delta}(63360-23760\delta+2592\delta^2+168\delta^3-54\delta^4+\delta^6)\big)\Big\}\Big].$$

It is to mention here that using the following trial functions, which satisfy the other set of boundary conditions (6); *i.e. upper rigid and lower free boundary*

$$w = 2z^4 - 3z^3 + z\,, \quad \theta = z(z-1) \quad \text{and} \quad \varphi = \frac{z^2}{2} - \frac{z^3}{3}\,,$$

we have obtained the expressions analogous to (24) and (25), which yield almost same numerical values for Rayleigh numbers and hence are omitted here for the sake of compactness.

4 Results and Discussion

In the present analysis, we have investigated the effects of *linear* and *exponential* temperature-dependent viscosity on the onset of thermal convection in ferrofluids. As reported earlier that the *principle of exchange of stabilities* (PES) is valid for the problem with variable viscosity [17], when the values of the magnetic parameter ($M_2 \approx 0$) are very small, which yields that the onset of instability is through stationary modes only. Therefore, the expressions for Rayleigh numbers for stationary convection (for both *linear* and *exponential* cases of viscosity variations) for different cases of boundary conditions are derived using a single-term *Galerkin method* and the values of critical wave numbers and Rayleigh numbers are computed numerically from these eigenvalues.

In Table 1, the values of critical wave numbers a_c^2 computed numerically from expressions (13) for *linear* temperature-dependent viscosity with $\delta = 0.0, 0.2, 0.5, 0.7, 0.9$, $M_1 = 1, 5$ and fixed $M_3 = 10$ are presented, for the case of *both free boundaries.* Using these values of a_c^2 in expression (11), the values of the critical Rayleigh number $(R_{\text{ff}})_c$ are computed and are shown in Table 1. Similarly, the values of a_c^2 and Rayleigh numbers; $(R_{\text{rr}})_c$ and $(R_{\text{rf}})_c$ for both rigid and one rigid one free cases of boundary conditions are computed from expressions (22) and (24) respectively and are also presented in Table 1.

Also, for *linear* viscosity variation, the values of a_c^2 and analogous $(R_{\text{ff}})_c$, $(R_{\text{rr}})_c$ and $(R_{\text{rf}})_c$ which are computed from these expressions for the same set of values δ

Table 1: Values of critical wave number a_c^2 and Rayleigh numbers R with δ, M_1 and fixed M_3

$M_3 = 10$		Linear viscosity variation					
M_1	δ	a_c^2	$(R_{\text{ff}})_c$	a_c^2	$(R_{\text{rr}})_c$	a_c^2	$(R_{\text{rf}})_c$
1	0.0	5.174	342.52	9.921	889.15	7.336	581.78
	0.2	5.174	376.78	9.921	978.06	7.237	641.06
	0.5	5.174	428.16	9.921	1111.4	7.120	729.84
	0.7	5.174	462.41	9.921	1200.3	7.058	788.97
	0.9	5.174	496.79	9.921	1289.2	7.004	848.05
5	0.0	5.319	116.49	10.06	299.56	7.476	196.72
	0.2	5.319	128.14	10.06	329.51	7.378	216.81
	0.5	5.319	145.62	10.06	374.45	7.262	246.89
	0.7	5.319	157.26	10.06	404.40	7.199	266.93
	0.9	5.319	168.92	10.06	434.36	7.146	286.95

with $M_3 = 2$, 6 and fixed $M_1 = 1000$ are presented in Table 2. Further, in Table 3, the values of a_c^2 and analogous $(N_{\text{ff}})_c$, $(N_{\text{rr}})_c$ and $(N_{\text{rf}})_c$ are presented, which are computed from expression (20) and from analogous expressions for the same set of values of parameters. Figs. (1a)–(1f) depict the variations of Rayleigh number and *magnetic* Rayleigh number (R_{ff} and N_{ff}) *versus* the square of the wave number a^2 for

Table 2: Values of critical wave number a_c^2 and Rayleigh numbers R with δ, M_3 and fixed M_1

$M_1 = 1000$		Linear viscosity variation					
M_3	δ	a_c^2	$(R_{\text{ff}})_c$	a_c^2	$(R_{\text{rr}})_c$	a_c^2	$(R_{\text{rf}})_c$
2	0.0	6.582	0.849	11.443	2.014	8.792	1.368
	0.2	6.582	0.934	11.443	2.033	8.695	1.510
	0.5	6.582	1.062	11.443	2.518	8.581	1.723
	0.7	6.582	1.146	11.443	2.719	8.519	1.865
	0.9	6.582	1.231	11.443	2.933	8.467	2.00
6	0.0	5.366	0.731	10.383	1.84	7.793	1.219
	0.2	5.366	0.804	10.383	2.023	7.695	1.344
	0.5	5.366	0.914	10.383	2.302	7.580	1.531
	0.7	5.366	0.987	10.383	2.486	7.519	1.656
	0.9	5.366	1.061	10.383	2.671	7.466	1.781

Table 3: Values of critical wave number a_c^2 and magnetic Rayleigh numbers N with δ, M_3 and fixed M_1

$M_1 = 1000$		Linear viscosity variation					
M_3	δ	a_c^2	$(N_{\text{ff}})_c$	a_c^2	$(N_{\text{rr}})_c$	a_c^2	$(N_{\text{rf}})_c$
2	0.0	6.584	850.679	11.445	2017.1	8.794	1369.7
	0.2	6.584	935.747	11.445	2035.8	8.697	1512.0
	0.5	6.584	1063.35	11.445	2521.3	8.523	1725.2
	0.7	6.584	1148.42	11.445	2723.0	8.521	1867.2
	0.9	6.584	1233.48	11.445	2924.8	8.469	2009.2
6	0.0	5.637	732.049	10.383	1843.7	7.793	1220.8
	0.2	5.637	805.254	10.383	2028.1	7.696	1346.0
	0.5	5.637	915.061	10.383	2304.6	7.581	1533.6
	0.7	5.637	988.266	10.383	2489.0	7.519	1658.5
	0.9	5.637	1061.47	10.383	2673.4	7.466	1783.3

exponential viscosity variations (for the case of *both free boundaries*) with different values of δ and fixed values of M_1 and M_3.

It is found from the values presented in Tables 1–3 for the case of *linear* viscosity variation that the critical wave number (the cell size) is independent of temperature-dependent viscosity δ, but depend upon the *magnetic* parameters M_1 and M_3. It is observed that the values of the critical Rayleigh numbers increase for increasing values of δ for all cases of boundary conditions, which yields that the temperature-dependent viscosity has stabilizing effect on the onset of stationary convection in ferrofluid.

We observed that for increasing values of magnetic number (M_1), the values of critical Rayleigh numbers decrease, which implies that M_1 has destabilizing effect on the onset of stationary convection in ferrofluid layer. This behavior of the system may be due to the increase in the destabilizing magnetic force, as heat is transported more efficiently in magnetic fluids as compared to ordinary fluids. A similar behaviour of Rayleigh numbers plotted in Figs. (1a)–(1b) *versus* a^2 for *exponential* case of viscosity variation with different values of M_1 is also observed.

From Table 2, we observed that the values of Rayleigh numbers decrease with increasing values of M_3 (the measure of *nonlinearity of magnetization*) which yields that M_3 has a destabilizing effect on the onset of stationary convection for all cases of boundary conditions. This may be due to the large *pyromagnetic* coefficient or large *temperature gradient*. However, the destabilizing effect of nonlinearity of the fluid magnetization observed in the present analysis is marginal. Similar effect of M_3 on

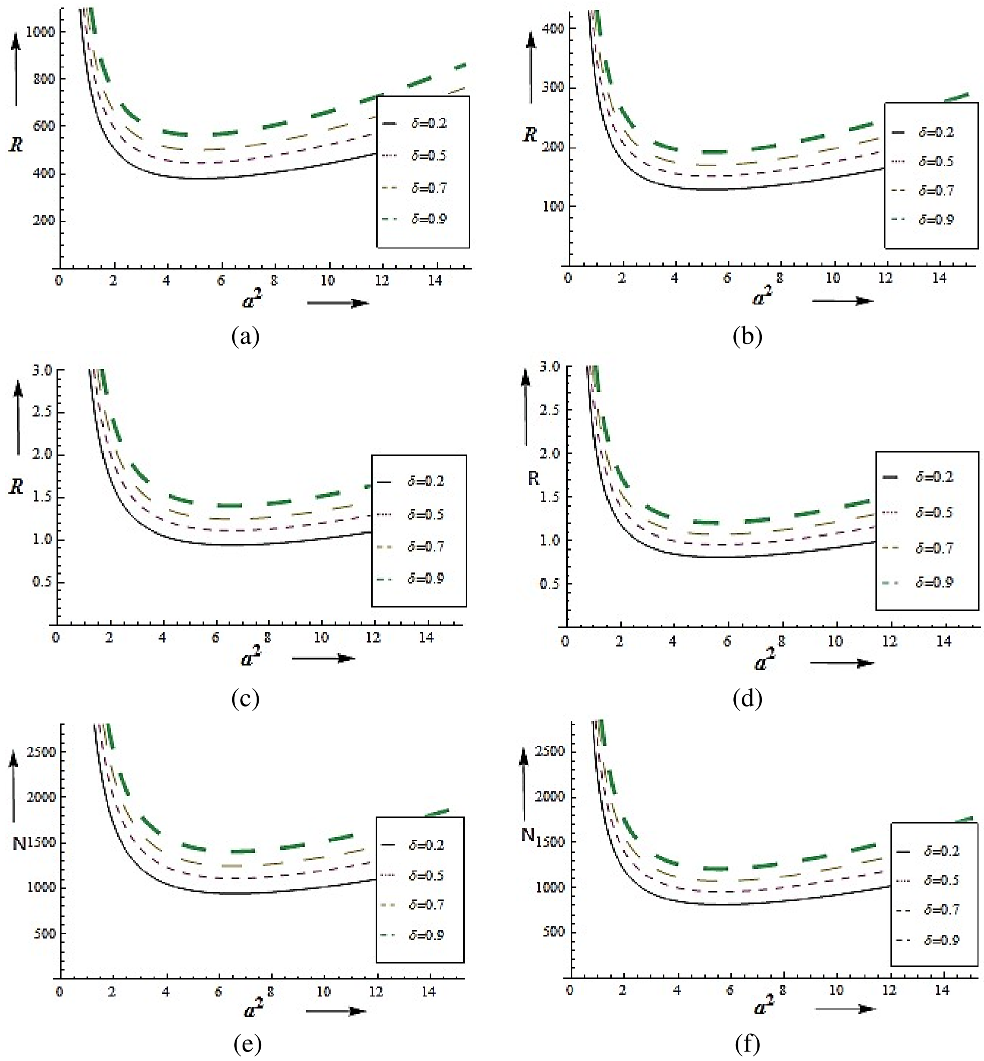

Fig. 1: Plots showing variations of Rayleigh numbers vs. wave number for exponential viscosity variations: Case of both free boundaries: (a) & (b) – variations of R vs. a^2 respectively for $M_1 = 1$ and 5 with fixed $M_3 = 10$; (c) & (d) – variation of R vs. a^2 respectively for $M_3 = 2$ and 6 with fixed $M_1 = 1000$; (e) & (f) – variation of N vs. a^2 respectively for $M_3 = 2$ and 6 with fixed $M_1 = 1000$.

Rayleigh number for *exponential* viscosity variation is also observed from Figs. (1c) & (1d). From Table 3, we observed that the values of critical *magnetic* Rayleigh number also decrease with the increasing values of M_3, for all cases of boundary conditions. Further, it is evident form Tables 2 and 3 that the values of critical *magnetic* Rayleigh numbers (N_c) are greater than the critical Rayleigh numbers (R_c) yielding that the system is more stable when magnetic mechanism is alone present (in the absence of *buoyancy forces*). From Figs. (1e) & (1f), we observed that the *magnetic* Rayleigh number admits the same behaviour with M_3, for *exponential* case of viscosity variation, also.

In general, it is evident from the computed values that values of critical wave numbers decrease slightly for increasing values of δ for each case of boundary conditions, however the values of critical Rayleigh numbers increase with the increasing δ. We also conclude from the comparison of the numerical values presented here that the *exponential* temperature-dependent viscosity is more stabilizing than the *linear* case of viscosity variation.

From the analysis of the values computed for Rayleigh numbers for *exponential* viscosity variation, we found that it follows analogous trends and variations with respect to the parameters as shown in the case of *linear* viscosity variation in Tables 1–3, hence the tables of values for *exponential* case are omitted here for the sake of compactness. Further, we have observed the same trends of variations of Rayleigh numbers for *linear* and *exponential* cases of viscosity variation, hence the graphs for the *linear* case of viscosity variations and for other two case of boundary conditions for the case of *exponential* viscosity variation are also omitted here.

Appendix A Derivation of Eigenvalue Problem

The basic hydrodynamic equations for thermal convection in ferrofluid in the presence of vertical magnetic field are given in Finlayson [3]. These equations can be rewritten for the case when the viscosity is temperature dependent following Chandrasekhar [2]. Thus, the system of basic equations governing the physical configuration (Section 2) under Boussinesq approximation are given by

$$\nabla \cdot \vec{q} = 0\,, \tag{A.1}$$

$$\rho_0 \frac{D\vec{q}}{Dt} = -\nabla P - \rho g \hat{k} + \frac{\partial}{\partial x_j}\Big[\mu\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big)\Big] + 2\frac{\partial \mu}{\partial z}\nabla(\vec{q}\cdot\hat{k}) + \nabla\cdot(\vec{H}\vec{B}), \tag{A.2}$$

$$\Big[\rho_0 C_{V,H} - \eta_0 \vec{H}\cdot(\frac{\partial \vec{M}}{\partial T})_{V,H}\Big]\frac{DT}{Dt} + \eta_0 T(\frac{\partial \vec{M}}{\partial T})_{V,H}\cdot\frac{D\vec{H}}{Dt} = k_1 \nabla^2 T\,, \tag{A.3}$$

$$\rho = \rho_0[1 - \alpha(T - T_0)]\,. \tag{A.4}$$

The above equations have been casted in a Cartesian coordinate system (x, y, z) with origin at the bottom layer and z-axis is considered vertically upward. In these equations: $\vec{q} = (u, v, w)$ is the velocity vector, T is the temperature, P is the pressure, g is the gravity, $\hat{k}$ is the unit vector in the z-direction, $\vec{H} = (H_x, H_y, H_z)$ is the magnetic field intensity, $\vec{B} = (B_x, B_y, B_z)$ is the magnetic induction, $\vec{M} = (M_x, M_y, M_z)$ is the magnetization. Also, $\frac{D}{Dt} \equiv \left(\frac{\partial}{\partial t} + \vec{q} \cdot \nabla\right)$ is the material derivative, $\mu = \mu_0 f(T)$ is the temperature dependent viscosity, $C_{V,H}$ is the specific heat at constant volume and magnetic field, k_1 is the thermal conductivity and α is the coefficient of thermal expansion. Further, ρ_0, η_0 and μ_0 are the density, magnetic permeability and viscosity at the reference temperature T_0.

Equation (A.1) represents the continuity equation, (A.2) is the momentum equation, (A.3) is the energy equation, and (A.4) is the equation of state. Further, the Maxwell's equations for a non-conducting ferromagnetic fluid with no displacement current are given as (Finlayson [3])

$$\nabla \cdot \vec{B} = 0\,, \quad \nabla \times \vec{H} = 0\,, \quad \vec{H} = \nabla \cdot \varphi\,, \tag{A.5}$$

where φ is the magnetic scalar potential. Further, in the Chu formation (see [3]) the magnetic field $\vec{H}$ and magnetization $\vec{M}$ are related to magnetic induction $\vec{B}$ as

$$\vec{B} = \eta_0(\vec{H} + \vec{M})\,. \tag{A.6}$$

First of the equations (A.5), *i.e.* $\nabla \cdot \vec{B} = 0$, upon using equation (A.6), yields

$$\eta_0\big(\nabla \cdot (\vec{H} + \vec{M})\big) = 0\,. \tag{A.7}$$

Since the magnetization is assumed to be aligned with the magnetic field and is a function of temperature and magnetic field, we have

$$\vec{M} = \frac{\vec{H}}{H} M(H, T)\,. \tag{A.8}$$

In addition to the above equations, the magnetic equation of state is given by

$$M = M_0 + \chi(H - H_0) - K_2(T - T_0)\,. \tag{A.9}$$

Here, the magnetic equation is linearized about the magnetic field H_0 and the average temperature T_0. In the above equations; M_0 is the reference magnetization at $H = H_0$ and $T = T_0$, $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_0,T_0}$ is the magnetic susceptibility, $K_2 = \left(\frac{\partial M}{\partial T}\right)_{H_0,T_0}$ is the pyromagnetic co-efficient and H_0 is the uniform magnetic field of the fluid layer when placed in an *external* magnetic field $\vec{H} = H_0^{\text{ext}}$.

To investigate the instability of equilibrium system, let us add small perturbations to the initial state solution and the perturbed quantities are denoted as

$$\text{(A.10)}\qquad \begin{aligned} &\vec{q} = \vec{q}_b(0,0,0) + \vec{V}\,; && P = P_b(z) + \delta p\,;\\ &T = T_b(z) + \theta\,; && \rho = \rho_0(1 - \alpha(T_b(z) + \theta - T_0))\,;\\ &\vec{H} = \vec{H}_b(z) + \vec{H}'\,; && \vec{M} = \vec{M}_b(z) + \vec{M}'\,, \end{aligned}$$

where $\vec{V}(u, v, w)$, δp, θ, $\vec{H}'$ and $\vec{M}'$ are the respective perturbations in the initial velocity, pressure p temperature T, magnetic field and magnetization and $\vec{q}_b$, $P_b(z)$, $T_b(z)$ and $\vec{H}_b(z)$ are the basic state solutions. Also, $\kappa = \dfrac{k_1}{\rho_0 C_1}$ is the effective thermal diffusivity, $\beta = \dfrac{T - T_0}{d}$ is maintained uniform temperature gradient.

Now following the usual steps of linear stability (Chandrasekhar [2], Finlayson [3]), substituting the perturbed quantities (A.10) in equations (A.1)–(A.3) and (A.7), we obtain the following linearized perturbation equations:

$$\text{(A.11)}\qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,,$$

$$\text{(A.12)}\qquad \rho_0 \frac{\partial u}{\partial t} = -\frac{\partial \delta p}{\partial x} + \mu_0 f(T)\nabla^2 u + \mu_0 \frac{\partial f(T)}{\partial z}\Big(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\Big) + \eta_0(H_0 + M_0)\frac{\partial H_1(z)}{\partial z}\,,$$

$$\text{(A.13)}\qquad \rho_0 \frac{\partial v}{\partial t} = -\frac{\partial \delta p}{\partial y} + \mu_0 f(T)\nabla^2 v + \mu_0 \frac{\partial f(T)}{\partial z}\Big(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\Big) + \eta_0(H_0 + M_0)\frac{\partial H_2(z)}{\partial z}\,,$$

$$\text{(A.14)}\qquad \rho_0 \frac{\partial w}{\partial t} = -\frac{\partial \delta p}{\partial z} + \mu_0 f(T)\nabla^2 w + 2\mu_0 \frac{\partial f(T)}{\partial z}\frac{\partial w}{\partial z} + g\alpha\theta\rho_0 + \eta_0(H_0 + M_0)\frac{\partial H_3(z)}{\partial z} + \eta_0(M_3 + H_3)\frac{\partial}{\partial z}\Big(H_0 - \frac{K_2\beta z}{1+\chi}\Big)\,,$$

$$\text{(A.15)}\qquad \rho_0 C_1 \frac{\partial \theta}{\partial t} - \eta_0 T_0 K_2 \frac{\partial}{\partial t}\Big(\frac{\partial \varphi}{\partial z}\Big) = k_1 \nabla^2 \theta + (\rho_0 C_1 - \frac{\eta_0 T_0 K_2^2}{1+\chi})\beta w\,,$$

$$\text{(A.16)}\qquad \frac{\partial}{\partial x}(H_1 + M_1) + \frac{\partial}{\partial y}(H_2 + M_2) + \frac{\partial}{\partial z}(H_3 + M_3) = 0\,.$$

Here φ is the perturbed magnetic potential and $\rho_0 C_1 = \rho_0 C_{V,H} + K_2 H_0 \eta_0$.

Now, substituting (A.10) in equations (A.8) and (A.9) and taking $K_2\beta d \ll (1+\chi)H_0$, we have

$$H_i + M_i = \begin{cases} (1+M_0/H_0)H_i & \text{for } i=1,2\,, \\ (1+\chi)H_i - K_2\theta & \text{for } i=3\,. \end{cases} \tag{A.17}$$

Upon using equations $\vec{H} = \nabla\cdot\varphi$ and (A.17), equations (A.14) and (A.16) yield

$$\begin{aligned} \rho_0\frac{\partial w}{\partial t} = -\frac{\partial\delta p}{\partial z} + \mu_0 f(T)\nabla^2 w + 2\mu_0\frac{\partial f(T)}{\partial z}\frac{\partial w}{\partial z} + g\rho_0\alpha\theta \\ + \eta_0(H_0+M_0)\frac{\partial^2\varphi}{\partial z^2} - \eta_0 K_2\beta H_3 + \frac{\eta_0 K_2^2\beta}{1+\chi}\theta\,, \end{aligned} \tag{A.18}$$

$$\Big(1+\frac{M_0}{H_0}\Big)\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\varphi + (1+\chi)\frac{\partial^2\varphi}{\partial z^2} - K_2\frac{\partial\theta}{\partial z} = 0\,. \tag{A.19}$$

Now, eliminating δp amongst the equation (A.11)–(A.13) and (A.18), we have

$$\begin{aligned} \rho_0\frac{\partial}{\partial t}\nabla^2 w = \rho_0 g\alpha\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\theta + \mu_0 f(T)\nabla^4 w \\ + 2\mu_0\frac{df(T)}{dz}\nabla^2\Big(\frac{\partial w}{\partial z}\Big) - \mu_0\frac{d^2 f(T)}{dz^2}\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\frac{d^2}{dz^2}\Big)w \\ - \eta_0 K_2\beta\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\frac{\partial\varphi}{\partial z} + \frac{\eta_0 K_2^2\beta}{1+\chi}\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\theta\,. \end{aligned} \tag{A.20}$$

Finally, using the *normal mode analysis* by assuming the time dependence periodic solution in horizontal plane of the form

$$(u,v,w,\delta p,\theta) = F^*(z)\exp[i(k_x x + k_y y) + nt] \tag{A.21}$$

we obtain the following system of non-dimensional linearized perturbation equations, after some algebraic manipulations

$$\begin{aligned} f(T)(D^2-a^2)^2 w - \frac{p}{\sigma}(D^2-a^2)w + 2Df(T)D(D^2-a^2)w \\ + D^2 f(T)(D^2+a^2)w = Ra^2[(1+M_1)\theta - M_1 D\varphi]\,, \end{aligned} \tag{A.22}$$

$$(D^2-a^2-p)\theta + pM_2 D\varphi = -(1-M_2)w\,, \tag{A.23}$$

$$(D^2-a^2M_3)\varphi = D\theta\,. \tag{A.24}$$

The above equations have been recasted into the non-dimensional forms by using the following non-dimensional quantities:

$$\hat{z} = \frac{z}{d}\,, \quad \hat{a} = kd\,, \quad \hat{D} = d\frac{d}{dz}\,, \quad \hat{p} = \frac{nd^2}{\kappa}\,, \quad \hat{w} = \frac{dw^*}{\nu}\,,$$
$$\hat{\theta} = \frac{\kappa\theta^*}{\beta\nu d}\,, \quad \hat{\varphi} = \frac{(1+\chi)\kappa\varphi^*}{K_2\beta\nu d^2}\,, \quad \sigma = \frac{\nu}{\kappa}\,, \quad \nu = \frac{\mu_0}{\rho_0}\,.$$

In the above equations; $R = \dfrac{g\alpha\beta d^4}{\kappa\nu}$ is Rayleigh number, a is wave number, p is complex *growth rate*, σ is the Prandtl number, $M_1 = \eta_0 K_2^2\beta/\rho_0 g\alpha(1+\chi)$ is the magnetic number, $M_2 = \eta_0 K_2^2 T_0/\rho_0 C_1(1+\chi)$ is the magnetic parameter and $M_3 = \left(1 + \dfrac{M_0}{H_0}\right)/(1+\chi)$ is the measure of nonlinearity of magnetization. Further, the caps have been dropping in the above equations for convenience in writing.

The equations (A.22)–(A.24) must seek solutions subject to certain boundary conditions based upon the nature of the bounding surfaces. The boundary surfaces shall be either rigid (with no slip condition) or dynamically free (where the tangential stresses vanish). Hence, the relevant non-dimensional boundary conditions (for $\chi \to \infty$, *i.e.* for very large magnetic susceptibility) considered here are (*cf.* [3])

(A.25) $\quad w = 0 = \theta = D\varphi = D^2 w \quad$ on dynamically free surfaces,

(A.26) $\quad w = 0 = \theta = D\varphi = Dw \quad$ on rigid surfaces.

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