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STRUCTURAL DYNAMICS

MODELING OF CROWD FLOWS FOR FOOTBRIDGE DESIGN

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ABSTRACT: Pedestrians exert different dynamic loads (with regard to their type, magnitude and frequency) on civil structures. The commonly used practice of designing such structures as lightweight leads to inevitable decrease in the structural mass and stiffness, thus making the structures less inert and less resistant to the loads induced by humans.

The currently available codes of practice, adopt simplified procedures for the vibration serviceability assessment of footbridges. Their procedures disregard the human-human interaction, which may have a substantial effect on the maximum acceleration levels of the structure under consideration. In this article, an extensive study of the human-human interaction is fulfilled. To describe this aspect of the human motion, the widely-applied social force model is calibrated such as to simulate normal traffic conditions on footbridges.

KEY WORDS: social force model, footbridge, bidirectional flow.

1 INTRODUCTION

As a part of a crowd flow, a pedestrian cannot be regarded as an independent particle. Rather, when headed toward a certain destination, coming across other participants in the traffic, the pedestrian adjusts his motion to the ambient situation, influenced by intrinsic psychological considerations. The phenomena described here is defined as human-human interaction and is a basis for a realistic representation of the crowd behaviour, with regard to their significance for civil structures.

The current paper is with strong accent on the human-human interaction, employing the social force model [1]. It can be referred to as a micro-scale model, meaning that the behaviour of every single pedestrian is defined and the respective characteristic temporal and spatial variables (velocities and positions) are acquired.

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2 Social Force Model

Pedestrian motion, when one moves through a crowd, can be described quite accurately by employing the social force model [1]. It is adopted that pedestrians are undergoing these social forces which are not physical forces in their essence, but represent the motivation (or lack thereof) of pedestrians to react in a certain manner to given circumstances.

It is widely accepted that human motions are random by nature. Contrary to this conventional assumption human motion is not a chaotic phenomenon – when being part of a crowd, a pedestrian is subjected to certain behavioural rules which dictate his movements [1,2]. For some complex and extreme situations crowd behaviour cannot be explicitly described but as regards some trivial and common situations pedestrians move in a rather organized way. The social force model can be conveniently used for assessing the human influence on pedestrian facilities such as bridges, overpasses etc. [3].

2.1 General concept of the social force model

The "social force model" of a crowd represents an aggregate of simulations of every single pedestrian motion [1]. From these simulations, the model quantities such as pedestrian positions $\mathbf{r}_\alpha(t)$ and pedestrian velocities $\mathbf{v}_\alpha(t)$ can be derived. Social forces exhibit the alteration in the pedestrian actual velocity in the course of time. They are not directly applied to the human body, but are quantities describing acceleration or deceleration forces which are subconsciously generated by a pedestrian considering the conditions of the surrounding environment. The resultant force consists of three components:

- Driving force – $\mathbf{f}^0_\alpha(\mathbf{v}_\alpha)$
- Repulsive force due to pedestrian interaction – $\mathbf{f}_{\alpha\beta}(\mathbf{r}_\alpha, \mathbf{v}_\alpha, \mathbf{r}_\beta, \mathbf{v}_\beta)$
- Repulsive force due to the presence of boundaries and obstacles – $\mathbf{f}_{\alpha B}(\mathbf{r}_\alpha)$

2.2 Mathematical formulation

Considering the above-stated, the basic quantities of the model can be simply derived by solving a couple of differential equations. As $\mathbf{r}_\alpha(t)$ represents the current position of pedestrian α at the particular instant t and $\mathbf{v}_\alpha(t)$ is the respective velocity, the following equation can be written:

$$\frac{d\mathbf{r}_\alpha(t)}{dt} = \mathbf{v}_\alpha(t) \tag{1}$$

A similar relationship can be used for the velocity and respective acceleration at moment t which is, actually, the social force $\mathbf{f}_\alpha(t)$ – it takes into consideration all ambient conditions which affect human movements. A further term should be added to account for random fluctuation from irregular behaviour – $\xi_\alpha(t)$

$$\frac{d\mathbf{v}_\alpha(t)}{dt} = \mathbf{f}_\alpha(t) + \xi_\alpha(t) \tag{2}$$

As mentioned above the total social force is the result of the summation of the different influences on the pedestrians

$$\mathbf{f}_\alpha(t) = \mathbf{f}^0_\alpha(\mathbf{v}_\alpha) + \sum_{\beta\neq\alpha}^{n_p} \mathbf{f}_{\alpha\beta}(\mathbf{r}_\alpha, \mathbf{v}_\alpha, \mathbf{r}_\beta, \mathbf{v}_\beta) + \sum_{B}^{n_B} \mathbf{f}_{\alpha B}(\mathbf{r}_\alpha), \tag{3}$$

with n_p being the number of pedestrians and n_B – the number of boundaries

2.2.1 Driving force term

It describes the intention of a given pedestrian to move toward his desired destination with a definite desired velocity

$$\mathbf{f}^0_\alpha(\mathbf{v}_\alpha) = \frac{v^0_\alpha(t)\mathbf{e}_\alpha(t) - \mathbf{v}_\alpha(t)}{\tau_\alpha}, \tag{4}$$

with $v^0_\alpha(t)$ being desired velocity, m/s; $\mathbf{v}_\alpha(t)$ – actual velocity, m/s; $\mathbf{e}_\alpha(t)$ – desired direction corresponding to the desired destination; τ_α – relaxation time, s.

As a pedestrian heads in a certain direction, he will move with some desired velocity, if not disturbed. Any deviations of the actual velocity $\mathbf{v}_\alpha(t)$ from the desired one $v^0_\alpha(t)\mathbf{e}_\alpha(t)$ due to the presence of obstacles or interaction with other pedestrians, are eliminated after a certain relaxation time τ_α, and the pedestrian reaches again his desired velocity.

The desired destination vector can be defined depending on the current position of the pedestrian $\mathbf{r}_\alpha(t)$ and his desired destination $\mathbf{r}^k_\alpha$

$$\mathbf{e}_\alpha(t) = \frac{\mathbf{r}^k_\alpha - \mathbf{r}_\alpha(t)}{||\mathbf{r}^k_\alpha - \mathbf{r}_\alpha(t)||} \tag{5}$$

In normal situations – low crowd densities and unobstructed motion - the desired velocity $v^0_\alpha(t)$ has a mean value of 1.34 m/s, but delays can always occur especially for the cases of bidirectional flow. In condition of restricted flow – high densities (above 1 ped./m^2) or extreme (panic) conditions - the mean velocity would acquire much smaller values (Fig. 8). As a result there is a trend of increasing the desired velocity. This relation can be described employing the expression

$$v_\alpha^0(t) = [1 - n_\alpha(t)]v_\alpha^0(0) + n_\alpha(t)v_\alpha^{\max}, \tag{6}$$

where $v_\alpha^0(0)$ is initial desired velocity; $v_\alpha^{\max}$ – maximum desired velocity; $n_\alpha(t)$ – a quantity which is changing in the course of time and accounts for the possible nervousness and impatience of the pedestrian to attain his desired destination.

$$n_\alpha(t) = 1 - \frac{\bar{v}_\alpha(t)}{v_\alpha^0(0)}, \tag{7}$$

where $\bar{v}_\alpha(t)$ is the average speed into the desired direction – $\bar{v}_\alpha(t) = \dfrac{||\mathbf{r}_\alpha(t) - \mathbf{r}_\alpha(0)||}{t}$.

2.2.2 Repulsive forces due to interaction with other pedestrians

When moving through a crowd, a pedestrian will feel uncomfortable if being too close to the other pedestrians. Therefore, he will try to keep a certain distance from them depending on the specific circumstances – the desired velocity, the density of the crowd, etc. This psychological effect is described by the interaction repulsive force which is formulated as

$$\mathbf{f}_{\alpha\beta}(t) = A_\alpha^1 e^{\frac{r_{\alpha\beta} - d_{\alpha\beta}}{B_\alpha^1}} \mathbf{n}_{\alpha\beta}\Big(\lambda_\alpha + (1-\lambda_\alpha)\frac{1+\cos(\phi_{\alpha\beta})}{2}\Big) + A_\alpha^2 e^{\frac{r_{\alpha\beta} - d_{\alpha\beta}}{B_\alpha^2}} \mathbf{n}_{\alpha\beta}, \tag{8}$$

with A_α^1 being interaction strength related to the territorial effect, m/s^2; B_α^1 – repulsive interaction range related to the territorial effect, m; A_α^2 – interaction strength related to physical interactions, m/s^2; B_α^2 – repulsive interaction range related to physical interactions, m; $r_{\alpha\beta} = r_\alpha + r_\beta$ – sum of pedestrians' radii, m; $d_{\alpha\beta}(t) = ||\mathbf{r}_\alpha(t) - \mathbf{r}_\beta(t)||$ – distance between pedestrians' centers of mass, m; $\mathbf{r}_\alpha(t)$ – coordinates of the centers of mass of the respective pedestrian; $\mathbf{n}_{\alpha\beta}(t)$ – normalized vector directed from pedestrian β toward pedestrian α; λ_α – coefficient taking into consideration the anisotropic nature of pedestrians interaction as pedestrians are far more influenced by the situation in front than the one behind; $\phi_{\alpha\beta}$ – the angle between the direction vector $\mathbf{e}_\alpha(t)$ and the $-\mathbf{n}_{\alpha\beta}(t)$ vector pointing from pedestrian β to pedestrian α (Fig. 1).

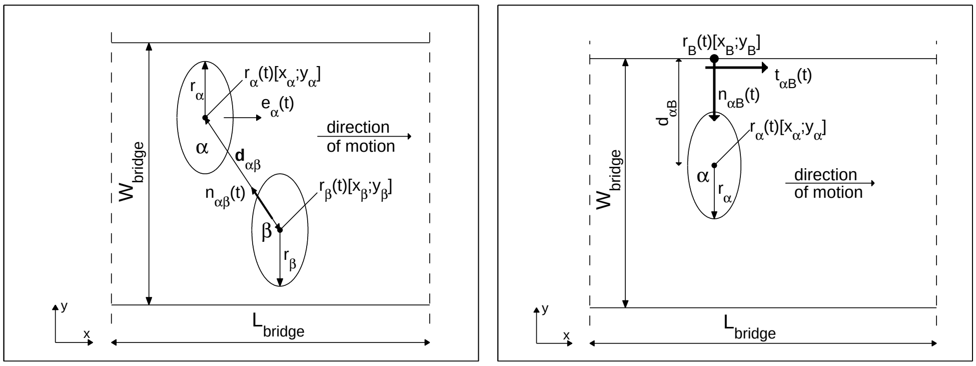


Fig. 1: Depiction of the parameters related to the interaction repulsive forces (left) and to the boundary repulsive forces due to the footbridge boundaries (right)

2.2.3 REPULSIVE FORCE DUE TO BOUNDARIES

Similar to the interaction repulsive force, there is a repulsive force due to the presence of the footbridge boundaries (Figure 1). Each of the pedestrians will keep a certain distance from boundaries aiming to avoid injuries and not to feel confused. For the evaluation of the force an exponential relation, similar to the one used for the interaction between pedestrians, can be adopted

$$\mathbf{f}_{\alpha B}(t) = A_\alpha e^{\frac{r_\alpha - d_{\alpha B}}{B_\alpha}} \mathbf{n}_{\alpha B}, \tag{9}$$

where A_α is interaction strength due to boundaries; B_α – repulsive interaction range due to boundaries; r_α – pedestrian's radius; $d_{\alpha B}(t) = ||\mathbf{r}_\alpha(t) - \mathbf{r}_B(t)||$ – distance from pedestrian's center of mass to the closest point of the boundary; $\mathbf{r}_B(t)$ – coordinates of the nearest boundary point; $\mathbf{n}_{\alpha B}(t)$ – normalized vector pointing from the closest point B to the pedestrian α (Figure 1).

2.3 KEY PARAMETERS REFERRING TO THE DRIVING FORCE TERM

The parameters essential for the driving force term are the desired velocity and the relaxation time. In the following, all parameters defining the driving force term are described:

- Initial desired velocity – $v_\alpha^0(0)$

  For the cases with normal pedestrian densities the desired velocity has a Gaussian distribution N(1.34 m/s, 0.26 m/s) according to [1].

- Desired velocity – $\mathbf{v}_\alpha^0(t)$

The values of the desired velocity depend on the particular situation. They are calculated at each time instant depending on the eventual nervousness or impatience to reach the desired destination.

- Relaxation time – τ_α

The relaxation time represents the time for adapting the actual velocity to the desired one. Its value should be taken in a way the pedestrian motion is realistic and not abrupt.

2.4 KEY PARAMETERS REFERRING TO THE INTERACTION REPULSIVE FORCE

With respect to the main formulation $\mathbf{f}_{\alpha\beta}(t) = Ae^{\frac{r_{\alpha\beta}-d_{\alpha\beta}}{B}}\mathbf{n}_{\alpha\beta}(t)$ of the interaction repulsive forces, it can be seen that the principal variable determining its magnitude is the pure distance between each two pedestrians $(r_{\alpha\beta} - d_{\alpha\beta})$. Therefore, in order to obtain realistic pedestrian behaviour the constants A and B should be adjusted properly.

- Interaction strength due to interaction between pedestrians – $A^2_{\alpha\beta}$

The constant $A^2_{\alpha\beta}$ gives the amplitude of the respective interaction repulsive force if multiplied by the mass of the pedestrian under consideration. Clearly, the larger $A^2_{\alpha\beta}$, the larger the repulsive force - the pedestrians try to stay further from each other.

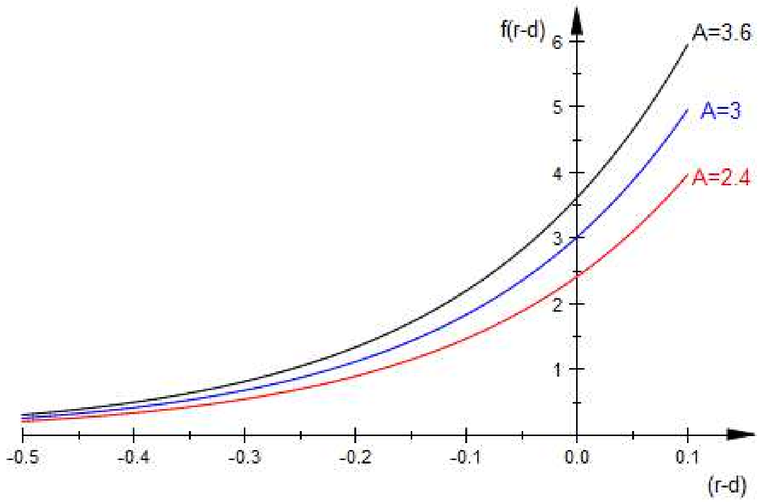


Fig. 2: Interaction force $Ae^{\frac{r-d}{B}}$, with $B^2_{\alpha\beta} = 0.2$ m, for three different values of the constant $A^2_{\alpha\beta}$.

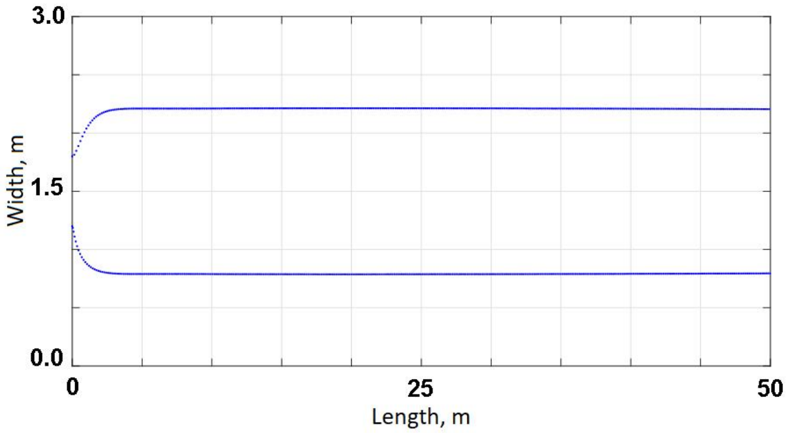


Fig. 3: Trajectories of two pedestrians with $A^2_{\alpha\beta} = 3$ m/s^2 moving from left to right on a footbridge with $L = 50$ m and $W = 3$ m.

Hence, the value for $A^2_{\alpha\beta}$ should be taken such that the resultant interaction force gets reasonable values in cases of physical contact, i.e. $r_{\alpha\beta} = d_{\alpha\beta}$. The value given by Helbing [2] is $A^2_{\alpha\beta} = 3$ m/s^2 for normal situations. Figure 2 explicitly shows the alteration of the interaction forces as a function of the distance between pedestrians $(r_{\alpha\beta} - d_{\alpha\beta})$.

From the graph, it can be concluded that when two pedestrians have increased sufficiently the distance between each other, they do not tend to increase it any more as the interaction force converges to zero.

Figure 3 shows the trajectories for two pedestrians walking in one and the same direction (from left to right) on a simple geometry bridge with length $L = 50$ m and width $W = 3$ m. In the incipient moments, the pedestrians try to increase the distance in between and after reaching a reasonable distance they continue walking along straight line trajectories. This distance could be explained by a force equilibrium of the forces between pedestrians and between pedestrians and boundaries.

Figure 4 displays the paths of four pedestrians – two of these pedestrians walk from left to right (blue dots) and the other two – from right to left (red dots).

- Repulsive interaction range due to interaction between pedestrians – $B^2_{\alpha\beta}$.

The parameter $B^2_{\alpha\beta}$ shows how quickly the interaction force reduces, i.e. to what extent the pedestrians are influenced by each other depending on the distance between them. For bigger values of $B^2_{\alpha\beta}$ the values of d should be significantly greater (respectively $r - d$ should be smaller) so that the repulsive force stops being exerted by the pedestrians.

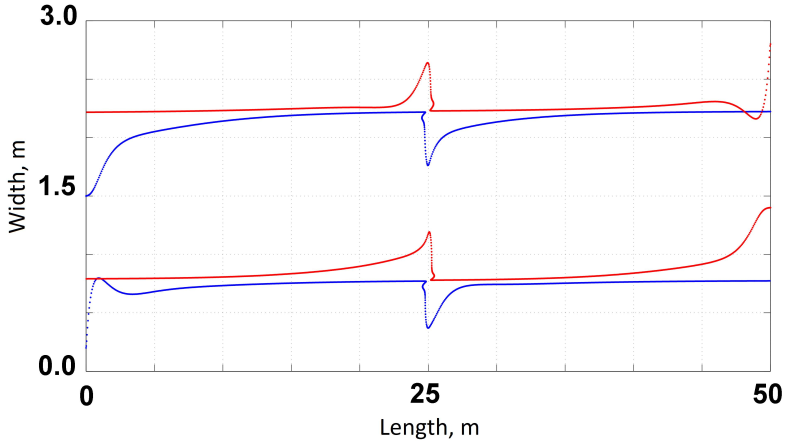


Fig. 4: Trajectories of four-pedestrian crowd with $A^2_{\alpha\beta} = 3$ m/s^2, two pedestrians moving from left to right (blue dots) and the other two moving from right to left (red dots) on a footbridge with $L = 50$ m and $W = 3$ m

Increasing the values of $B^2_{\alpha\beta}$, for $r - d > 0$ decreases the repulsive force itself – the participants in the traffic get less affected by each other. On the other hand, for $r - d < 0$, increasing $B^2_{\alpha\beta}$ leads to bigger values for the interaction force - the participants in the traffic get affected by each other even at large distances (Fig. 5).

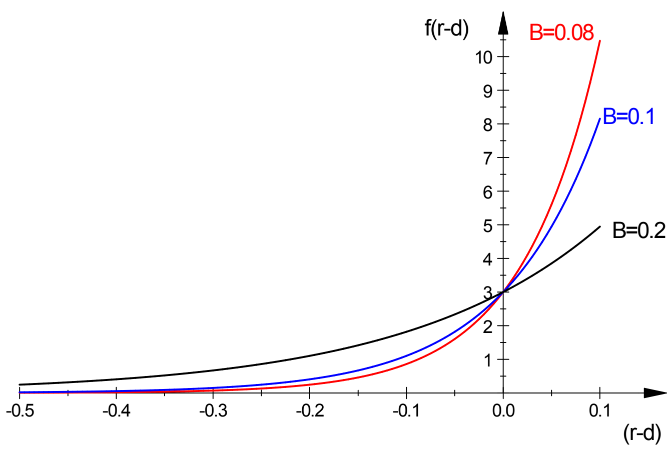


Fig. 5: Interaction force $Ae^{\frac{r-d}{B}}$, with $A^2_{\alpha\beta} = 3$ m/s^2, for three different values of the constant $B^2_{\alpha\beta}$.

- Anisotropic parameter – λ_α

The coefficient λ_α defines the anisotropic way the pedestrians interact. It is clear that the pedestrians being exactly in front of the pedestrian under consideration have the strongest influence on him, whereas these who are exactly behind him have the lowest influence (Fig. 6).

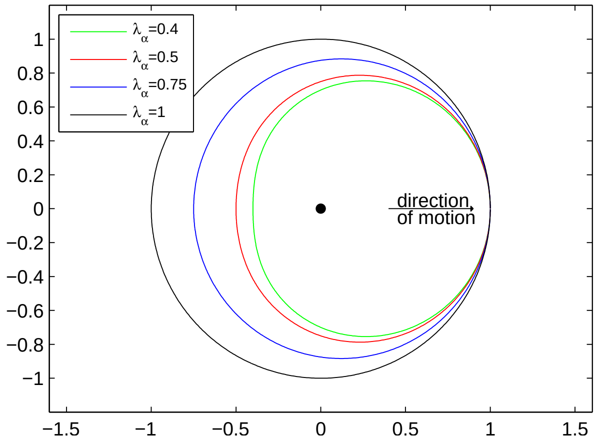


Fig. 6: Anisotropic area of interaction

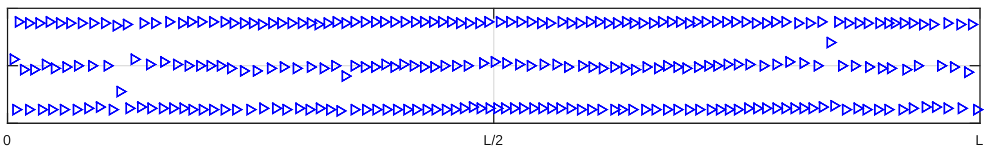
2.5 Key parameters referring to the boundary repulsive force

As the expression for the boundary repulsive force $\mathbf{f}_{\alpha B}(t) = A_{\alpha B} e^{\frac{r_{\alpha B} - d_{\alpha B}}{B_{\alpha B}}} \mathbf{n}_{\alpha B}(t)$ is of the same type as the expression for the interaction repulsive force, similar conclusions can be drawn for the crucial constants, namely $A_{\alpha B}$ and $B_{\alpha B}$.

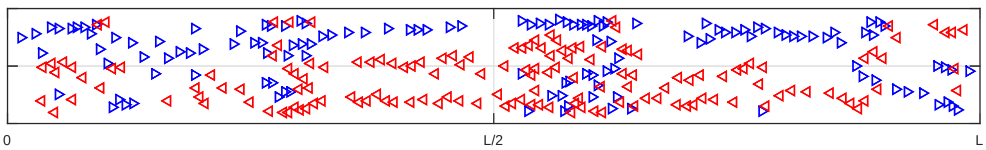
3 Initialization of the Flow

During the initial stage of the process, until the desired crowd density is reached, the arrival times of the pedestrians for a certain crowd density are assumed to follow a Poisson distribution. Thereafter every pedestrian leaving the bridge is replaced by a new one, thereby keeping both the continuity of the flow and the desired density constant. The arrival rate is calculated as: $\lambda = N_{\text{ped}}/TL = A_{\text{eff}} d/TL$ with N_{ped} being number of pedestrians on the bridge; A_{eff} – effective area of the bridge deck; d – desired flow density; TL is average time needed by a pedestrian to cross the bridge ($TL = L/v_{\text{Weidmann}}$); L – length of the bridge; v_{Weidmann} – average walking velocity defined by Weidmann (Fig. 8).

Figure 7 illustrates unidirectional and bidirectional flows with density $d = 1.0$ ped/m^2 on an exemplary footbridge with $L = 50$ m and $W = 3$ m. Distinct lane formations



(a) Unidirectional flow



(b) Bidirectional flow

Fig. 7: Instantaneous positions of the pedestrians being part crowd flow with density $d = 1.0$ ped./m^2 at a certain point in time.

for the unidirectional flows can be observed in Figure 7a – for such a high density $d = 1.0$ ped/m^2 namely due to fact the pedestrians' behaviour is dictated by confined disposable space the lanes are rarely disrupted. The lane formation here is not that explicit as it is for unidirectional flows. It can be qualified as *temporal* lane formation, since disruption happens due to the newly arrived pedestrians moving in opposite direction. This may lead to uneven distribution over the whole area of the footbridge and clustering of pedestrians trying to pass each other, especially for high densities, contrary to the unidirectional flow where the distribution of pedestrians is mostly uniform for high densities.

4 Calibration of the Model Parameters for Use on Footbridges

The values of the governing parameters of the social force model defined in the literature sources [1] differ depending on the traffic situation under consideration. The parameters related to the repulsive interaction force $A^1_{\alpha\beta}$, $B^1_{\alpha\beta}$, λ_α are calibrated for the unidirectional flow [4], so that the average speed of the flow for a certain crowd density converges with the reference values defined by Weidmann [5] (Fig. 8).

The extensive research performed regarding the unidirectional flow confirms that the flow density and its velocity are inversely proportional [4]. Using the calibrated values for the determining parameters in the social force model (Table 2), the results acquired for the unidirectional flow can be referred to as stable and even constant (Fig. 9a).

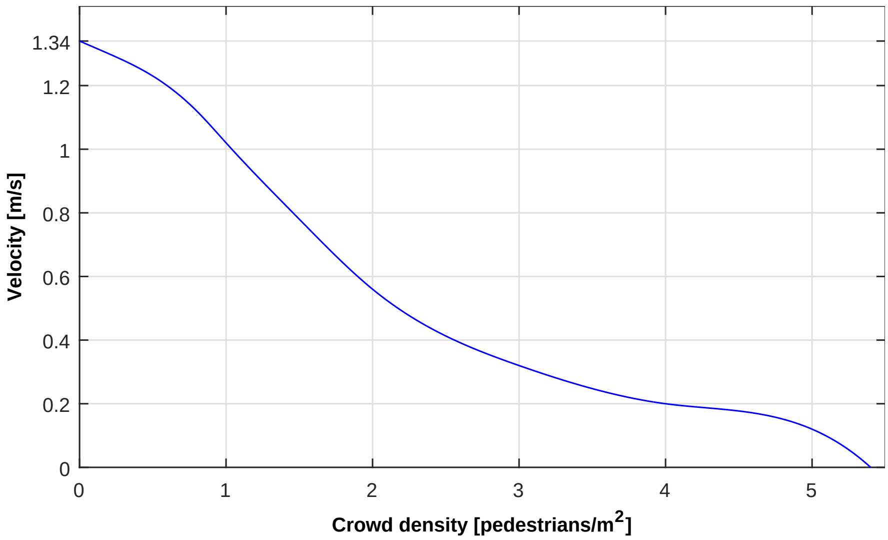
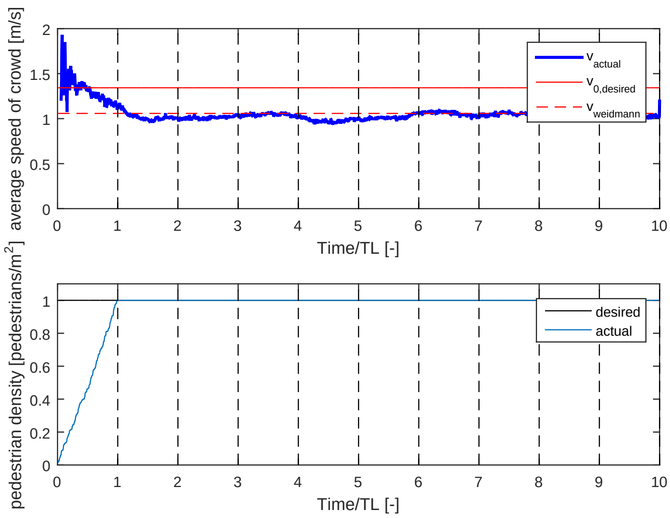


Fig. 8: Density-velocity relation according to [5].

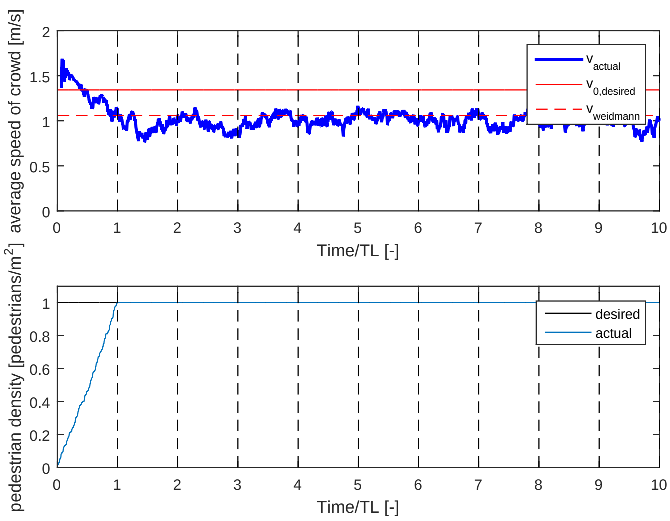
Similar simulations implemented for bidirectional flows with the calibrated values for the unidirectional flow manifest a significant decrease in the actual speed value compared to the unidirectional flow (Table 1). This can be justified by the different nature of the two processes. When considering the unidirectional flow, a settled process is present – there are no premises for abrupt movements which may violate the established monotonous motion, respectively speed. On the other hand, bidirectional flow cannot be defined as an uniform process due to the frisk movements undertaken

Table 1: Values for the actual speed according to Weidmann (Fig. 8) and using the calibrated values for unidirectional flow [4] for crowd densities according to [6,7]

Crowd density,		Unidirectional flow		Bidirectional flow		Bidirectional flow calibrated	
ped/m^2	v_{weidmann}, m/s	v_{actual}, m/s	Δ, %	v_{actual}, m/s	Δ, %	v_{actual}, m/s	Δ, %
0.10	1.32	1.28	-3.03	1.14	-13.64	1.24	-6.01
0.20	1.30	1.28	-1.54	1.00	-23.08	1.22	-6.08
0.50	1.23	1.24	0.81	0.59	-52.02	1.16	-5.79
0.80	1.12	1.14	1.79	0.40	-64.29	1.08	-3.38
1.00	1.02	1.02	-0.42	0.32	-68.63	1.00	-1.68
1.50	0.78	0.79	1.28	0.13	-83.33	0.81	3.99



(a) Unidirectional flow



(b) Bidirectional flow

Fig. 9: Average speed of crowd (top) and pedestrian density (bottom) as a function of the dimensionless time for unidirectional flow with density $d = 1.0$ ped./m^2 for calibrated values of the main parameters according to [4] (left) and for bidirectional flow for calibrated values according to Table 2 (right).

Table 2: Calibrated parameters for unidirectional flows according to [4] and bidirectional flows

Parameter	λ_α	Δ, %	A^1_α, m/s^2	Δ, %	B^1_α, m	Δ, %	τ_α, s	Δ, %
Unidirectional flow	0.80	–	9.43	–	0.35	–	0.50	–
Bidirectional flow	0.92	+15.00	2.00	-78.79	0.20	-42.86	0.43	-14.00

by the pedestrians coming across pedestrians walking in opposite direction.

The values of the mean speeds for bidirectional flow are evidently much smaller than the respective values for unidirectional flow for one and the same values of the characteristic parameters of the social force model. From Table 1, it is obvious that the set of parameter values, as calibrated for unidirectional flows, generate bidirectional flows which are unrealistic. The differences between the acquired values for the mean speed and the reference ones derived by Weidmann are inadmissible, reaching value of 83.33% for density 1.5 ped./m^2. These flows may be characterized by significant clustering of pedestrians being obstructed to pass by each other due to high magnitude of the repulsive interaction force.

The most important parameters influencing the pedestrians behaviour (with reference to the other participants in motion) and therefore their speed, are the interaction strength A^1_α, the repulsive interaction range B^1_α, the anisotropic parameter λ_α and the relaxation time τ_α.

A certain number of simulations is performed for period of $TL = 100$ and the calibrated values for the bidirectional flow are summarized in Table 2. The derived values for the actual speed do not coincide completely with the reference values defined by Weidmann, but do differ with margin of maximum 6.08% (Table 1). On the other hand, it can be seen that no stable area for the process occurs over the selected time window.

5 CONCLUCIONS

The extensive study conducted by the authors in the present article shows the fundamental difference between unidirectional and bidirectional flows. As a result the graphs of the actual speed versus time for the two kind of flows differ greatly in shape. The calibrated values of the crucial model parameters have significant differences for the two types of flows. The final bidirectional flow gives mean speed values for the regarded crowd densities approaching the ones derived by Weidmann. Thus obtained the determining parameters of the social force model result into reasonable crowd flows.

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