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CRANE TROLLEY START OPTIMIZATION

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ABSTRACT: This paper presents the statement and the solving of the optimal control problem for a crane trolley start. In the calculation has been used two-mass model of the trolley with a load on the flexible suspension. The problem has been solved with the variation calculus. Some generalization of the obtained solution made it possible to find the solution of the complex optimization problem. All the obtained results have been investigated with the estimation indexes. Their analysis showed the superiority of the complex optimization problem solution. All the optimal laws of the trolley motion provide the load oscillation elimination at the end of the start mode. It causes an increase of the crane operation rate. At the end of the article, a hypothesis has been set. It concerns the influence of the weight coefficient in the complex optimization criterion.

KEY WORDS: optimal control, crane, load oscillations, variation calculus, complex criterion, generalization.

1 INTRODUCTION

1.1 PROBLEM STATEMENT

Bridge cranes are widespread in modern factories of chemical and metallurgical industries, mechanical engineering, log dumps, sea and river ports, building yards, etc. One of the major factors, which harm the efficiency of the bridge crane, are oscillations of the load. They cause increasing the dynamic loads in the metal structure of the crane, decline its controllability, and provide a high level of energy losses. It is important to eliminate the mentioned oscillations during transient modes of the crane movement. That problem has been studied in many scientific works. In the present article, we refer to those of the works in which various methods of optimal control have been applied to the problem. Note, that such an approach provides the best solution of the problem in the sense of criterion (movement duration, energy losses, dynamic loads). That is why the choice of an optimization criterion is crucial.

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1.2 Analysis of recent studies and publications

The problems of the linear-quadratic controllers' synthesis have been investigated in many scientific works [1–5]. Indeed, the "crane-load" system can be presented as a plant to control. In these works, the optimal control of the crane is obtained in the closed-loop form, which means that the control is a function of the actual phase coordinates of the system. Similar results may be obtained with dynamic programming [6]. If the main requirement for the crane operation is the high performance it is expedient to use time-optimal control [7–10] (such requirement is usual for the longshore cranes).

Thus, these two classes of optimal control are significantly different. The first class [1–6] provides "smooth" modes of crane motion. It does not cause a high level of the dynamic loads in a crane structure and continues a relatively long time. The features of the second class of control [6–9] are the opposite.

Note, that if the external disturbances are not significant it is reasonable to use the program optimal control. The variations calculus or direct variation methods allow obtaining such control [11, 12]. However, in the known scientific publication there is a lack of the optimal control problems for crane movement with the energy criteria. The present article deals with such a problem.

1.3 The purpose and tasks statement

The purpose of the study is the optimization of the crane trolley start with the load on the flexible suspension. In order to achieve the purpose, the following tasks should be solved: 1) to state the problem of the "trolley-load" optimal control and to find its solution via variation calculus method; 2) to generalize the obtained results; 3) to analyze the plots of the optimal functions and a complex of corresponding estimation indicators; 4) to make conclusions and to give the perspectives of further investigations.

2 Main Text

2.1 Crane trolley movement

The two-mass dynamic model of the crane trolley with the load on the flexible suspension will be used in the current study (Fig. 1). The model shown in Fig. 1 may be considered as a mathematical pendulum with a movable pivot.

The movement of the above mentioned dynamic model (Fig. 1) is described with a system of differential equations:

$$\begin{cases} m_1\ddot{x}_1 = F - W - m_2\Omega_0^2(x_1 - x_2)\,, \\ m_2\ddot{x}_2 = m_2\Omega_0^2(x_1 - x_2)\,, \end{cases} \tag{1}$$

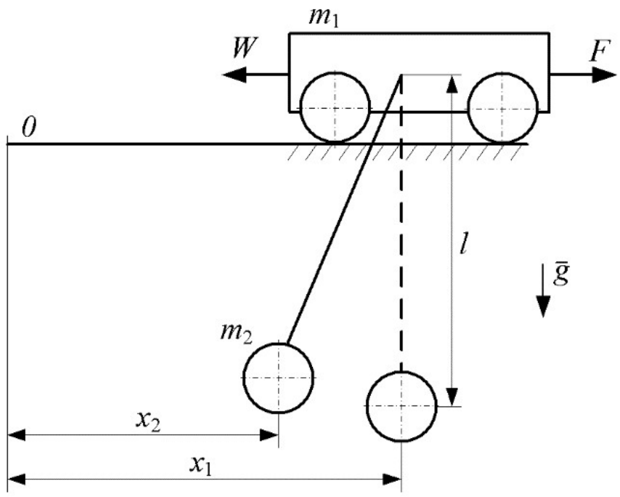

Fig. 1: A dynamic model of the system "crane trolley – load".

where m_1 and m_2 are reduced masses of the trolley and the load respectively; x_1 and x_2 are the coordinates of the trolley and of the load masses; g is the acceleration of the free fall; l is the length of the flexible suspension; Ω_0 is the own frequency of load oscillations relative to the fixed crane ($\Omega_0 = \sqrt{g/l}$); F is the driving force acting on the trolley; W is the force of all the resistances exerted to the crane trolley (it includes all the friction forces which take place in the trolley movement mechanism).

The simplifications that have been used during the model (1) development are the following: the wind ruches are absent as well as the air resistance; the load deviation is not bigger than 15 degrees; geometrical features of the system do not influence its movement; the length of the flexible suspension is a constant value. Acceptability of these simplifications is confirmed by the real conditions of cranes exploitation [9].

Mathematical model (1) describes the trolley movement and the load oscillations. For the wide range of optimal control problems of the system "crane trolley – load" it is reasonable to use the system of equations (1) or similar models [1, 2, 6, 9, 10].

Using the second equation of the system (1), we have written the first equation in the following form:

$$F = W + m_1\ddot{x}_1 + m_2\ddot{x}_2\,,$$

or

$$F = W + (m_1 + m_2)\ddot{x}_2 + m_1\Omega_0^{-2}\overset{IV}{x_2}\,. \tag{2}$$

As the optimization criterion, we have chosen the average power of the drive mechanism during the start of the crane trolley. The choice of this criterion is caused by the need of improving the energy efficiency of the process of the trolley start mode. The optimization criterion is presented in the following form:

$$\overline{P}_{[0;\,v]} = \int_0^v F d\dot{x}_1 \,, \tag{3}$$

where v is a steady velocity of the trolley at the end of the start.

Taking into account the expression (2) criterion (3) might be written in the following form:

$$\begin{aligned} \overline{P}_{[0;\,v]} &= \int_0^{v(T)} \Big(W + (m_1+m_2)\ddot{x}_2 + m_1\Omega_0^{-2}\overset{IV}{x_2}\Big)\frac{d\dot{x}_1}{dt}dt \\ &= \int_0^T \Big(W + (m_1+m_2)\ddot{x}_2 + m_1\Omega_0^{-2}\overset{IV}{x_2}\Big)\ddot{x}_1 dt \\ &= \int_0^T \Big(W + (m_1+m_2)\ddot{x}_2 + m_1\Omega_0^{-2}\overset{IV}{x_2}\Big)\Big(\ddot{x}_2 + \Omega_0^{-2}\overset{IV}{x_2}\Big)dt \,, \end{aligned} \tag{4}$$

where T is the duration of the trolley start.

Criterion (4) is an integral functional to minimize. The necessary condition of criterion (4) minimum is the Euler-Poisson equation [13]:

$$\sum_{i=0}^{4} (-1)^i \frac{d^i}{dt^i}\frac{\partial f}{\partial \overset{IV}{x_2}} = 0 \,, \tag{5}$$

where f is an integrand of the criterion (4). Executing intermediate mathematical transformations, we may present the equation (5) in the following form:

$$\overset{VIII}{x_2} + \Big(2 + \frac{m_2}{m_1}\Big)\Omega_0^2\overset{VI}{x_2} + \Big(1 + \frac{m_2}{m_1}\Big)\Omega_0^4\overset{IV}{x_2} = 0 \,. \tag{6}$$

Taking into account the notations $A_1 = (1 + m_2/m_1)\Omega_0^4$ and $A_2 = (2 + m_2/m_1)\Omega_0^2$ a solution of the differential equation (6) takes the following form:

$$x_2 = \sum_{i=1}^{4} C_i t^{i-1} + \frac{C_5}{A_2 + \sqrt{A_2^2 - 4A_1}} \exp\left(t\sqrt{\frac{-A_2 - \sqrt{A_2^2 - 4A_1}}{2}}\right) + \frac{C_6}{A_2 + \sqrt{A_2^2 - 4A_1}} \exp\left(-t\sqrt{\frac{-A_2 - \sqrt{A_2^2 - 4A_1}}{2}}\right) + \frac{C_7}{A_2 - \sqrt{A_2^2 - 4A_1}} \exp\left(t\sqrt{\frac{-A_2 + \sqrt{A_2^2 - 4A_1}}{2}}\right) + \frac{C_8}{A_2 - \sqrt{A_2^2 - 4A_1}} \exp\left(-t\sqrt{\frac{-A_2 + \sqrt{A_2^2 - 4A_1}}{2}}\right), \tag{7}$$

where $C_1, \ldots, C_8$ are constants of integration. In order to find them, it is necessary to specify the boundary conditions of the reduced masses of the system. They are written as follows:

$$\begin{cases} x_1(0) = x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0\,, \\ x_1(T) = x_2(T) = vT/2\,, \quad \dot{x}_1(T) = \dot{x}_2(T) = 0\,. \end{cases} \tag{8}$$

The boundary conditions (8) in mathematical form reflect the condition for eliminating the oscillations of the load on the flexible suspension at the end of the trolley start mode. The optimal law of the trolley's movement, which satisfies conditions (8), increases the crane performance since the crane operator should not control of the trolley movement to eliminate the load oscillations.

Taking into account the second equation of the differential equation system (1), the boundary conditions (8) can be written as follows:

$$\begin{cases} x_2(0) = \dot{x}_2(0) = \ddot{x}_2(0) = x_2(0) = 0\,, \\ x_2(T) = vT/2\,, \quad \dot{x}_2(T) = v\,, \quad \ddot{x}_2(T) = x_2(T) = 0\,. \end{cases} \tag{9}$$

We formed a system of eight linear algebraic equations on the basis of the boundary conditions (8) and solved them. As a result, we obtained unknown constants of integration $C_1, \ldots, C_8$. They aren't listed in the work because they have a significant volume. Thus, we have provided a link to find them [14]. We may present only their dimensions: C_1 – m, C_2 – m/s, C_3 – m/s^2, C_4 – m/s^3, $C_5, \ldots, C_8$ – m/s^2. Substituting the obtained expressions in formula (7), we obtain the law of optimal crane trolley acceleration.

2.2 SOME GENERALIZATION OF THE PROBLEM SOLUTION AND FURTHER RESEARCHES

One may notes, that changing the coefficients A_1 and A_2 in a certain manner in expression (7), can bring to an extremal of the criterion

$$(10)\qquad F_{\text{RMS}} = \left(T^{-1} \int\limits_0^T \left(W + (m_1 + m_2)\ddot{x}_2 + m_2 \Omega_0^{-2} \overset{IV}{x_2} \right)^2 dt \right)^{1/2} \to \min \, .$$

The criterion (10) reflects the average force of the trolley drive during the start. Minimizing of the criterion (10) reduces undesired dynamic forces in the crane trolley drive.

The necessary condition of criterion (10) minimum is the Euler-Poisson equation. It is written as follows:

$$(11)\qquad \overset{VIII}{x_2} + \Omega_0^2 \overset{VI}{x_2} + \Omega_0^4 \overset{IV}{x_2} = 0 \, .$$

Indeed, we can obtain the extremal (7) of criterion (10) by changing the notations $A_1 = \Omega_0^4$ and $A_2 = \Omega_0^2$. Note, that the corresponding changes of the coefficients A_1 and A_2 should be made in the expressions of constant integration $C_1, \ldots, C_8$.

Consequently, we can generalize the obtained result. Any problem of optimal control of the system “crane trolley – load” that can be reduced to the boundary problem

$$(12)\qquad \begin{cases} \overset{VIII}{x_2} + A_2 \overset{VI}{x_2} + A_1 \overset{IV}{x_2} = 0 \, , \\ \begin{cases} x_2(0) = \dot{x}_2(0) = \ddot{x}_2(0) = x_2(0) = 0 \, ; \\ x_2(T) = vT/2 \, , \quad \dot{x}_2(T) = v \, , \quad \ddot{x}_2(T) = x_2(T) = 0 \end{cases} \end{cases}$$

is solved. In order to obtain the extremal of an optimal problem, in formula (7) and in the expressions of constant integration $C_1, \ldots, C_8$ the corresponding notations of the coefficients A_1 and A_2 have to be used.

The above generalization allows carrying out a complex optimization of the acceleration mode of the trolley with the load on the flexible suspension. For instance, we have chosen a complex criterion

$$(13)\qquad I = \overline{P}_{[0;\,v]} \alpha_1 \beta + F_{\text{RMS}} \alpha_2 (1 - \beta) \, ,$$

where β is a weight coefficient, which determines the importance of average trolley drive power in the structure of the criterion (13); α_1 and α_2 are coefficients that reduce the dimensions of the individual components of the criterion (13) to one type (for example, to dimensionless form).

From expression (13) it follows that:

$$\begin{cases} \lim\limits_{\beta \to 0} I = F_{\text{RMS}} \,, \\ \lim\limits_{\beta \to 1} I = \overline{P}_{[0;\, v]} \,. \end{cases} \tag{14}$$

In order to minimize the criterion (13) the coefficients A_1 and A_2 in the expression (7) should be set as follows:

$$A_1 = \left(\left(1 + \frac{m_2}{m_1} \right) \alpha_1 \beta + \alpha_2 (1 - \beta) \right) \Omega_0^2 \quad \text{and}$$

$$A_2 = \left(\left(2 + \frac{m_2}{m_1} \right) \alpha_1 \beta + \alpha_2 (1 - \beta) \right) \Omega_0^4 \,.$$

These expressions are obtained by taking into account the weights of the coefficients of the sixth and fourth order derivatives in the Euler-Poisson equations (6) and (11). Thus, the impact of the weight of the criteria (3) and (10) in the complex criterion (13) reduces to the weighting of the respective coefficients in the Euler-Poisson equations.

The proposed approach can be extended to complex optimization problems with criteria that include two and more components.

3 Analysis of the Results

In order to characterize the movement of the system during the acceleration of the trolley, the estimation indicators have been calculated. All of them reflect the undesired features of the trolley acceleration process. Therefore the preference should be given to the trolley motion law, which is characterized by lower values of the indicators. All calculations have been made for the following parameters: $T = 3$ s; $m_1 = 500$ kg; $m_2 = 2000$ kg; $v = 1.2$ m/s; $\beta = 0.5$. The numerical values of indicators are recorded in Table 1 (bold numbers represent the best values of the indicators). All indicators are calculated for two cases of the length of the flexible suspension. In practice, the trolley starts occur at different lengths of the flexible suspension. In the cells of Table 1, the upper values correspond to the case $l = 5$ m, the lower ones correspond to the case $l = 2.5$ m.

The data analysis, which is presented in Table 1, shows that the optimal motion law of the trolley by the complex criterion (13) occupies an “intermediate” position between the criteria (3) and (10) for the most of indicators. The maximum value of the dynamic component of the drive power for the case $l = 5$ m and the maximum deviation of the load from the vertical for the case $l = 2.5$ m are exceptions.

Table 1: Values of estimation indicators of the optimal motion laws of the crane trolley

Estimation indicators	Units	Criterion		
		$\overline{P}_{[0;\ ,v]}$	F_{RMS}	I
Maximum value of the dynamic component of the drive power	W	1060	1340	**949**
		1167	1150	**1132**
Final value of the dynamic component of the drive power	W	**153**	1310	762
		390	1093	1079
Maximum value of the dynamic component of the drive force	N	1561	1566	**1551**
		1610	**1106**	1139
Start value of the dynamic component of the drive force	N	**128**	1091	635
		325	910	900
Final value of the dynamic component of the drive force	N	**128**	1091	635
		325	910	900
RMS of the dynamic component of the drive force	N	1098	**1048**	1062
		1094	**1003**	1006
Maximum deviation of the load from the vertical	m	0.46	**0.30**	0.37
		0.16	0.20	0.22

The criterion (13) represents a compromise in terms of conflicting objectives for minimizing energy (3) and dynamic (10) criteria. Thus, when some indicators decrease, others increase. The criterion (13) has an advantage over both criteria (3) and (9) for the indicator of maximum drive power (the case $l = 5$ m).

We will illustrate the results that have been obtained in the previous paragraphs with plots (Fig. 2), which are built for the case $l = 5$ m. As may be inferred from Fig. 2 (a), the load oscillations at the end of the trolley start are eliminated. The phase-point begins to move from the origin. It returns there at the end of the trolley start. The plots in Fig. 2 (b) show the ride quality of the trolley. It, undoubtedly, has a positive effect on the durability of its operation. The trolley optimal start leads to a decrease of the dynamic loads in the trolley drive and in the metal construction of the crane.

The maximum value of the drive force for all motion laws of the trolley is almost the same (Fig. 2 (c)). However, the initial and final values of the forces vary. The desired feature of the start – is the least values of the final drive force (at moment $t = T$). This requirement is caused by the need of elimination of the dynamic forces in the trolley transmission and in the metal construction. The symmetry of the plots (Fig. 2) is caused by the symmetry of the boundary conditions (8).

The plot of drive power (Fig. 2 (d)) for the complex optimal acceleration law of

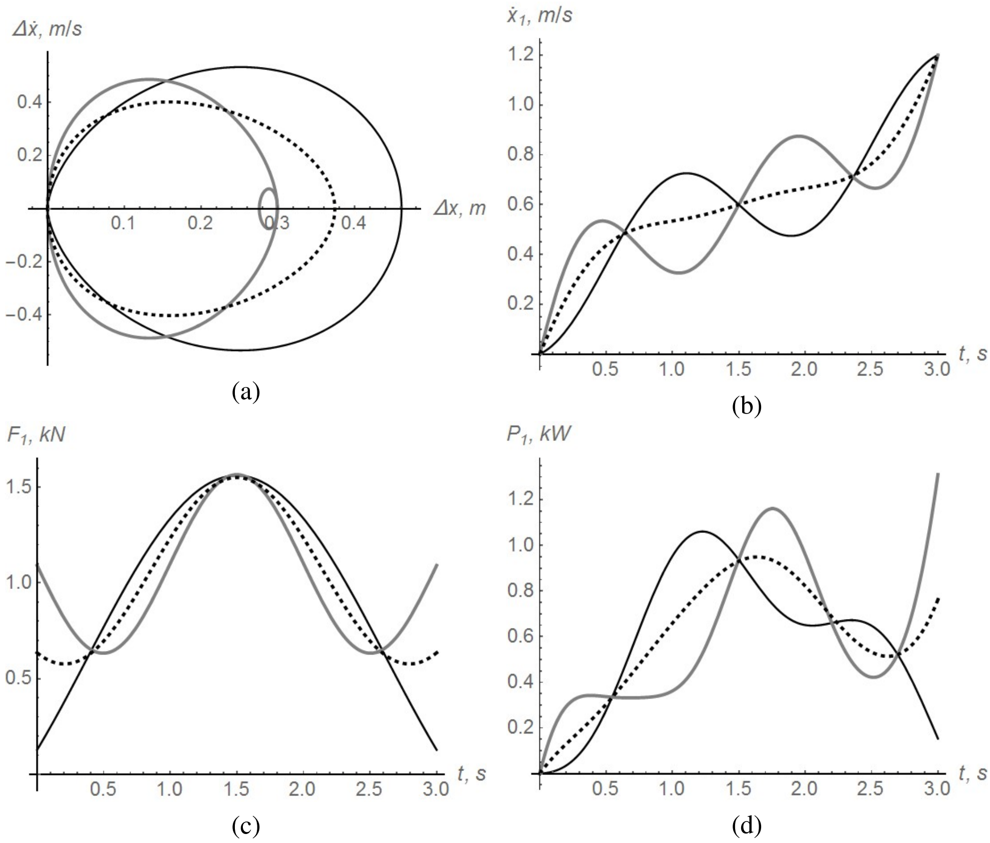

Fig. 2: Plots of the system's motion characteristics under the optimal control: (a) phase portrait of load's oscillations on the flexible suspension; (b) velocity of the trolley; (c) dynamic component of the drive force; (d) dynamic component of the drive power.

the trolley has small variations: the power is slowly changing during the start of the trolley and has no significant peaks. Undoubtedly, it positively affects the life duration of the trolley electric drive and increases its energy efficiency.

The data analysis, which are presented in Table 1, and analysis of the plots (Fig. 2), allows us to put forward a hypothesis. The variation of the weight coefficient β, which shows the impact of the criterion (3), improves the desired characteristics of the trolley start. In other words, there is the value of the weight coefficient β, which allows combining the advantages of the optimal control laws with the absence of their disadvantages.

The proof of the hypothesis lies outside of the article scope. It will be investigated in the further authors' research.

4 Conclusions

The optimal control problem of the trolley with the load on the flexible suspension has been solved in the work. The analysis of the problem solution made it possible to carry out some generalizations. They, in turn, have led to the solution of the complex optimization problem. It includes energy and dynamic factors.

The analysis of the results showed that the optimization of the trolley motion control is accompanied by the elimination of the load oscillations and a decrease of undesired dynamic and energy features. The analysis of the estimated indicators confirmed that the approach proposed in the paper is effective for the complex problems of the dynamical systems' optimal control. It was suggested that the developed approach might be applied for solving more complicated problems of complex optimization. In addition, this approach can be extended to other optimization modes, since it does not require specific boundary conditions of the dynamic systems' motion.

The hypothesis is put forward in the work. It is as follows. The proper choice of the weight coefficient β in the optimization criterion (13) allows achieving the desired dynamic and energy characteristics of the trolley with the load on the flexible suspension. It will be studied in further authors' research.

Obtained in the article results are connected with their integration to the software of the crane control system. In that system sensors provide all the needed input information (the weight of the load, length of the flexible suspension, etc.), the microcontroller calculates the law of crane optimal motion (particularly for obtained from sensors values of parameters), and the drive changes crane's velocity according to the calculated law.

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