

A TILTED LORENTZ FORCE EFFECT ON POROUS MEDIA FILLED WITH NANOFLUID

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[Received 4 January 2017. Accepted 6 January 2018]

ABSTRACT: This paper is intended to investigate the effects of an inclined magnetic field on the mixed convection flow in a lid-driven porous enclosure filled with nanofluid. Both the left and right vertical walls of the cavity are thermally insulated while the bottom and top horizontal walls are maintained at constant but different temperatures. The governing equations are solved numerically by using finite volume method on a uniformly staggered grid system. The computational results are obtained for various combinations of Richardson number, Darcy number, Hartmann number, inclination angle of magnetic field, and solid volume fraction. It is found that the presence of magnetic field deteriorates the fluid flow, which leads to a significant reduction in the overall heat transfer rate. The inclination angle of magnetic field plays a major role in controlling the magnetic field strength and the overall heat transfer rate is enhanced with the increase of inclination angle of magnetic field. Adding the nanoparticles in the base fluid significantly increases the overall heat transfer rate in the porous medium whether the magnetic field is considered or not.

KEY WORDS: Mixed convection; porous cavity; nanofluid; inclined magnetic field.

NOMENCLATURE

g	gravitational acceleration	C_p	specific heat
B_0	magnetic field	Gr	Grashof number
Ha	Hartmann number	H	enclosure height
h_{nl}	thickness of nano-layer	K	permeability
k	thermal conductivity	Nu_{avg}	average Nusselt number
Nu	local Nusselt number	p	fluid pressure
P	dimensionless pressure	Pr	Prandtl number
r_s	radius of nanoparticles	Re	Reynolds number
Ri	Richardson number	θ	temperature
T	dimensionless temperature	$\mathbf{u}$	velocity vector
U, V	dimensionless velocities in X- and Y-direction respectively	u, v	velocities in x- and y-direction respectively

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U_0	lid velocity	$\Delta\theta$	temperature difference
X, Y	dimensionless Cartesian coordinates	x, y	Cartesian coordinates
γ	inclination angle of magnetic field	Da	darcy number
d_p	average particle size	F	Forchheimer coefficient
ϵ	porosity	α	thermal diffusivity
β	thermal expansion coefficient	μ	dynamic viscosity
ν	kinematic viscosity	ρ	density
η	ratio of thermal conductivity of nano-layers	χ	solid volume fraction
ω	ratio of the thickness of nano-layer	σ	electrical conductivity

Subscripts

c	cold wall	eff	effective	eq	equivalent	f	fluid
h	hot wall	nf	nanofluid	nl	nano-layer	s	solid

1 INTRODUCTION

Thermal properties of fluids play a vital role in heating as well as cooling applications in the development of new energy-efficient heat transfer equipment. In particular, the thermal conductivity of fluids is an important physical property that decides the heat transfer performance. The fluids such as water, mineral oils, and ethylene glycol have a low thermal conductivity and do not meet the growing demand in the heat transfer exchange. Consequently, an important need still exists to develop the new types of fluids, which should be more effective in heat transfer performance. Nanofluids, base pure fluids with suspended metallic nanoparticles, are such a new kind of fluids to fulfill the rising demand in the heat transfer exchange and were introduced by Choi [1]. Eastman et al. [2] observed the increased heat transfer in Nanofluids and have shown that the effective thermal conductivity of ethylene glycol has tendency to increase the heat transfer up to 40% for nanofluid consisting of ethylene glycol. Xuan and Li [3] have performed an experimental study on convective heat transfer feature of Cu-water nanofluids in a tube. It is found that the nanofluids remarkably enhance the heat transfer process than the base fluid under the same Reynolds number. Maiga et al. [4] made a numerical study on convective heat transfer behaviors of nanofluids in a uniformly heated tube. Their results have shown that the heat transfer enhancement of ethylene glycol– γ Al_2O_3 mixture is higher than that of water– γ Al_2O_3 mixture. Tiwari and Das [5] presented a numerical approach on mixed convection of nanofluids in a two-sided lid-driven differentially heated square cavity. Mansour et al. [6] studied on mixed convection flows in a cavity partially heated from below using nanofluid. Ghasemi et al. [7] have reported a numerical study on convective heat transfer in a cavity filled with nanofluids in the presence of magnetic field. It is found that the effect of the solid volume fraction on the heat transfer rate strongly depends on the values of the Rayleigh number and the Hartmann number. Muthamilselvan

and Doh [8] conducted a numerical analysis on mixed convection flow of heat generating nanofluid in an enclosure with non-uniform heating along the bottom wall. It is observed that the variation of average Nusselt number is nonlinear with the increase of solid volume fraction for the natural convection case whereas it is linear for the forced and mixed convection regimes.

Convective heat transfer in an enclosure filled with porous media has attracted many researchers in the area of heat and mass transfer and has been given substantial attention over the several years due to the wide range of applications in engineering and geophysical system, such as dynamics of lakes, solar ponds, electronic equipment cooling, heating and drying process, float glass production. Al-Amiri [9] investigated a numerical approach on momentum and energy transfer in a lid-driven cavity filled with a water-saturated porous medium. It has been revealed that the stable stratification restrains the flow motion, which is further reinforced in the presence of porous media in the enclosure. Mahmud and Pop [10] reported a two dimensional steady mixed convection flow in a vented enclosure saturated with a porous medium. It is concluded that a unicellular flow pattern is destroyed and a multi-cellular flow is formed when the physical parameters, namely, Pe , Ra , and D/H are varied. Wang et al. [11] performed a numerical analysis on natural convection of fluid in a tilted cavity saturated with porous medium under magnetic field. They showed that the flow field and heat transfer in porous medium are affected for both magnetic force and inclination angle. Kandaswamy et al. [12] numerically studied the transient free convection in a water saturated porous enclosure with internal heat generation. Jeng and Tzeng [13] analyzed the flow and thermal field in lid-driven enclosure filled with water-saturated aluminum foams. Chattopadhyay et al. [14] presented a numerical simulation on mixed convection flow in a double lid-driven sinusoidally heated porous enclosure. It is found that the flow field is strongly affected in the presence of porous medium.

Very recently, the convective heat transfer of nanofluids in porous media has received considerable attendance due to its wide applications in various fields of engineering and geophysical system. Only few literatures have been conducted to analyze the convective heat transfer of nanofluids in porous media. Ghazvini and Shokouhmand [15] focused on nanofluid-cooled microchannel heat sink using fin model and porous media approach. They showed that an increase in porosity is to enhance the dimensionless temperature in both fin and porous media approaches. Rosca et al. [16] made a numerical study on non-Darcy mixed convective heat transfer over a horizontal plate embedded in a porous medium saturated by a nanofluid. Rohni et al. [17] analyzed combined convection boundary layer flow along a vertical cylinder embedded in a porous medium saturated by a nanofluid. Natural convection of nanofluids in porous media has been numerically investigated by Bourantas [18]. In

the presence of a porous medium and for different Ra , the overall heat transfer rate is increased as the solid volume fraction increases. Zhang et al. [19] presented a numerical study on MHD flow and radiation heat transfer of nanofluids in porous media with the effects of variable surface heat flux and chemical reaction. It is found that velocity and temperature fields are strongly affected in the presence of magnetic field and radiation effects. Nguyen et al. [20] made a numerical analysis on laminar natural convective flow in non-Darcy porous enclosure filled with nanofluid. Their results have revealed that increasing the solid volume fraction decreases the overall heat transfer rate in Darcy flow regime at a high Rayleigh number and low Darcy number. Sureshkumar and Muthamilselvan [21] studied on mixed convection flow of nanofluid in an inclined square cavity saturated with porous medium. They found that the overall heat transfer rate increases with the increase of solid volume fraction in the porous enclosure. Sheikholeslami and Shehzad [22] reported a numerical study on magnetohydrodynamic nanofluid convective flow in a porous enclosure. Their outputs showed that temperature gradient reduces with increase of Ha while it increases with augment of permeability of porous media and buoyancy forces.

Nanofluid convection in porous media has been widely studied in recent years because of its wide-spread applications in porous foam, microchannel heat sinks (used for electronic cooling) and cancer treatment. Especially, in cancer treatment, the objective is to induce the maximum damage on the tumor (this requires elevating the temperature of at least 90% of the tumor above 43°C) with the minimum damage to the normal tissues. Since a living tissue is a type of fluid-saturated porous medium, the development of optimal protocols for this type of treatment again requires fundamental understanding of nanofluid convection in porous media. Motivated by these concepts and on paying a great attention to convection of nanofluid in a porous cavity, we propose this study to explore the flow and heat transfer characteristics in a lid-driven porous cavity filled with nanofluid in the presence of magnetic field with various inclination angles.

2 MATHEMATICAL ANALYSIS

Fig. 1 presents a schematic diagram of physical configuration of the present study. The system is considered to be steady, laminar, incompressible mixed convective flow and heat transfer in a 2D square porous cavity of height H filled with nanofluid. The top and bottom walls are assumed to have the uniform temperatures θ_c and θ_h , respectively, with $\theta_h > \theta_c$. The side walls are considered to be thermally insulated. The velocity components u and v are taken in x and y directions respectively. No slip condition is assumed between the fluid and two vertical walls. Further, the tangential fluid velocity is considered to equal the moving wall velocity. The top wall of the cavity is moving from left to right with constant speed U_0 . The inclined mag-

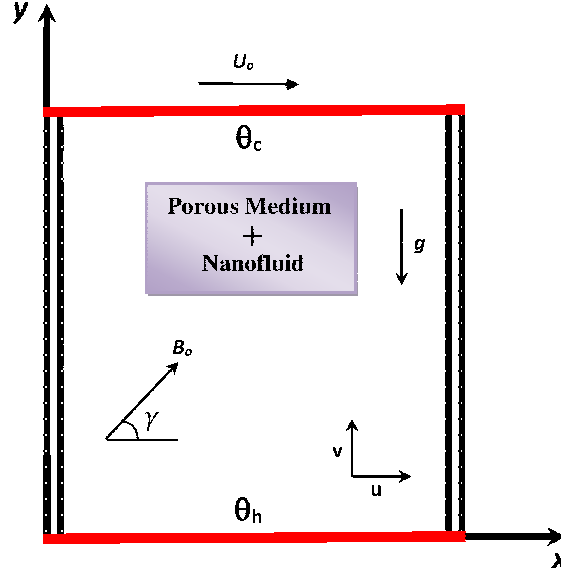


Fig. 1. A schematic view of the considered cavity in the present study.

netic fields with different strengths and inclination angles are imposed. The porous cavity is filled with nanofluid, which is made of a base fluid (water) and spherical nanoparticles (Cu). The fluid in the cavity is Newtonian and incompressible. The porous medium is taken as hydrodynamically and thermally isotropic and saturated with a fluid that is local thermal equilibrium (LTE) model with the solid matrix. It is assumed that the base fluid (water) and spherical nanoparticles (Cu) are in thermally equilibrium and no-slip velocity between them. The fluid physical properties are assumed to be constant except the density variation in the buoyancy term. The Boussinesq approximation is valid. The thermophysical properties of the base fluid and nanoparticles are shown in Table 1. Further, the effects of joule heating, displacement currents and induced magnetic field are assumed to be negligible. The governing equations consisting of mass, momentum and energy equations for the

Table 1. Thermophysical properties of water and nanoparticles

	ρ (kg m ⁻³)	C_p (J kg ⁻¹ K ⁻¹)	k (W m ⁻¹ K ⁻¹)	$\beta \times 10^{-5}$ (K ⁻¹)
water	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67

mixed convection of the nanofluid in the 2D porous cavity can be written in the dimensional form as follows:

Continuity equation

$$(1) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0;$$

Momentum equations

$$(2) \quad \frac{u}{\epsilon} \frac{\partial u}{\partial x} + \frac{v}{\epsilon} \frac{\partial u}{\partial y} = -\frac{\epsilon}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} (\nabla^2 u) - \frac{\mu_{nf}}{\rho_{nf}} \frac{\epsilon}{K} u - \frac{F}{K^{\frac{1}{2}}} \epsilon u \sqrt{u^2 + v^2} + \sigma_{nf} B_0^2 (v \sin(\gamma) \cos(\gamma) - u \sin^2(\gamma))$$

$$(3) \quad \frac{u}{\epsilon} \frac{\partial v}{\partial x} + \frac{v}{\epsilon} \frac{\partial v}{\partial y} = -\frac{\epsilon}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} (\nabla^2 v) - \frac{\mu_{nf}}{\rho_{nf}} \frac{\epsilon}{K} v - \frac{F}{K^{\frac{1}{2}}} \epsilon v \sqrt{u^2 + v^2} + \frac{(\rho\beta)_{nf}}{\rho_{nf}} \epsilon g(\theta - \theta_c) + \sigma_{nf} B_0^2 (u \sin(\gamma) \cos(\gamma) - v \cos^2(\gamma));$$

Energy equation

$$(4) \quad u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \alpha_{nf} (\nabla^2 \theta).$$

The appropriate dimensional forms of the boundary conditions are given as follows:

$$(5) \quad \begin{aligned} u = v = 0, \quad \theta = \theta_h, \quad y = 0; \\ u = U_0, \quad v = 0, \quad \theta = \theta_c, \quad y = H; \\ u = v = 0, \quad \frac{\partial \theta}{\partial x} = 0, \quad x = 0; \\ u = v = 0, \quad \frac{\partial \theta}{\partial x} = 0, \quad x = H. \end{aligned}$$

The above dimensional form of mass, momentum and energy equations can be converted to non-dimensional form by using the following dimensionless parameters:

$$(6) \quad \begin{aligned} X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \\ T = \frac{\theta - \theta_c}{\theta_h - \theta_c}, \quad Gr = \frac{g\beta\Delta\theta H^3}{\nu_f^2}, \quad Ri = \frac{Gr}{Re^2}, \quad P = \frac{p}{\rho U_0^2}, \\ Re = \frac{U_0 H}{\nu_f}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Da = \frac{K}{H^2}, \quad Ha = B_0 H \sqrt{\frac{\sigma_f}{\mu_f}} \end{aligned}$$

After non-dimensionalization, the above dimensional form of governing equations can now be written as follows:

$$(7) \quad \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 ;$$

$$(8) \quad \frac{1}{\epsilon^2} \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = - \frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\epsilon Re \rho_{nf} \nu_f} (\nabla^2 U) \\ - \frac{\mu_{nf}}{\rho_{nf} \nu_f} \frac{U}{Re Da} - \frac{1.75}{\sqrt{150}} \frac{(U^2 + V^2)^{1/2}}{\sqrt{Da}} \frac{U}{\epsilon^{3/2}} \\ + \frac{\rho_f}{\rho_{nf}} \frac{\sigma_{nf}}{\sigma_f} \frac{Ha^2}{Re} (V \sin(\gamma) \cos(\gamma) - U \sin^2(\gamma)) ;$$

$$(9) \quad \frac{1}{\epsilon^2} \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = - \frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\epsilon Re \rho_{nf} \nu_f} (\nabla^2 V) \\ - \frac{\mu_{nf}}{\rho_{nf} \nu_f} \frac{V}{Re Da} - \frac{1.75}{\sqrt{150}} \frac{(U^2 + V^2)^{1/2}}{\sqrt{Da}} \frac{V}{\epsilon^{3/2}} + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} (RiT) \\ + \frac{\rho_f}{\rho_{nf}} \frac{\sigma_{nf}}{\sigma_f} \frac{Ha^2}{Re} (U \sin(\gamma) \cos(\gamma) - V \cos^2(\gamma)) ;$$

$$(10) \quad U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Pr.Re} (\nabla^2 T) .$$

The non-dimensional boundary conditions of the considered problem are given as follows:

$$(11) \quad \begin{aligned} U = V = 0, \quad T = 1, \quad Y = 0; \\ U = 1, \quad V = 0, \quad T = 0, \quad Y = 1; \\ U = V = 0, \quad \frac{\partial T}{\partial X} = 0, \quad X = 0; \\ U = V = 0, \quad \frac{\partial T}{\partial X} = 0, \quad X = 1. \end{aligned}$$

The properties of the nanofluids are given in Table 2.

The local and average heat transfer rates of the cavity can be presented by means of the local and average Nusselt numbers. The local Nusselt number for the consider problem is calculated along the top heated wall and the average Nusselt number

Table 2. Applied formulae for the nanofluid properties

Nanofluid properties	Applied model
Density	$\rho_{nf} = (1 - \chi)\rho_f + \chi\rho_s$
Thermal diffusivity	$\alpha_{nf} = \frac{k_{eff}}{(\rho C_p)_{nf}}$
Heat capacitance	$(\rho C_p)_{nf} = (1 - \chi)(\rho C_p)_f + \chi(\rho C_p)_s$
Dynamic viscosity	$\mu_{nf} = \frac{\mu_f}{(1 - \chi)^{2.5}}$
Thermal expansion coefficient	$(\rho\beta)_{nf} = \chi\rho_s\beta_s + (1 - \chi)\rho_f\beta_f$

(Nu_{avg}) for overall heat transfer rate is obtained by integrating the local Nusselt number.

$$(12) \quad Nu = -\frac{k_{eff}}{k_f} \frac{\partial T}{\partial Y},$$

$$(13) \quad Nu_{avg} = \int_0^1 Nu dX.$$

Many researchers have cited the classical Maxwell [22] model in which only the particle volume concentration and the thermal conductivities of the particle are considered but the Brownian motion of the nanoparticles and the effect of solid like nano-layers formed around nanoparticles are not taken into the account. To execute the above need the modified Maxwell [23] model considers a nano-layer with a solid like structure formed by the liquid molecules close to solid surface. As far as the modified Maxwell model is concerned, k_{eff} is the effective thermal conductivity of the nanofluid for spherical nanoparticles, which is given as follows:

$$(14) \quad \frac{k_{eff}}{k_f} = \frac{(k_{eq} + 2k_f) + 2(k_{eq} - k_f)(1 + \omega)^3\chi}{(k_{eq} + 2k_f) - (k_{eq} - k_f)(1 + \omega)^3\chi},$$

where ω is the ratio of the thickness of nano-layer to the original radius of nanoparticles (h_{nl}/r_s) and k_{eq} is the equivalent thermal conductivity of nanoparticles and their layers

$$(15) \quad \frac{k_{eq}}{k_s} = \eta \frac{2(1 - \eta) + (1 + \omega)^3(1 + 2\eta)}{-(1 - \eta) + (1 + \omega)^3(1 + 2\eta)},$$

where η is the ratio of thermal conductivity of nano-layer upon the thermal conductivity of the nanoparticles ($\eta = k_{nl}/k_s$). In this study, it is assumed that $h_{nl} = 2$ nm,

$r_s = 3$ nm and $k_{nl} = 100k_f$. Yu and Choi [23] obtained that for these conditions, the result of the modified Maxwell model are in good agreement with experimental results.

3 NUMERICAL TECHNIQUE AND VALIDATION

3.1 NUMERICAL METHOD

The governing equations (6)–(9) subject to the boundary conditions are discretized by the finite volume method (FVM) on a uniform staggered grid arrangement using SIMPLE algorithm of Patankar [24]. The third order accurate deferred QUICK scheme of Hayase et al. [25] and central difference scheme are applied for the convection and diffusion terms in the both momentum and energy equations. The uniform grid is selected in both X and Y directions. The solution domain consists of a number of control volumes at which discretization equations are applied. The governing equations are transferred into a system of algebraic equations through integration over each control volume. The solution of the discretized equations for each variable is obtained by TDMA line-by-line method. The grid independence tests are carried out using the grid points from 21×21 to 101×101 for $Ri = 100$, $Pr = 6.2$, $\epsilon = 0.4$, $Da = 10^{-2}$, $Ha = 30$, $\gamma = 0^\circ$, and $\chi = 0.06$. The grid independence test, which is shown in Fig. 2, demonstrates that an 81×81 grid system is enough to obtain the desired accuracy of results.

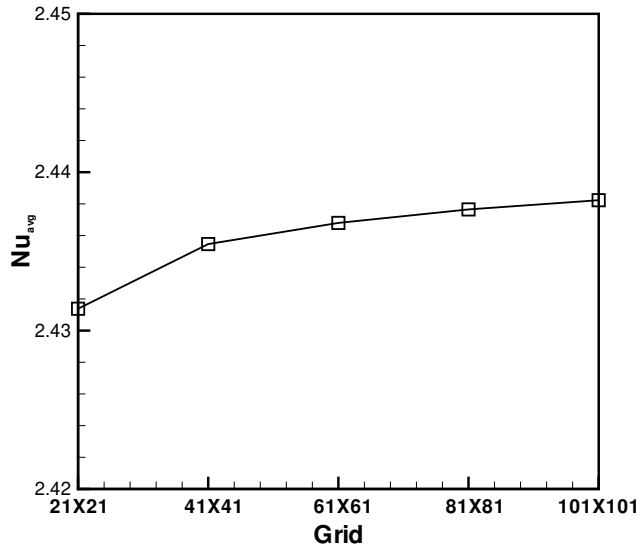


Fig. 2. Average Nusselt number for different mesh sizes at $Ri = 100$, $Da = 10^{-2}$, $\epsilon = 0.4$, $Ha = 30$, $\gamma = 0^\circ$ and $\chi = 0.06$.

3.2 CODE VALIDATION

In order to check the accuracy of the present results, the present computational code has been validated by two published work in the literature. The results for mixed convection in differentially heated square cavity filled with fluid saturated porous medium were compared with Khanafer and Chamkha [26]. The results of mid-plane velocity profiles are presented in Fig. 3(a) and a excellent agreement is observed

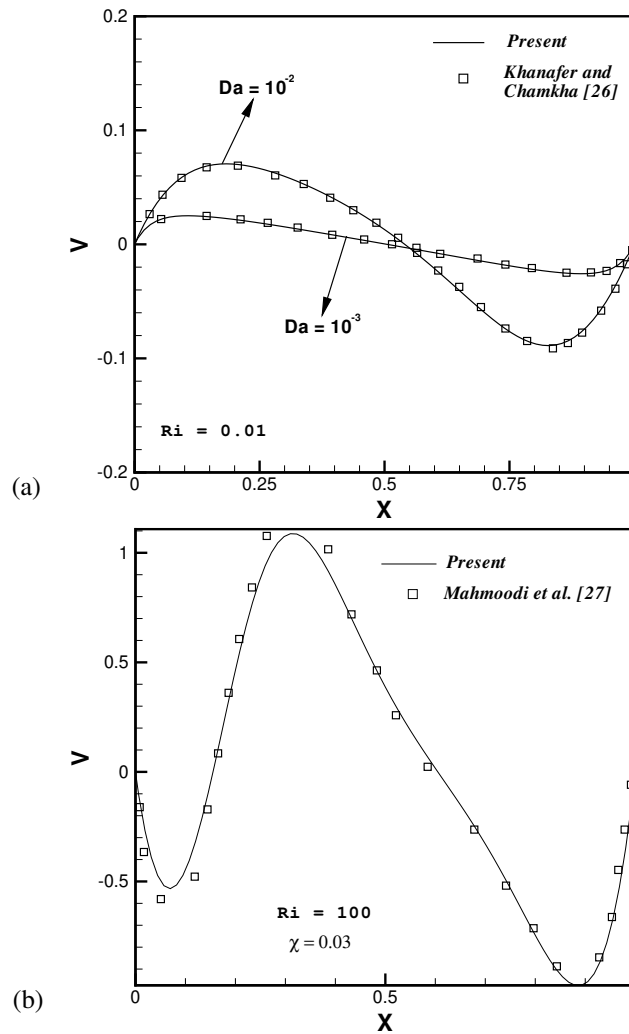


Fig. 3. Comparison of the mid-plane velocity profiles of Khanafer and Chamkha [26] and Mahmoodi et al. [27]

between the two numerical results. The results for combined convection in a lid-driven enclosure filled with nanofluid were also compared with Mahmoodi et al. [27]. A comparison of the dimensionless vertical velocity component along the horizontal centerline of the cavity for $Ri = 100$ and $\chi = 0.03$ are executed in Fig. 3(b) and shows a good agreement. The results compared with the previous literature provide confidence to the accuracies of the present numerical solutions.

4 RESULTS AND DISCUSSION

In this article, mixed convection flow of a nanofluid in a lid-driven porous cavity with an inclined magnetic field has been investigated numerically. Throughout the study, the Cu-water nanofluid is chosen as working fluid with the Prandtl number $Pr = 6.2$. The computations are carried out to investigate the effects of the controlling

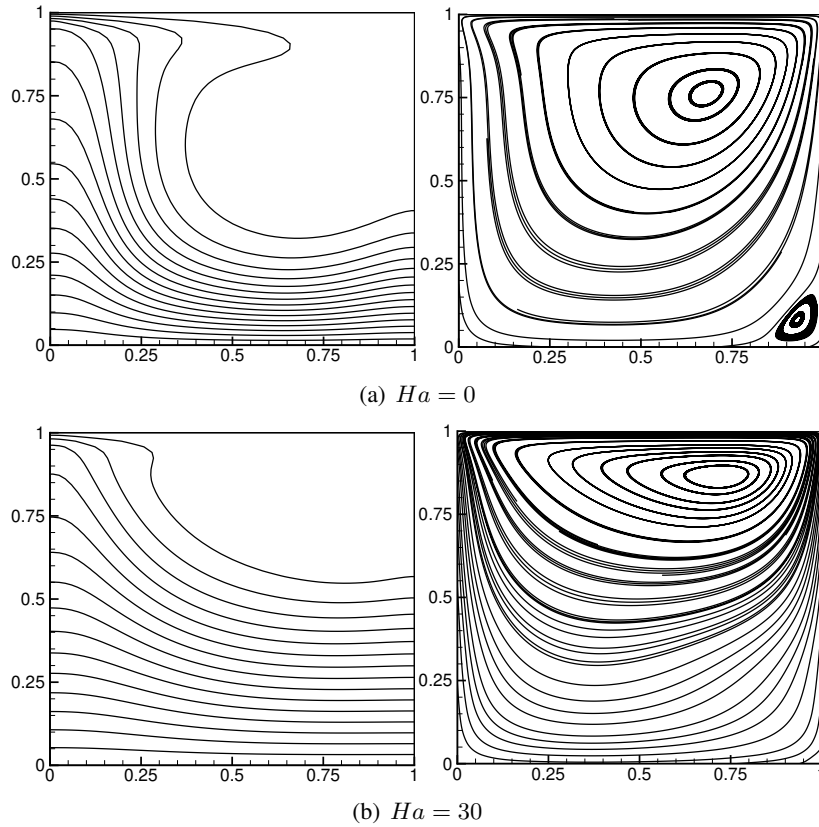


Fig. 4. Isotherms (left) and Streamlines (right) for different Hartmann number with $Ri = 1$, $Da = 10^{-2}$, $\gamma = 0^\circ$ and $\chi = 0.06$.

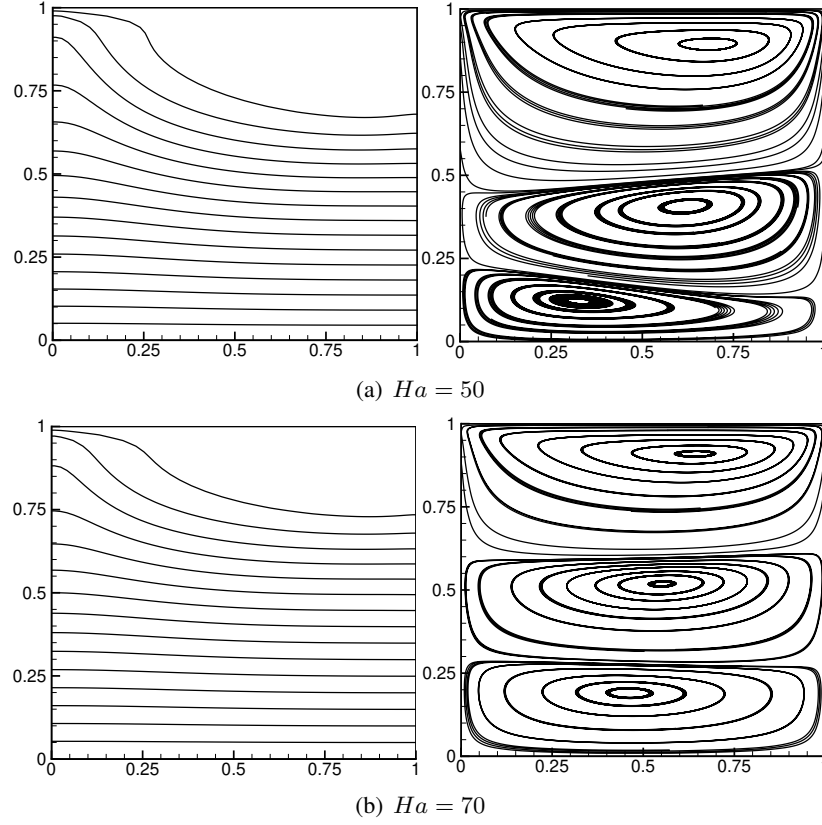


Fig. 4. Continued from previous page.

parameters on the fluid flow, namely, Richardson number ($Ri = 0.01 - 100$), Darcy number ($Da = 10^{-4} - 10^{-1}$), Hartmann number ($Ha = 0 - 70$), inclination angle of magnetic field ($\gamma = 0^\circ - 90^\circ$), and solid volume fraction ($\chi = 0 - 0.06$). The numerical results have been presented in terms of isotherm, streamline, mid-plane velocity, and local as well as average Nusselt numbers.

Fig. 4 shows the effect of the applied magnetic field, Ha , varied from 0 to 70 on the flow field and temperature distributions for the fixed values of $Ri = 1$, $\gamma = 0^\circ$, $Da = 10^{-2}$, and $\chi = 0.06$. When $Ha = 0$, i.e., in the absence of magnetic field, a primary recirculating eddy occupies the entire cavity while a minor cell is appeared near the right-bottom corner of the cavity. The temperature distributions are more prominent near the bottom and left vertical walls of the cavity while it is very weak in the interior region of the cavity due to the vigorous effects of the shear-driven circulations. For $Ha = 30$, the applied magnetic field retards the fluid motion

gradually except the region close to the top moving wall of the cavity (see Fig. 4(b)). As Ha is increased to 50 and 70, a primary recirculating eddy is broken up and a tri-cellular structure appears in the cavity. The physical phenomenon for this behaviour is that while increasing the Hartmann number, the resistive type force called Lorentz force increases, which causes a reduction in the fluid motion. On the other hand, as Ha increases, the isotherms become parallel to the bottom wall of the cavity, which indicates the conduction dominant mode. It can clearly be understood from these contour plots that the magnetic field has the tendency to transform the heat transfer mechanism from convective to conductive mode.

The effect of the magnetic field inclination angle on the flow field and temperature distributions is displayed in Fig. 5 for $Ri = 1$, $Ha = 50$, $Da = 10^{-2}$, and $\chi = 0.06$. When the magnetic field is not inclined, i.e., at $\gamma = 0^\circ$, a transverse magnetic field

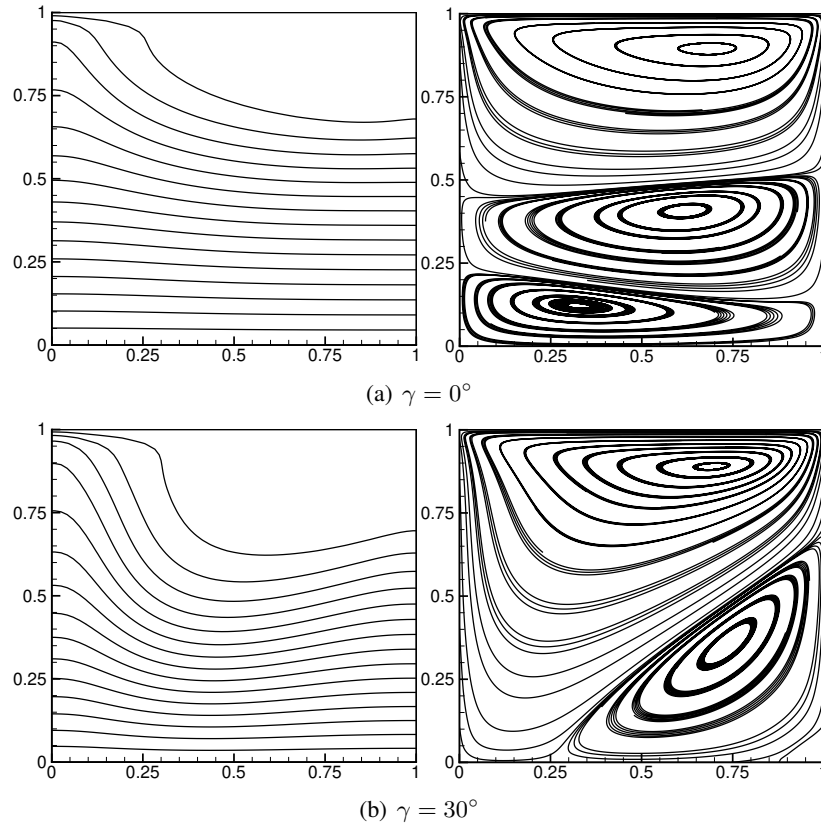


Fig. 5. Isotherms (left) and Streamlines (right) for different γ with $Ri = 1$, $Da = 10^{-2}$, $Ha = 50$ and $\chi = 0.06$.

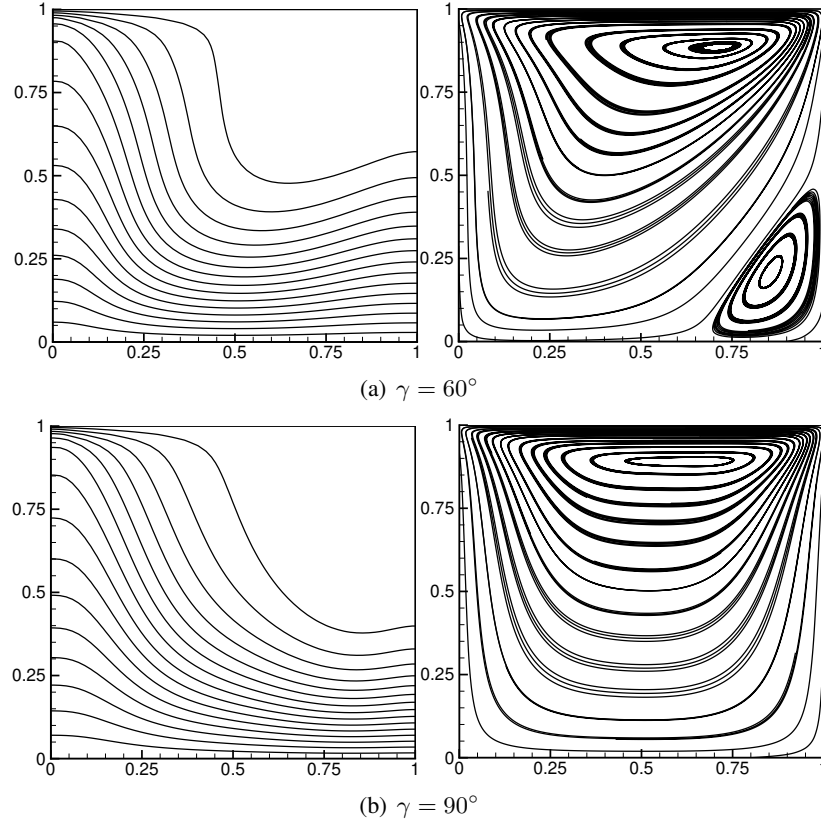


Fig. 5. Continued from previous page.

is applied in the horizontal direction normal to the vertical walls of the enclosure. In this case, the Lorentz force will act perpendicularly to the direction of the applied magnetic field, i.e., opposite to the buoyancy force. As a result, the applied magnetic field directly affects the motion of the fluid inside the cavity when $\gamma = 0^\circ$, as can be seen in Fig. 5(a). In this figure, the streamlines specify the flow structure of a primary eddy near the top moving lid and two weaker cells appeared one below the other in the rest of the cavity. Moreover, the fluid remains stagnant in the middle and bottom of the cavity and conduction heat transfer occurs inside the cavity. Consequently, the temperature distributions are almost parallel to the horizontal walls of the cavity. As the inclination angle of the magnetic field is increased from 0° to 30° , the primary recirculating eddy gets strengthen in its size and covers the three-fourth of the cavity. Meanwhile, the two weaker eddies are merged and a secondary cell appears near the right-bottom corner. For $\gamma = 60^\circ$, the size of the primary recirculating eddy is

enlarged further whereas the size of the secondary eddy is decreased. Further increase in γ merges the primary eddy and secondary vortex, and forms a major cell for the whole cavity. It is due to the fact that the Lorentz force associated with inclined magnetic field enhances the fluid circulation inside the enclosure. The isotherms show that the temperature distributions for the non-inclined magnetic field totally collapse when the magnetic field is inclined. It can be seen that an increase in γ starts pushing the isotherms towards the bottom of the cavity and builds up a large temperature gradients there.

Figs. 6 and 7 display the effect of the solid volume fraction for $Ha = 0$ and 50 while the other parameters are kept fixed. When $Ha = 0$, an increase in χ decreases the flow intensity as the solid volume fraction has the tendency to retard the strength of the buoyancy force. The isotherms show that a slight thicker thermal boundary

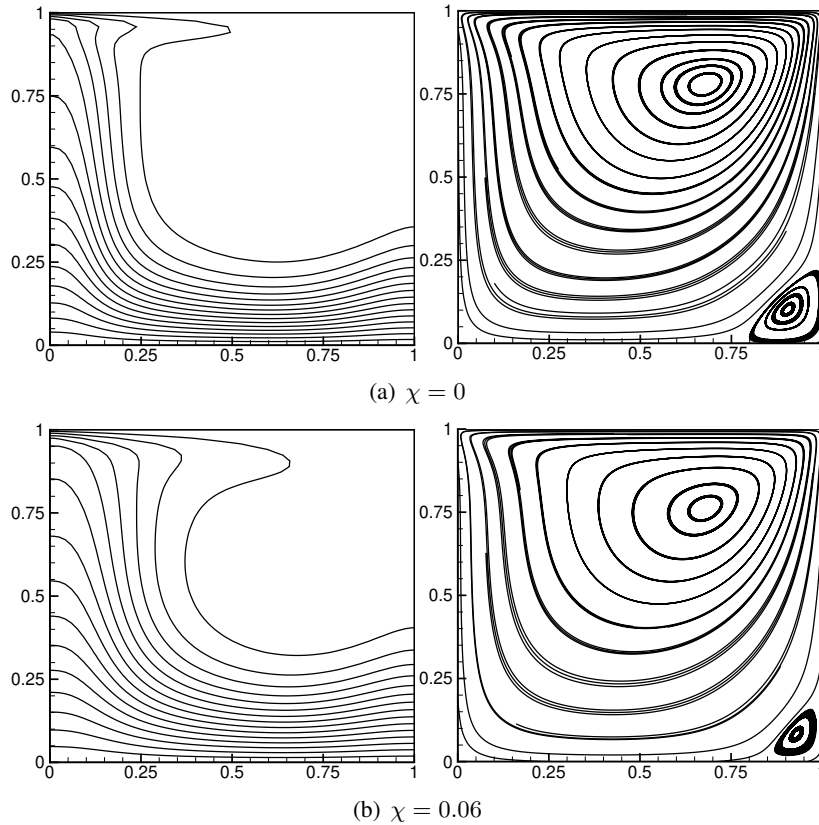


Fig. 6. Isotherms (left) and Streamlines (right) for different χ for $Ha = 0$ with $Ri = 1$, $Da = 10^{-2}$, and $\gamma = 0^\circ$.

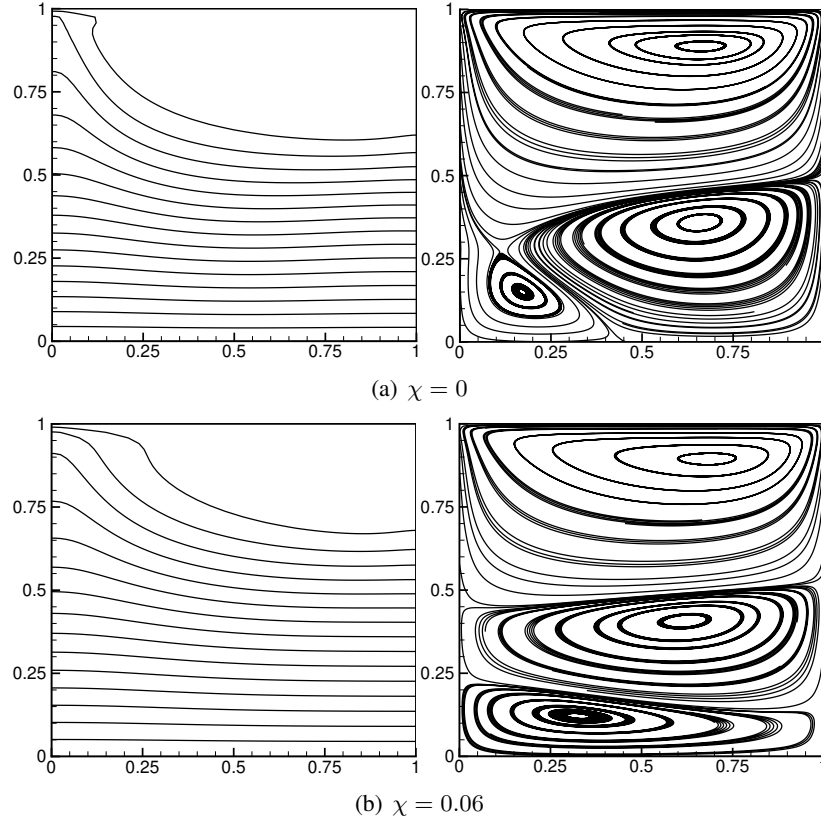


Fig. 7. Isotherms (left) and Streamlines (right) for different χ for $Ha = 50$ with $Ri = 1$, $Da = 10^{-2}$, and $\gamma = 0^\circ$.

layers are developed for $\chi = 0.06$. This reveals the fact that an increase in the solid volume fraction results in an improvement in thermal conductivity. Furthermore, more fluid is heated with the increase of solid volume fraction of nanoparticles. The streamlines and isotherms for $Ha = 50$ are shown in Fig. 7. In this case, for $\chi = 0$, the fluid structure is characterized by a primary vortex near the top moving wall and two secondary vortices along the bottom corners. In addition, the secondary eddy of the right-bottom corner is bigger than the one in the left-bottom corner. When χ is increased to 0.06, the secondary eddy near the left-bottom corner enlarges and occupies the bottom of the cavity. The temperature distributions are almost the same as the solid volume fraction is increased. The general conclusion based on Figs. 6 and 7 is that the flow intensity decreases with an increase in the solid volume fraction whether the magnetic field is considered or not.

The effects of Ha , γ , and χ on the mid-plane horizontal velocity are shown in Fig. 8. The effect of the magnetic field on the velocity profiles is exhibited in Fig. 8(a). The magnitude of velocity decreases with an increase in Ha . When $Ha = 70$, the horizontal velocity is almost zero in the half of the cavity, which indicates that much of the fluid remains stagnant in the middle and bottom of the enclosure (see Fig. 4(d)). This observation is physically justified by the fact that the magnetic field provides the resistive type force to the fluid flow and therefore the velocity of the fluid is decreased with increasing of Hartmann number. Fig. 8(b) displays the effects of the inclination angle of magnetic field on the horizontal velocity component at the

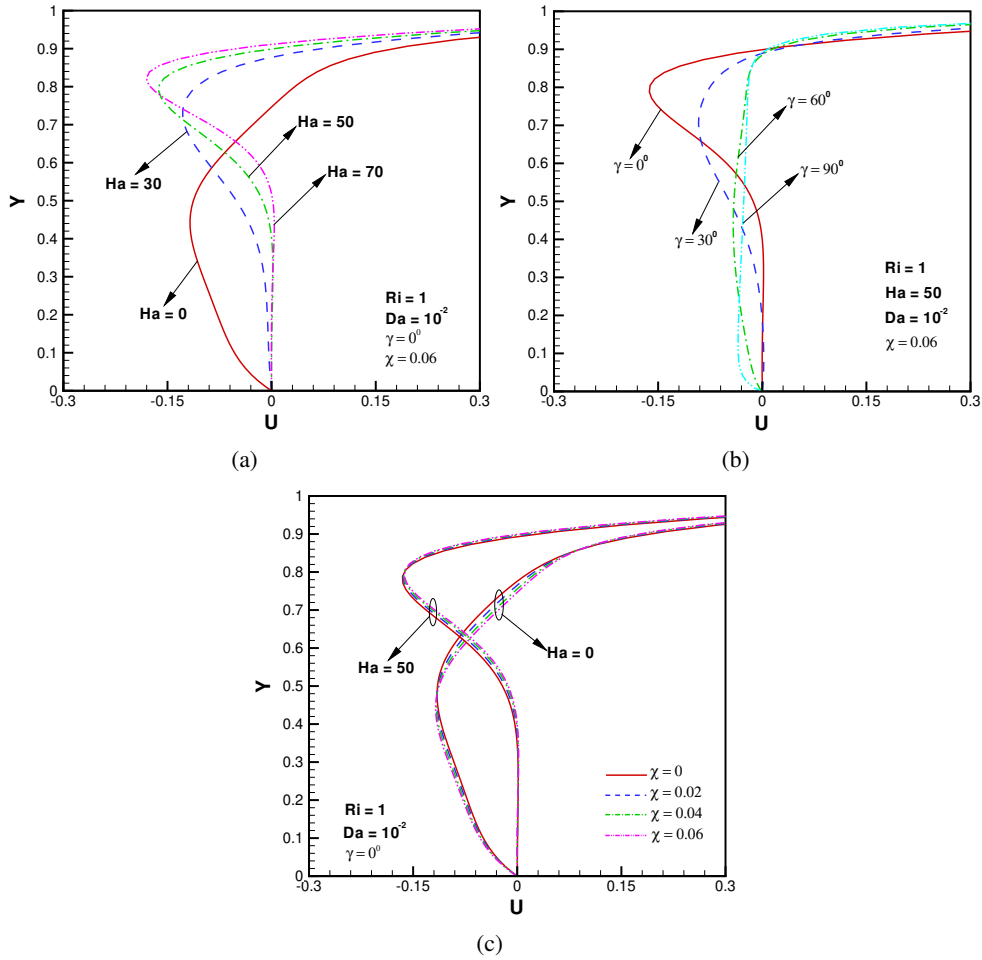


Fig. 8. Mid-plane velocity profiles of the cavity.

enclosure mid-section. It is to be noted that when $\gamma = 0^\circ$, the applied magnetic field results in a force (Lorentz force) opposite to the buoyancy force, which affects the flow intensity directly. Consequently, the flow velocity is zero up to $Y = 0.5$ for the non-inclined magnetic field. When the magnetic field is inclined, the effect of the force opposite to the buoyancy force is reduced. Hence, the magnitude of velocity increases with an increase in γ . An increase in the solid volume fraction leads to a decrease in the flow velocity whether the magnetic field is considered or not. This can be verified from Fig. 8(c).

Fig. 9 illustrates the effects of Richardson number, Darcy number, Hartmann number, inclination angle of magnetic field, and solid volume fraction on the average Nusselt number. In Fig. 9(a), the overall heat transfer rate augments slightly for $Ri > 1$ whereas it increases much more rapidly for $Ri < 1$. It should be noted that the applied magnetic field strongly affects the heat transfer characteristics inside the cavity. For all Ri , the overall heat transfer rate has been decreased significantly with the increase of the applied magnetic field strength. As Ri is fixed, an increase in γ enhances the overall heat transfer rate for all Ha considered. When the magnetic field is inclined to 60° , a better heat transfer rate is obtained for the forced convection dominated regime as to compare with the natural convection dominated regime. The effect of Hartmann number for different Darcy number is depicted in Fig. 9(b). For $Da = 10^{-1}$, the average Nusselt numbers drastically decrease with increasing Ha whereas it is almost the same for $Da = 10^{-4}$. This results show that the magnetic field suppression effect on the overall heat transfer rate is not significant in the low permeability when compared with the high permeability. Fig. 9(c) displays the effect of the magnetic field inclination angle γ for different Da . It is noted that an increase in the inclination angle of magnetic field accelerates the temperature distribution and thus, the overall heat transfer rate increases with an increase in γ for all the Darcy number varied from 10^{-3} to 10^{-1} . However, the variation of average Nusselt numbers is almost identical for $Da = 10^{-4}$. This reveals the fact that the inclination angle of magnetic field does not affect the flow field and heat transfer characteristics when $Da = 10^{-4}$. In Fig. 9(d), generally, the addition of nanoparticles in the base fluid causes an enhancement in the overall heat transfer rate for all Ha . In the other words, the average Nusselt numbers monotonically increase with the increase of the solid volume fraction for all Ha varied. The convection suppression effect is not significant for the non-inclined magnetic field whereas it increases with an increase in the inclination angle of magnetic field. As a result, there is a huge enhancement in the overall heat transfer rate on increasing γ for both pure ($\chi = 0$) and nanofluid ($\chi > 0$).

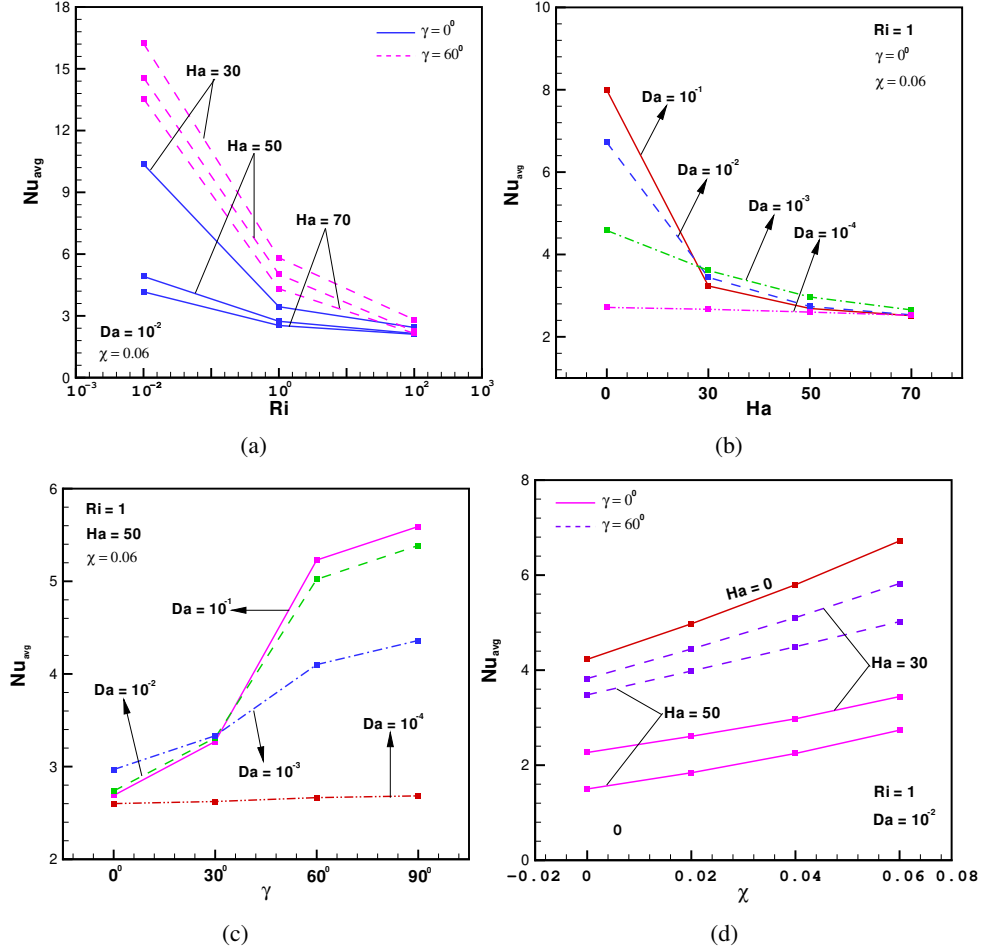


Fig. 9. Average Nusselt numbers.

5 CONCLUSION

The problem of mixed convection in a lid-driven porous cavity filled with Cu-water nanofluid with an inclined magnetic field has been numerically investigated. From this investigation, the following conclusions are drawn.

- An increase in the magnetic field increases the Lorentz force, which leads to a significant reduction in the overall heat transfer rate for all the three configurations, namely, natural, mixed, and forced convection.
- The magnetic field strength is reduced by the inclination angle of magnetic field

and the overall heat transfer rate is enhanced with the increase of inclination angle of magnetic field.

- When the magnetic field is inclined from 0° to 90° , the Lorentz force associated with inclined magnetic field assists to enhance the overall heat transfer rate for all Da considered.
- Both the magnetic field and its inclination angle do not make an significant impact on the overall heat transfer rate when the permeability of the porous medium is very low.
- Addition of nanoparticles in the base fluid enhances the overall heat transfer rate significantly whether the magnetic field is considered or not.

Acknowledgment

This work was supported by SERB-DST, India. The authors thank SERB-DST, India for its financial support through this research project (SR/FTP/MS-025/2009).

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