

ON THE WEAR OF A TRIBOSYSTEM

V. DIAMANDIEV

When a body moves on a surface, under pressure, a wear occurs as a consequence of friction [1]. Here we consider the wear of a round plate given in fig. 1. The plate is made of a soft material and moves on an ideally hard surface. The plate rotates around its axis with a constant angular velocity. Clearly the plate is worn out with a

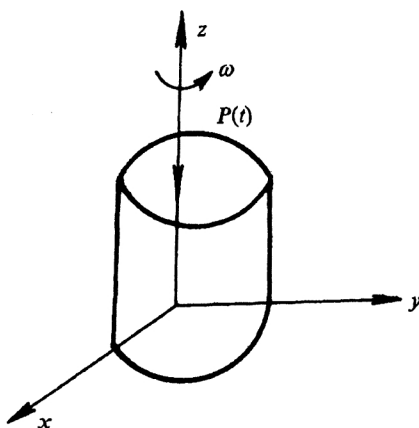


Fig. 1

height $w(t)$ as a function of the time t . The initial thickness of the plate h_0 changes with t as $h(t) = h_0 - w(t)$. According to the Preston's law (1), the conformation $w(t)$ is given by

$$(1) \quad w(r, t) = \int_0^t m(\tau) f(\tau) p(r, \tau) v(r, \tau) d\tau$$

where $m(t)$ is the wear coefficient, f is the friction coefficient, $p(r, t)$ is the normal pressure and $v(r, t)$ is the relative plate velocity. Due to the constant angular velocity

of the plate one has:

$$(2) \quad v = r\omega$$

where ω is the angular velocity.

We suppose, that the friction coefficient depends on time according to the formula (2)

$$(3) \quad f(t) = f_0[1 - k_1 T_*(t)]$$

where f_0 , k_1 are constants and $T_*(t)$ is the temperature of the surface of the plate ($z=0$).

The distribution of the pressure with respect to r is hyperbolic, i. e.

$$(4) \quad p(r, t) = \frac{k(t)}{r}.$$

The above formula (4) corresponds to the case of a concentrated force. We determine $k(t)$ if in the centre of the plate acts a force $\mathfrak{P}(t)$. The elementary force $\Delta\mathfrak{P}$ at the point (r, ϑ) is given by the expression

$$(5) \quad \Delta P = p(r, t) r dr d\vartheta$$

we substitute (4) into (5) and integrate over the surface of the plate, i. e.

$$(6) \quad P(t) = k(t) \int_0^a dr \int_0^{2\pi} d\vartheta$$

where a is the radius of the plate. Then (6) implies

$$(7) \quad k(t) = \frac{P(t)}{2\pi a}.$$

As a consequence of (7) and (1) we get the formula:

$$(8) \quad p(r, t) = \frac{P(t)}{2\pi ar}.$$

According to (3), (2), (8) and (1) we have:

$$(9) \quad w(r, t) = \frac{\omega f_0}{2\pi a} \int_0^t m(\tau) [1 - k_1 T_*(\tau) P(\tau)] d\tau.$$

We determine the temperature $T_*(t)$ at the base of the plane ($z=0$). The Fourier equation has the form (9)

$$(10) \quad \frac{\partial T}{\partial t} = a \frac{\partial T}{\partial z^2}$$

where a is a constant coefficient. We assume that the temperature T varies slowly

with time t , i. e. $\frac{\partial T}{\partial t} \approx 0$. Then (10) implies

$$\frac{\partial^2 T}{\partial z^2} = 0$$

or we get

$$(11) \quad T(z, t) = C_1 z + C_2.$$

We have the following boundary conditions

$$(12) \quad \begin{aligned} T(0, t) &= T_*(t) \\ T(h_0, t) &= 0. \end{aligned}$$

The latter condition expresses the constancy of the environment temperature. Then (11) and (12) imply

$$(13) \quad C_2 = T_*(t), \quad C_1 = -\frac{T_*(t)}{h_0}.$$

After a substitution of (13) in (11) we get

$$(14) \quad T(z, t) = -\frac{T_*(t)}{h_0} z + T_*(t).$$

According to the Fourier's law the quantity of heat per unit surface and time is given by the formula (4)

$$(15) \quad Q = -\lambda \frac{\partial T}{\partial z}, \quad (z = 0).$$

Then (14) and (15) give

$$(16) \quad Q = \frac{\lambda}{h_0} T_*(t).$$

On the other hand, the same quantity Q appears as a result of the action of the friction forces. The mechanical energy of friction is given by the following expression

$$(17) \quad A = \int_0^a dr \int_0^{2\pi} \frac{fP(t)r^2\omega}{2\pi ar} d\vartheta = \frac{f\omega P(t)}{2} a.$$

The quantity Q per unit surface is derived from the expression

$$(18) \quad Q = \frac{IA}{\pi a^2}$$

where I is heat equivalent of the mechanic energy. Then (17) and (18) give:

$$(19) \quad Q = \frac{If_0\omega P(t)[1 - k_1 T_*(t)]}{2\pi a}.$$

Comparing (16) and (19) we get

$$(20) \quad T_*(t) = \frac{Ih_0f_0\omega P(t)}{2\pi a\lambda + Ih_0f_0\omega k_1 P(t)} < \frac{1}{k_1}.$$

Formula (20) gives the surface temperature of the plate. After a substitution of (20) in (9) we have

$$(21) \quad w(t) = f_0\omega\lambda \int_0^t \frac{m(\tau)P(\tau)d\tau}{2\pi a\lambda + Ih_0f_0\omega k_1 P(\tau)}.$$

Consider the special case

$$(22) \quad m(t) = m_0 = \text{const}, \quad P(t) = P_0 e^{-kt}, \quad k > 0.$$

According to (22) we get from (21)

$$(23) \quad w(t) = \frac{m_0\lambda}{Ih_0kk_1} \ln \frac{2\pi a\lambda + Ih_0f_0\omega k_1 P_0}{2\pi a\lambda + Ih_0f_0\omega k_1 P_0 \exp(-kt)}.$$

The time of the full wear of the plate is determined by (23), when $w(t_*) = h_0$, i. e. we have

$$(24) \quad h_0 = \frac{m_0\lambda}{Ikk_1h_0} \ln \frac{2\pi a\lambda + Ih_0f_0\omega k_1 P_0}{\pi a\lambda + Ih_0\omega k_1 f_0 P_0 \exp(-kt_*)}.$$

The equation (24) implies

$$(25) \quad t_* = \frac{1}{k} \ln \frac{1}{\exp\left(-\frac{Ih_0^2kk_1}{m_0\lambda}\right) (2\pi a\lambda + Ih_0f_0\omega k_1 P_0) - 2\pi a\lambda}.$$

We consider the following numerical example using the following data:

$$I = 10^{-4} / 4.27 \frac{\text{cal}}{\text{kg} \cdot \text{cm}}, \quad f_0 = 0.1, \quad \lambda = 1.08 \cdot 10^{-4} \frac{\text{cal}}{\text{cm} \cdot \text{sec} \cdot \text{grad}}, \quad h_0 = 5 \text{cm}, \quad a = 5 \text{cm},$$

$$\omega = 100 \text{sec}^{-1}, \quad P_0 = 500 \text{kgf}, \quad k_1 = 2 \cdot 10^{-3} \text{grad}^{-1}, \quad m_0 = 0.1 \frac{\text{cm}^2}{\text{kgf}}, \quad k = 0.01 \text{sec}^{-1}.$$

Using (25) we get:

$$t_* = 51 \text{sec}.$$

We see that the full wear of the plate takes a small interval of time, since the force P_0 and the angular velocity are large.

REFERENCES

- [1] ЦЕСНЕК, Л. С. Механика и микрофизика истирания поверхностей, Москва, Машиностроение, 1979.

- [2] Чичинадзе, А. В. Тепловая динамика внешнего трения. Новое в теории трения, Москва, 1966.
- [3] Тихонов, А. Н., А. А. САМАРСКИЙ. Уравнения мат. физики, Москва, Наука, 1966.
- [4] Ликов, А. В. Теория теплопроводности, Москва, Вишняя школа, 1967.
- [5] АЛЕКСАНДРОВ, В. М., А. В. АНКЕКУЛОВА. Контактная задача термоупругости. *Трение и износ*, № 1 (1990), 24-28.