Journal of Theoretical
and Applied Mechanics
Sofia 1996, Year XXVI No 4

CONTINUUM MECHANICS

STUDY OF MECHANICAL BEHAVIOUR OF SELF-STANDING INFLATABLE DAMS

M. POPOVA, N. VELKOV

Introduction

The membrane structures are modern super-light structures with perfect distribution of the weight in the space. Their basic element is the membrane, the material of which can bear only tensile tensions.

Such kinds of structures were known even in the antiquity, however, only now they are implemented throughout the world in amazing numbers, types and shapes. This is due to the progress of the material science, to the development of high-strength polymer membrane materials. Especially promising is their application in the hydroengineering. They have contributed to the rational solution of numerous retaining, regulating and conveying structures ([1], [2] and [3]).

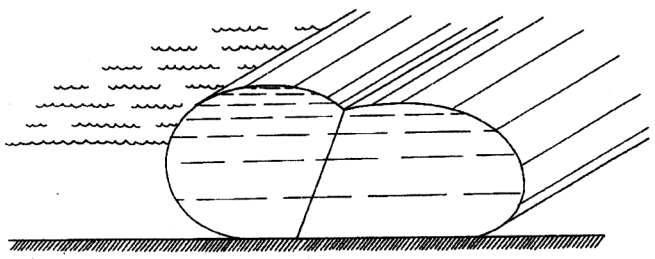


Fig. 1. Self-standing two-chamber membrane dam

Self-standing dams that are able to carry a considerable loading without external support (Fig. 1) are of particular interest. Their efficiency as temporary water retaining structures is undoubtful.

The mechanical behaviour of this type of structures is a relatively new and little investigated problem. Their practical implementation requires a wide range of experimental and theoretical investigations concerning the conditions of self-standing and load carrying capacity of the bearing membrane.

Object of study

The behaviour of non-fixed, water-inflated membrane submitted to one-sided hydrostatic loading is completely different from the behaviour of a solid body with similar shape. It begins a motion like a caterpillar chain, preserving its shape, but on the other hand, the distances between the points of its contour are varying constantly. Therefore the motion of the membrane can be prevented by fixing the distance between at least two points of its contour located sufficiently far from each other. The idea of creating of self-standing membrane dams by limiting the variation of the distance between the points of the membrane by internal ties arises (Fig. 1). Since the internal ties divide the volume into chambers, these structures are also named "multi-chamber".

A series of physical experiments is led for the establishment of the work principles of these structures. The influence of the number and the location of the chambers is ascertained experimental. The way of loosing stability of two-and three-chamber dams loaded by one sided hydrostatic pressure is found. The term "critical load" H_1^{cr} is defined as a criterion of carrying capacity. The results obtained by the model test are used for creation of the mathematical model of self-standing dams.

Mathematical model

Although the membrane structures are simple in design and operation, they present complex non-linear objects. Their mathematical modelling is very complicated. The obtained by the general theory of the flexible shells system of differential equations [4], [5] shows only the existence of a solution of that problem in principle, but this system is not solved for practical cases yet, despite the existence of powerful computers.

The features of the self-standing dams, which length is usually much longer than their height allows in cases of practical calculation to be neglected the influence of the tensions in direction of the length of the membrane. So the calculation of the dam can be simplified to calculation of non-circular flexible cylinder with unit length, loaded by hydrostatic pressure P.

The hydrostatic pressure is normal to the flexible ring (curve $y=f(x)$) and the stress T is constant throughout the whole length of the contour, or:

$$T=pR=\text{const}$$

where R is the radius of the curve in the given point, determined from the equation:

$$R=\frac{\left[1+\left(y^{\mathrm{I}}\right)^2\right]^{\frac{3}{2}}}{y^{\mathrm{II}}}.$$

The differential equation:

$$\frac{y^{\mathrm{II}}}{\left[1+\left(y^{\mathrm{I}}\right)^2\right]^{\frac{3}{2}}}=\frac{P}{T} \tag{1}$$

express the dependence of the profile of the membrane on the resultant load P. Analytical solution of equation (1) can be found by using elliptical integrals. Other way for solving these structures is using the graphical and graph-analytical methods. In the earliest publications in this area [1], [2] as well as in the latest [6] there are solutions only for one-chamber structures in case of neglecting the deformations in the material. The equations for form-finding of multi-chamber structures and the conditions of stability in this article are obtained on the basis of the represented method.

Fig. 2 shows a four-chamber self-standing membrane dam. It contains the two basic node types-with single C_2 and with double C_1 internal ties. Therefore every other configuration can be considered as derived from the four-chamber structure.

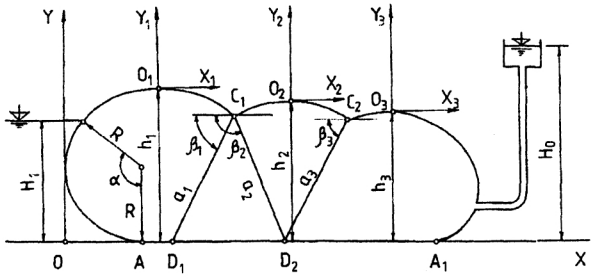


Fig. 2. Computation scheme

The forces for unit length in the material in the separate chamber are calculated from the conditions of equilibrium of flexible ring [2]:

$$T_i=0{,}25\ (2H_0h_i-h_i^2-H_1^2)\ \rho g \tag{2}$$

where:

h_i — height of i-chamber
H_1 — depth of the water upstream of the respective chamber
ρ — density of water
g — acceleration of gravity
$\rho g \approx 1$

The equation (1) is developed for each one of the sections separately, because of the difference of the load type.

Section AB (the resultant load is constant)

The resultant load p^{res} in this section is determined by the expression:

$$p^{res}=p^{int}-p^{ext}$$

where:

$p^{int}=(H_0\text{-}h_1\text{-}y_1)\,\rho g$ — internal pressure in j-point from first chamber

$p^{ext}=(H_1\text{-}h_1\text{-}y_1)\,\rho g$ — external pressure in j-point from first chamber

hence:

$p^{res}=H_0\text{-}H_1=\text{const.}$

Then that section is a circular bow tangential to the bottom with radius:

$$R=\frac{T_1}{H_0-H_1}. \tag{3}$$

Section free from external load

The resultant pressure in the sections free from external load is:

$$p^{res}=p^{int}=(H_0-h_i-y_i)\rho g=K_i-y_i$$

where

$$K_i=H_0-h_i$$

or:

$$\frac{y_i^{II}}{\left[1+\left(y^{I}\right)^2\right]^{\frac{2}{3}}}=\frac{K_i-y_i}{T_i}. \tag{4}$$

After integration of the equation (4) the following expression is obtained:

$$\frac{1}{\sqrt{1+(y_i)^2}}=\frac{K_i}{T_i}y_i-\frac{1}{2T_i}y_i^2+C. \tag{5}$$

Having in mind that:

$$\operatorname{tg}\varphi_i=\frac{dy_i}{dx_i} \tag{6}$$

for the integration constant is received:

$$C=\cos\varphi_0=\cos\pi=-1.$$

The solution of the equation (5) is:

$$y_{ij}=K_i-\sqrt{K_i^2+2T_i(1+\cos\varphi_{ij})}. \tag{7}$$

After replacing (7) in (6) is obtained:

$$x_{ij} = \int_{\pi}^{\varphi_{ij}} \frac{T_i \cos\varphi_i d\varphi_i}{\sqrt{K_i^2 + 2T_i(1+\cos\varphi_i)}} \tag{8}$$

for the length of the membrane in the considered section is received:

$$l_{ij} = \int_{\pi}^{\varphi_{ij}} \frac{T_i d\varphi_i}{\sqrt{K_i^2 + 2T_i(1+\cos\varphi_i)}}. \tag{9}$$

Conditions for self-standing of the dam

As a necessary condition for self-standing of the dam is assumed the equilibrium of the nodes C_1 and C_2 (Fig. 2).

It follows from the equilibrium equations for node with one internal tie (C_2, Fig. 3a):

$$\begin{vmatrix} T_2\cos\varphi_{22} + T_3\cos\varphi_{32} - P_3\cos\beta_3 = 0 \\ T_2\sin\varphi_{22} + T_3\sin\varphi_{32} - P_3\sin\beta_3 = 0 \end{vmatrix} \tag{10}$$

where all notations are according to the scheme.

The equilibrium equations of nodes with double internal tie (C_1, Fig. 3b) can be written as follows:

$$\begin{vmatrix} T_1\cos\varphi_{11} + T_2\cos\varphi_{21} - P_1\cos\beta_1 - P_2\cos\beta_2 = 0 \\ T_1\sin\varphi_{11} + T_2\sin\varphi_{21} - P_1\sin\beta_1 - P_2\sin\beta_2 = 0. \end{vmatrix} \tag{11}$$

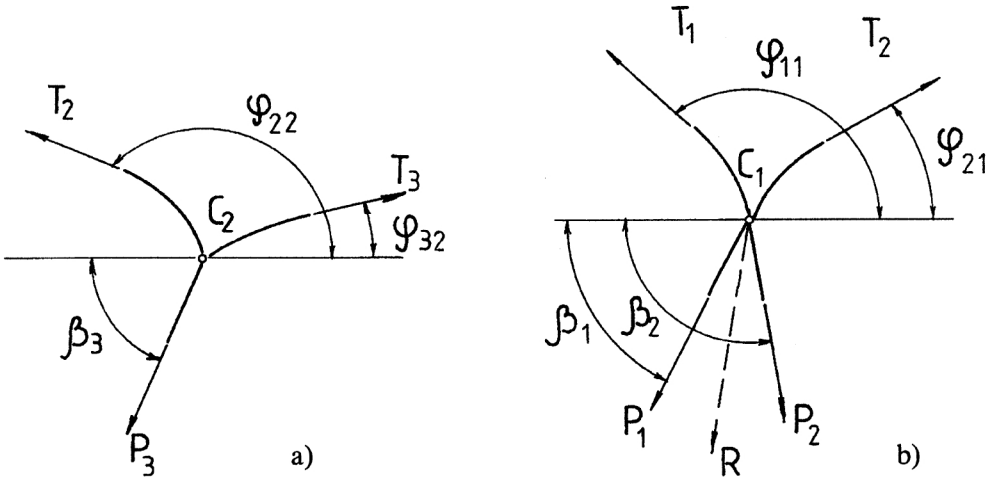


Fig. 3. Note types and forces in them

Computer simulation

On the basis of the above given relations a program system for simulating the mechanical behaviour of self-standing dams is developed. The influence of the geometric and force characteristics on the carrying capacity is numerical investigated.

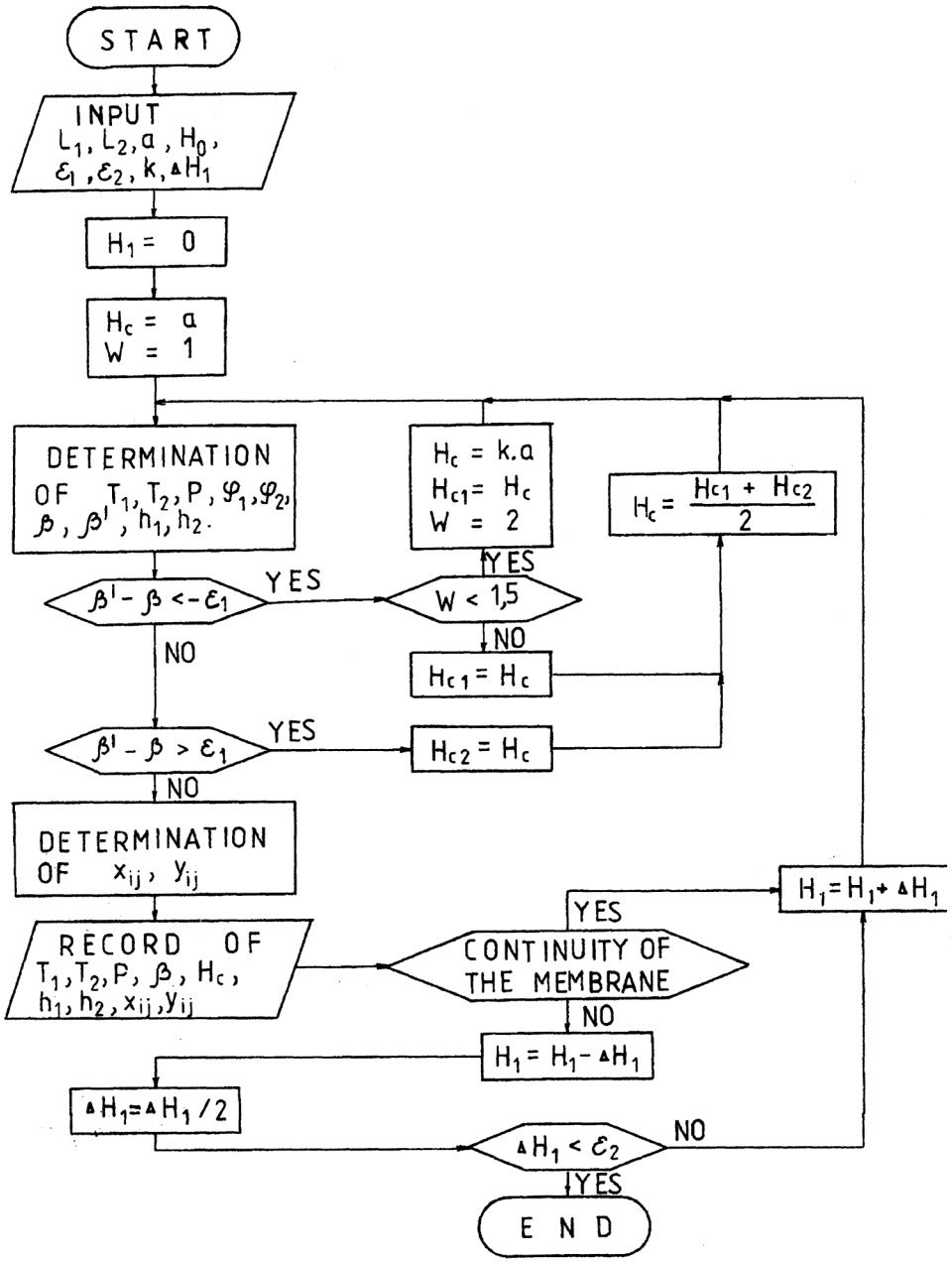


Fig. 4. Block-scheme of the basic computation programme

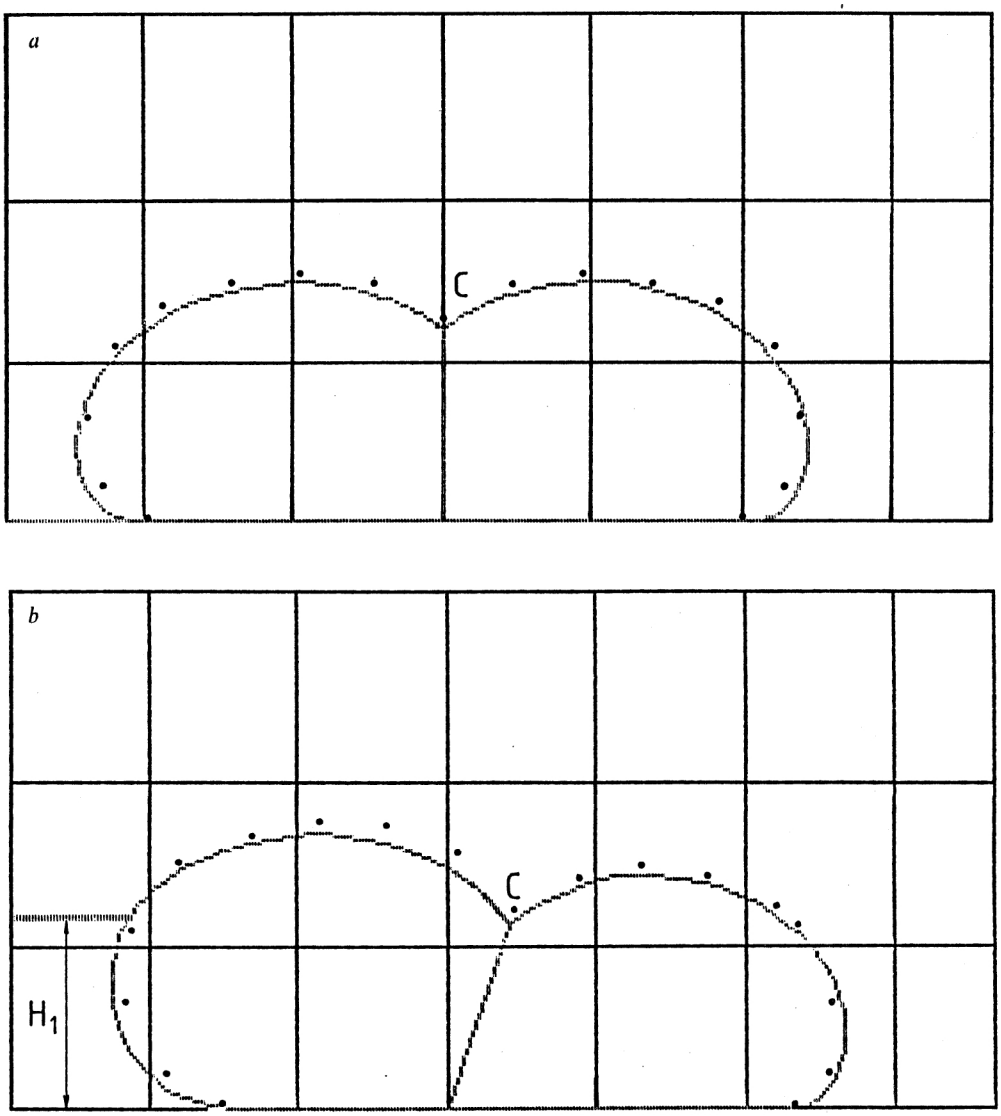


Fig. 5. Calculated cross-section of the contour of two-chamber dam

The algorithm of the main calculating program for two-chamber dams is shown in Fig. 4. The evaluation of the carrying capacity gets iterative step-by-step in depth H_1 (outside cycle, Fig. 4). The step ΔH_1 decreases with the increasing of the load till it reaches the necessary precision ε_2. The form of the membrane is determined for each value of the load by equations (7), (8) and (9). For this purpose the height of the node C, H_C is determined iterative by using the substitution method which in this

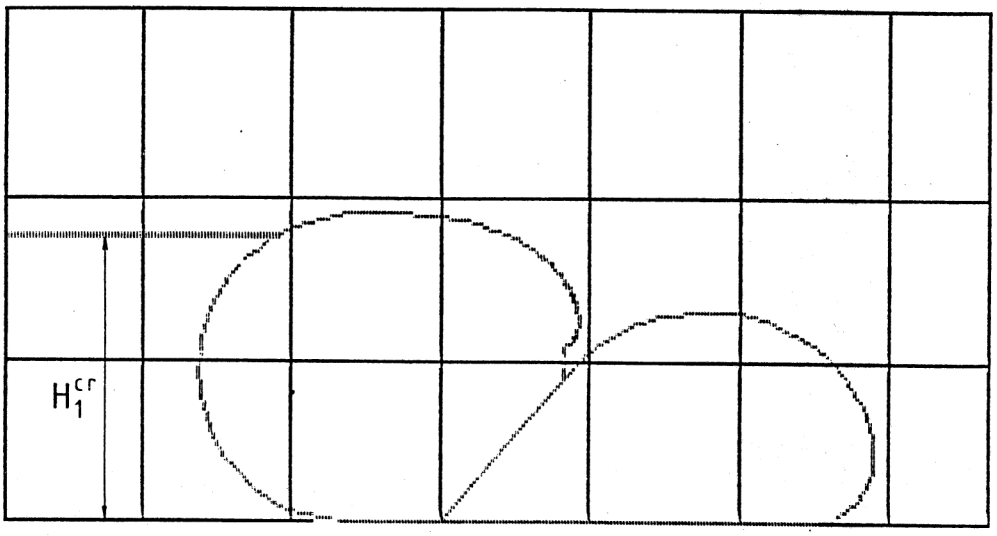


Fig. 6. Loss of stability of two-chamber dams

case proved to be the most rapidly convergent ones (inside cycle, Fig. 4). After that follows determination of the coordinates of sufficient number of points and their filing. The results can be illustrated on a display or printed (Fig. 5). That gives the possibility to observe the variation of the form respectively to the change of the load. The loss of stability results in violation of the conditions of smoothness and continuity of the membrane. For example the configurations of two-chamber dams in Fig. 6 show that their carrying capacity has been exhausted.

In three-chamber dams, that possess a node with double internal tie, the triangle formed by internal ties, that bears only tensile tensions, and the membrane represents a geometrically unchangeable figure. The condition for stability of the dams is that the resultant R of the forces in the internal ties to be located between them (Fig. 3b). In this case the problem of determining the carrying capacity of the dam is solved iterative and with the increasing of the load, the direction of the resultant R is observed (Fig. 7).

The results from the led solutions are compared with the experimental data from model experiments for identification of the created mathematical model.

Fig. 5 and 7 also show a comparison of the cross section obtained by the suggested mathematical model and those of physical models with the same parameters (points on the figures). The good coincidence of the forms as well as the results for the carrying capacity of the dams H_1^{cr} obtained by experimental and theoretical way (Fig. 8) proves the adequacy of the created mathematical model and its applicability in the studying the conditions for self-standing of multi-chamber non-fixed membrane dams under variation of their geometric and force characteristics.

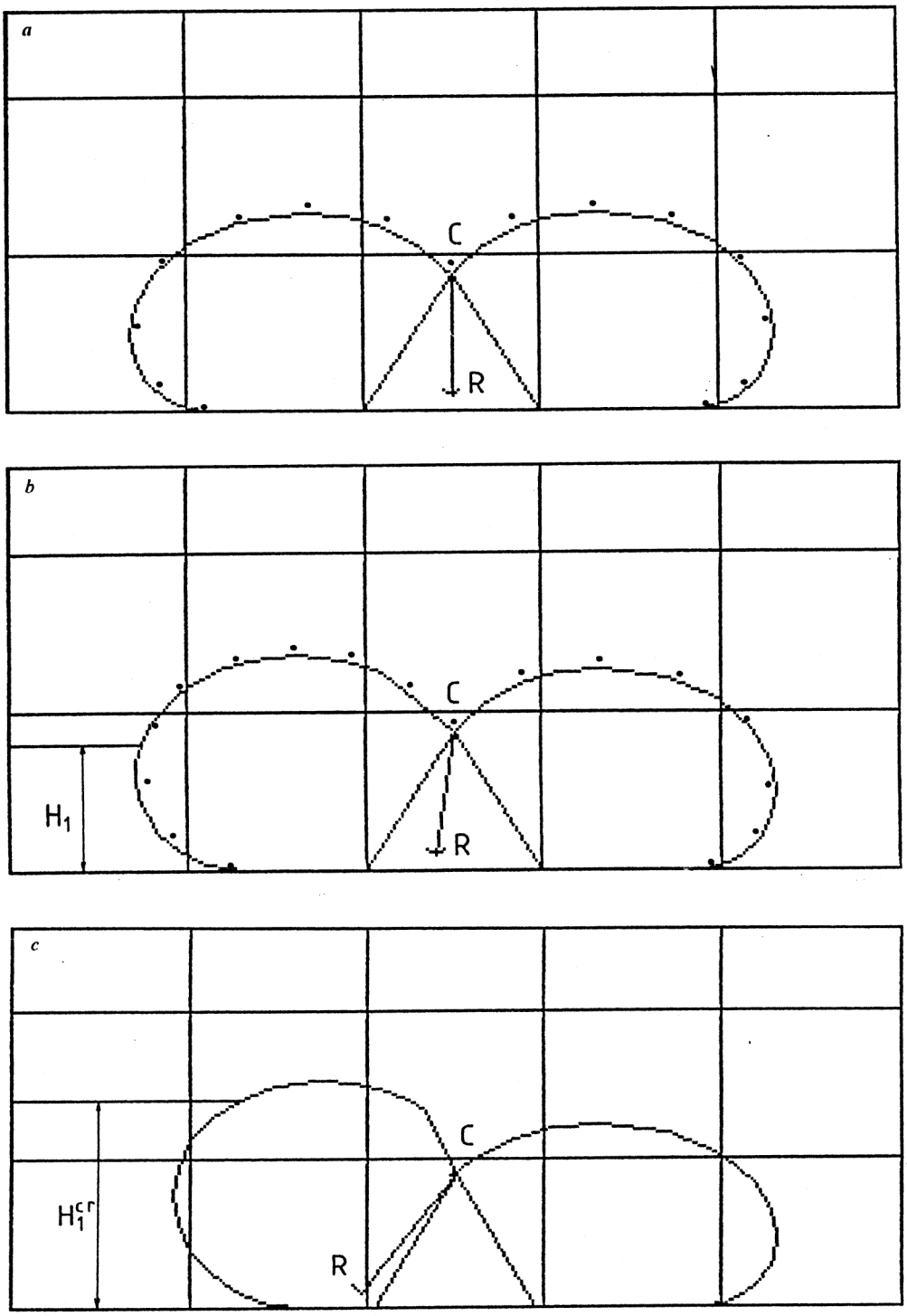


Fig. 7. Calculated cross-section of the contour of three-chamber dam

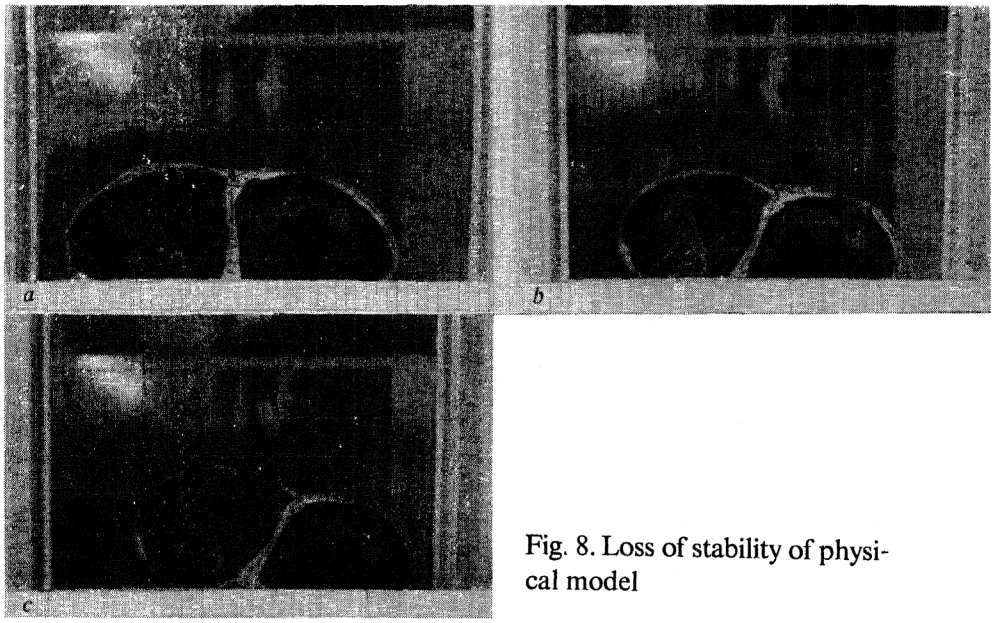


Fig. 8. Loss of stability of physical model

Results from the mathematical experiments

Study of two-chamber dams

The first series of investigations are carried out for establishment the influence of the relation of the length of the perimeters L_1 and L_2 of the two chambers (Fig. 9) on the carrying capacity of the dam.

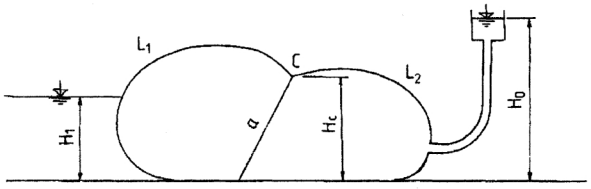


Fig. 9. Investigated parameters of two-chamber dam

The problem is solved for different values of the relation L_1/L_2 (L_1=6 m) with a constant internal pressure H_0. The procedure is repeated for values of the internal pressure H_0 from 2 m to 5 m. The results from the numerical experiment are presented graphically in the Fig. 10. The analysis of the diagrams shows that structures with approximately equal chambers have a maximum carrying capacity for each of the values H_0.

The further experiments are carried out for equal length of the perimeters of the chambers, i. e. L_1=L_2=L.

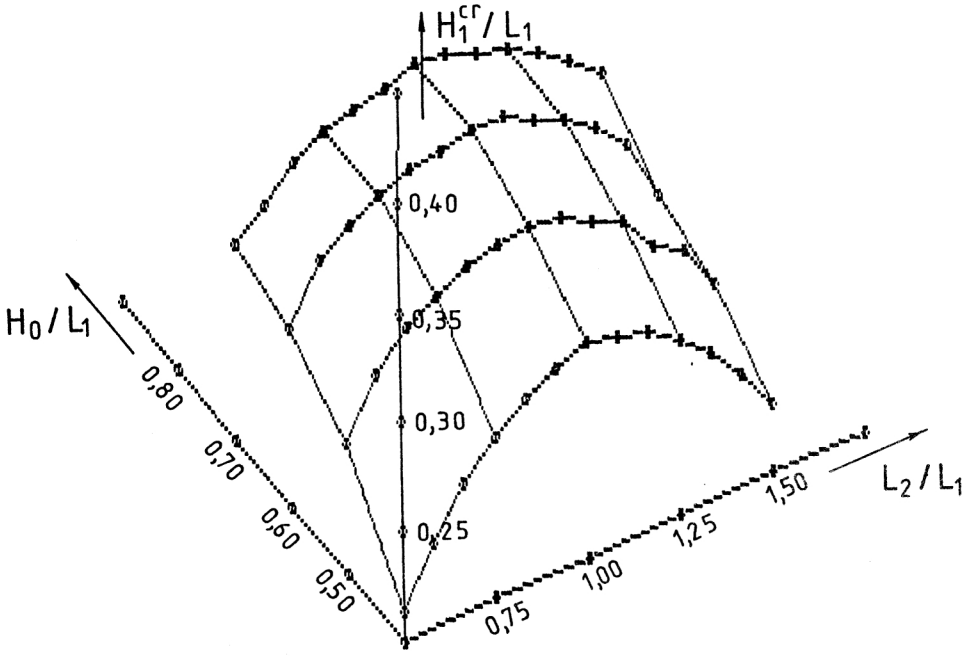


Fig. 10. Relation of the critical loading to the length of the perimeter of the second chamber and the internal pressure

An investigation for defining the influence of the length of the internal tie a is made. The problem for the determining the form of the membrane is solved for different values of the ratio a/l in the range 0,2÷0,4.

The graphically presented results from the numerical experiment (Fig. 11) show that with increasing of the length of the internal tie till a=0,35L, the carrying capacity of the dam increases too. For values of a>0,35L the internal tie does not perform its functions and the dam lose stability even for a minimum load.

So for realization in practice can be recommended:

$$a=(0{,}28\div 0{,}30)L.$$

It is obvious from the presented results that the internal pressure H_0 influences positive on the carrying capacity of the dam. Its values, however, are restricted from its influence on the tensions in the membrane material (equation 2). Having in mind the strength properties of the most used in practice rubberized fabrics [3] it can be recommended that H_0 in a real dams to be restricted to

$$H_0=(1{,}15\div 1{,}5)h$$

where h is the height of the dam.

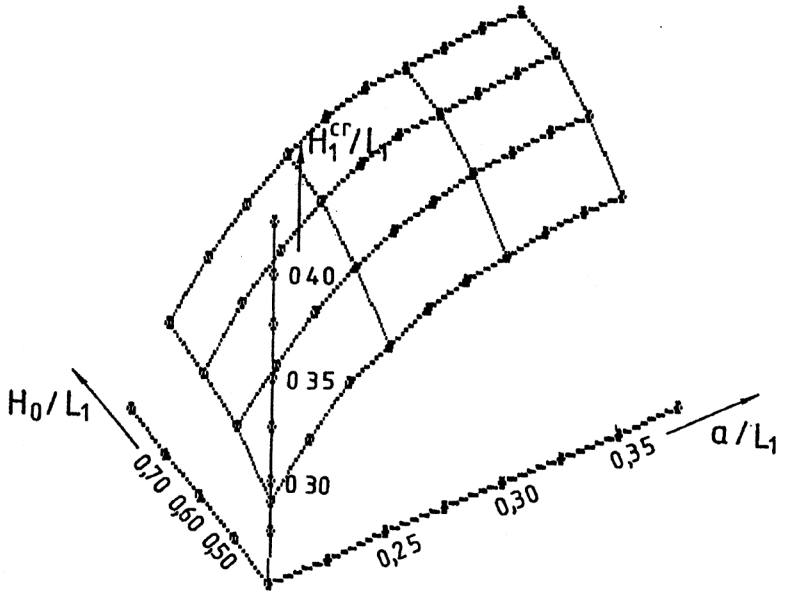


Fig. 11. Relation of the critical loading to the length of the internal tie and the internal pressure

Study of three-chamber dams

Analogous experiments for determining of the force and geometric characteristics of the structures on their stability are carried out for three-chamber dams with configuration shown in Fig. 12 too.

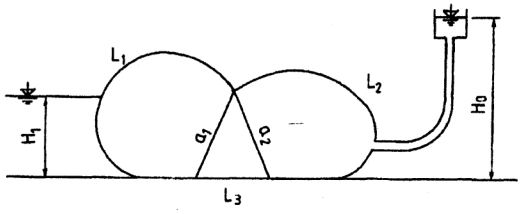


Fig. 12. Investigated parameters of three-chamber dam

The results obtained from that tests presented graphically in Fig. 13 and 14 give the possibility to be determined the recommendable from the point of view of realization in practice relations between the main parameters of the dam or:

$$L_1=L_2=L$$

$$L_3=a_1=a_2=0{,}25L$$

$$H_0=(1{,}15\div 1{,}5)h.$$

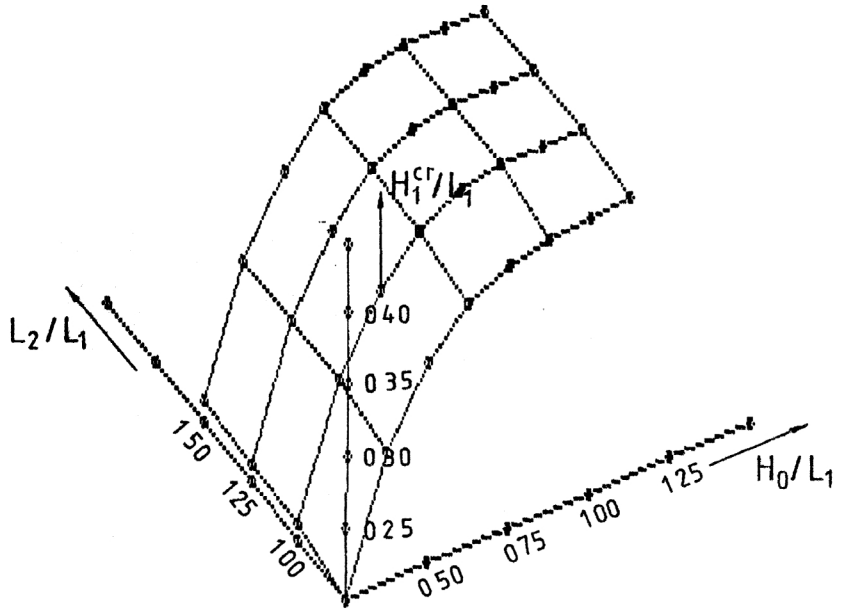


Fig. 13. Relation of the critical loading to the ratio between the lengths of the two end chambers and the internal pressure

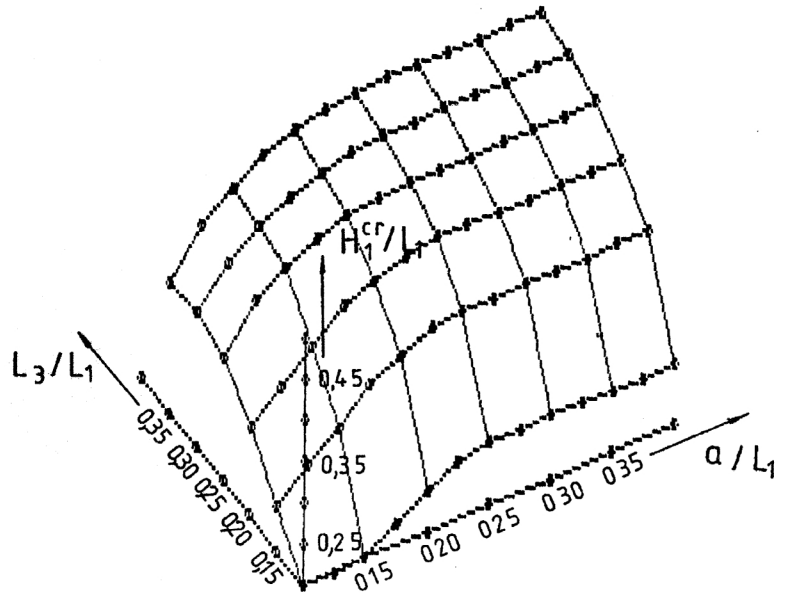


Fig. 14. Relation of the critical loading to the length of the internal ties and the perimeter of the third chamber

Conclusion

The comparison between the results obtained by the suggested mathematical model and the data from model test shows that, although set up under a number of simplifying prerequisites concerning the geometry of the membrane and the properties of the material, the model allows a sufficiently accurate description of the mechanical behaviour of the multi-chamber dams.

A wide-ranging numerical investigation of the form-building and self-standing of multichamber dams was carried out under varying combinations of the geometric and force characteristics. There are given the recommendable — from the point of view of self-standing — force and geometric characteristics for two- and three-chamber dams.

The results obtained give a basis for the design and introduction of effective portable dams in irrigation and drainage and for emergency cases.

REFERENCES

[1] ANWAR, H. O. Inflatable Dams. *Journal of Hydraulics, Division American Society of Civil Engineering,* 1967.

[2] BURG, H. De berkening van een stun van rubber, *Ingeniur*, 1961.

[3] ZATWORNIZKII, O. G. Flexible Dams in Hydroengineering, Energetica, Moscow, 1975 (in Russian).

[4] ALEKSSEEV, S. A. Basic Equations of Flexible Shells Theory, Calculation of Three-dimensional Structures, **11**, 1967 (in Russian).

[5] GOLDENWIESER, A. L. Theory of Thin Elastic Shells, Moscow, 1953 (in Russian).

[6] WATSON, R. A Note on the Shapes of Flexible Dams. *Journal of Hydraulic Research,* **23** (1985), No 2.