

## Unsteady MHD Flow in Rotating Fluid

V. M. Soundalgekar

### 1. Introduction

During recent years, the study of rotating fluid is receiving attention of many research workers. This is because of their wide industrial applications. The flow past an impulsively started infinite plate in a rotating fluid was studied by Soundalgekar and Pop [1] and the flow past an accelerated plate in a rotating fluid was presented by Pop [2]. It is now the object of the present paper to study the effects of the transverse magnetic field on the rotating flow past an accelerated plate. In section 2, the mathematical analysis is presented and the solutions are derived in terms of the parabolic cylinder functions, for the velocity and the skin-friction. They are shown graphically. In section 3, the conclusions are derived.

### 2. Mathematical analysis

Here a Cartesian coordinate system rotating uniformly with the fluid with the angular velocity  $\Omega$  about the  $z$ -axis, taken positive in the vertically upward direction, is considered. The  $x$  and  $y$  axes are assumed to coincide with the infinite horizontal plate. A magnetic field of uniform strength  $B_0$  is assumed to be applied parallel to  $z$ -axis. Then the flow of an electrically conducting incompressible, viscous fluid is governed by the following equations:

$$(1) \quad \frac{\partial u}{\partial t} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u,$$

$$(2) \quad \frac{\partial v}{\partial t} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v.$$

Here  $\nu$  is the kinematic viscosity,  $\rho$  — the density,  $\sigma$  — the scalar electrical conductivity of the fluid, and  $t$  is the time. The induced magnetic field is assumed to be negligible.

For an accelerated plate, the boundary conditions are:

$$(3) \quad \begin{aligned} u=0, \quad v=0 & \quad \text{at } z=0, \quad t=0, \\ u=At^a, \quad v=0 & \quad \text{at } z=0, \quad t>0, \\ u \rightarrow 0, \quad v \rightarrow 0 & \quad \text{as } z \rightarrow \infty, \quad t>0. \end{aligned}$$

On introducing the similarity variable  $\eta = z/2\sqrt{\nu t}$  in equations (1) and (2), the following non-dimensional form of equations are derived:

$$(4) \quad 4t \frac{\partial u}{\partial t} - 8\omega v = \frac{\partial^2 u}{\partial \eta^2} - 4M^2 u,$$

$$(5) \quad 4t \frac{\partial v}{\partial t} - 8\omega u = \frac{\partial^2 v}{\partial \eta^2} - 4M^2 v,$$

where  $\omega = \Omega t$  and  $M^2 = \sigma B_0^2 t / \rho$ .

As exact solutions to equations (4) and (5) are not possible, we derive a series solution. So in view of the boundary conditions, we assume for  $u$  and  $v$  the following series expansions in powers of  $\omega$ . This is possible for small values of the angular velocity. Thus

$$(6) \quad u(t, \eta) = At^\alpha \sum_{i=0}^{\infty} \omega^i u_i,$$

$$v(t, \eta) = At^\alpha \sum_{i=0}^{\infty} \omega^i v_i.$$

Substituting (6) in (4) and (5) and equating the coefficients of different powers of  $\omega$ , we have the following set of equations:

$$(7) \quad \begin{aligned} u_0'' + 2\eta u_0' - 4(\alpha + M^2)u_0 &= 0, \\ u_1'' + 2\eta u_1' - 4(\alpha + M^2 + 1)u_1 &= 8v_0, \\ u_2'' + 2\eta u_2' - 4(\alpha + M^2 + 2)u_2 &= -8v_1 \end{aligned}$$

and

$$(8) \quad \begin{aligned} v_0'' + 2\eta v_0' - 4(\alpha + M^2)v_0 &= 0, \\ v_1'' + 2\eta v_1' - 4(\alpha + M^2 + 1)v_1 &= 8u_0, \\ v_2'' + 2\eta v_2' - 4(\alpha + M^2 + 2)v_2 &= 8u_1 \end{aligned}$$

and the corresponding boundary conditions are:

$$(9) \quad \begin{aligned} u_0 &= 1, \quad v_0 = 0, \quad u_i = v_i = 0 \quad \text{at } \eta = 0, \\ u_0 &\rightarrow 0, \quad v_0 \rightarrow 0, \quad u_i = v_i = 0 \quad \text{at } \eta \rightarrow \infty. \end{aligned}$$

The solutions to these equations are derived in terms of parabolic cylinder functions as follows

$$(10) \quad u_0 = 2^{2(\alpha + M^2)} \Gamma(\alpha + M^2 + 1) g_{\alpha + M^2}(\eta),$$

$$(11) \quad v_0 = 0,$$

$$(12) \quad u_1 = 0,$$

$$(13) \quad v_1 = 2^{2(\alpha + M^2) + 3} \Gamma(\alpha + M^2 + 2) g_{\alpha + M^2 + 1}(\eta) - 2^{2(\alpha + M^2) + 1} \Gamma(\alpha + M^2 + 1) g_{\alpha + M^2}(\eta),$$

$$(14) \quad \begin{aligned} u_2 &= 2^{2(\alpha + M^2 + 2)} \Gamma(\alpha + M^2 + 2) g_{\alpha + M^2 + 1}(\eta) - 2^{2(\alpha + M^2) + 5} \Gamma(\alpha + M^2 + 3) g_{\alpha + M^2 + 2}(\eta) \\ &\quad - 2^{2(\alpha + M^2) + 1} \Gamma(\alpha + M^2 + 1) g_{\alpha + M^2}(\eta), \end{aligned}$$

$$(15) \quad v_2 = 0,$$

$$(16) \quad u_3 = 0,$$

$$(17) \quad v_3 = \frac{1}{3} 2^{2(\alpha+M^2+1)} \Gamma(\alpha+M^2+1) g_{\alpha+M^2}(\eta) - 2^{2(\alpha+M^2+2)} \Gamma(\alpha+M^2+2) g_{\alpha+M^2+1}(\eta) \\ + 2^{2(\alpha+M^2+3)} \Gamma(\alpha+M^2+3) g_{\alpha+M^2+2}(\eta) - \frac{1}{3} 2^{2(\alpha+M^2+4)} \Gamma(\alpha+M^2+3) g_{\alpha+M^2+3}(\eta),$$

where  $g$ 's are defined as

$$g_\alpha(\eta) = \frac{2}{\sqrt{\pi} \Gamma(2\alpha+1)} \int_\eta^\infty (s-\eta)^{2\alpha} e^{-s^2} ds.$$

Hence, the expressions for the non-dimensional velocity are to  $O(\omega^3)$ ,

$$(18) \quad U = u/At^\alpha = u_0 + \omega^2 u_2, \\ V = v/At^\alpha = \omega v_1 + \omega^3 v_3.$$

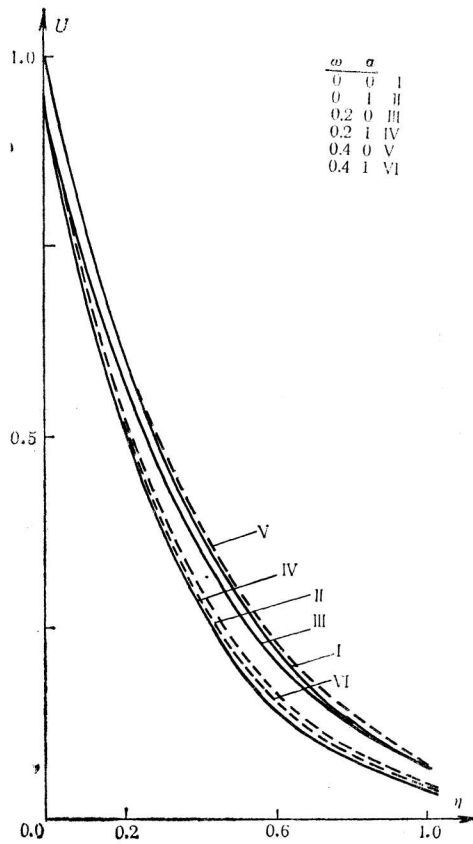


Fig. 1. Velocity profiles ( $M=1$ )

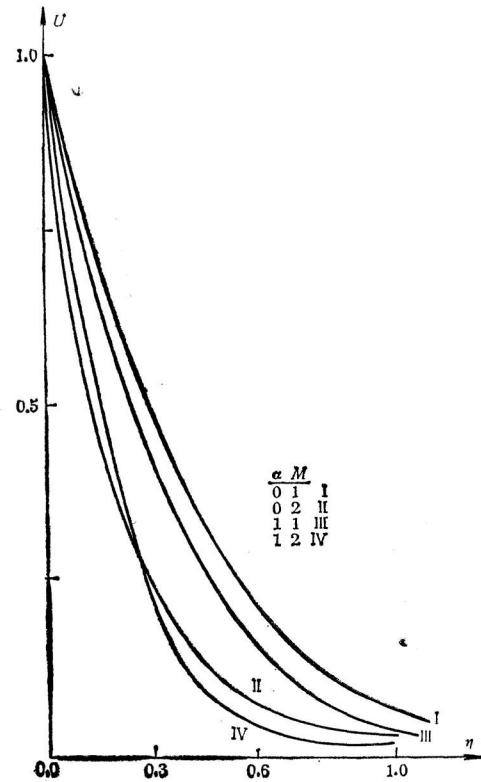


Fig. 2. Velocity profiles

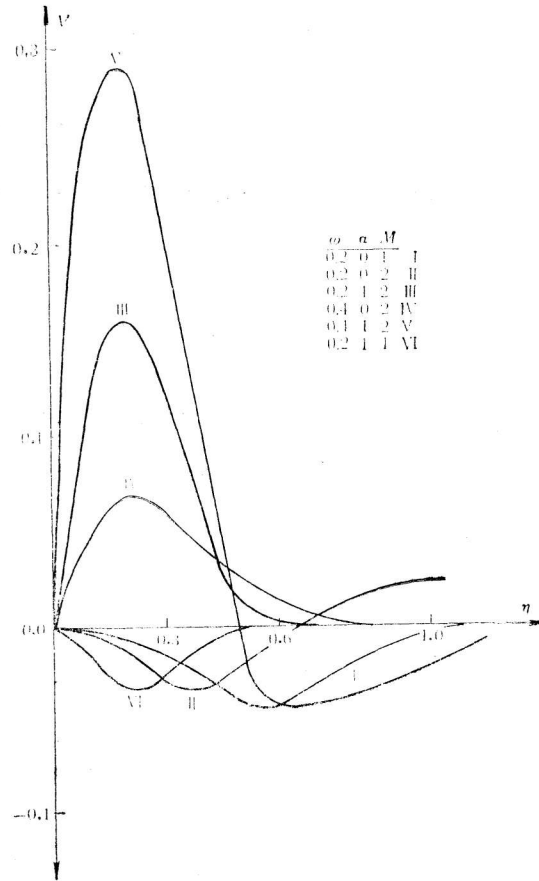


Fig. 3. Velocity profiles

The velocity profiles are calculated and are shown on Figs. 1—3. Here  $\alpha=0$  corresponds to impulsively started plate whereas  $\alpha=1$  corresponds to uniformly accelerated plate. We observe from Fig. 1 that due to a uniformly accelerated plate, the axial velocity  $U$  decreases for all values of  $\omega$ , and  $M$  is constant. An increase in  $M$  (Fig. 2), also leads to a decrease in the axial velocity. However, the transverse velocity  $V$  increases with increasing  $\omega$  or  $\alpha$ .

Knowing the velocity field, we now calculate the skin-friction. The axial skin-friction is given in non-dimensional form by

$$(19) \quad \tau_x = \frac{2\sqrt{\nu t}}{\mu A t^\alpha}, \quad \tau'_x = \frac{dU}{d\eta} \Big|_{\eta=0}$$

and the transverse skin-friction, in non-dimensional form is

$$(20) \quad \tau_y = \frac{2\sqrt{\nu t}}{\mu A t^\alpha}, \quad \tau'_y = \frac{dV}{d\eta} \Big|_{\eta=0}.$$

Remembering that

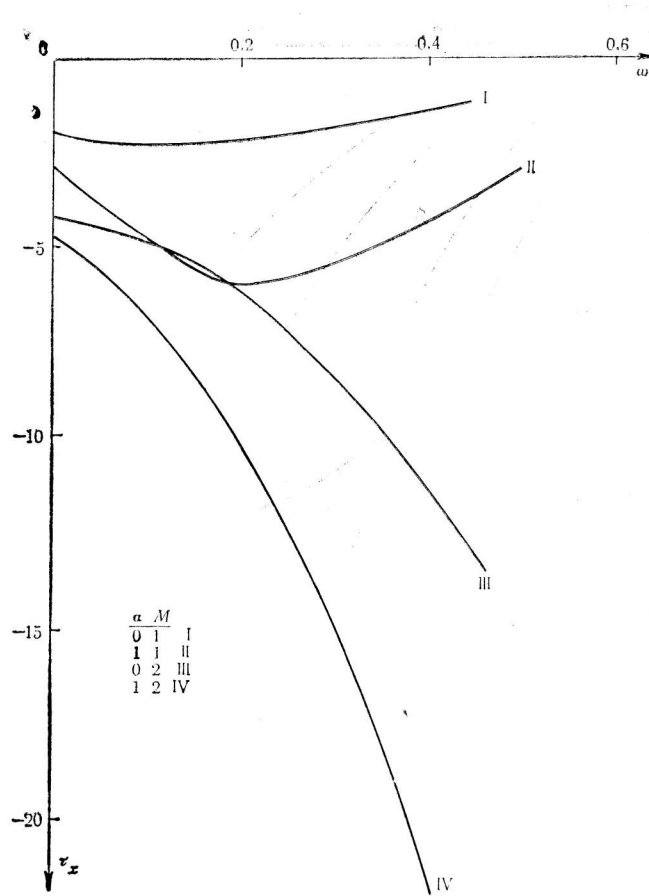


Fig. 4. Axial skin-friction

$$\frac{dg_n(\eta)}{d\eta} = -g_{n-1/2}(\eta), \quad g_n(0) = \frac{1}{2^{2n}\Gamma(n+1)}.$$

We have from (18), (19) and (20), after some algebraic simplification,

$$(21) \quad \tau_x = -\frac{2\Gamma(\alpha+M^2+1)}{\Gamma(\alpha+M^2+1/2)} + 4\omega^2 \left[ -\frac{2\Gamma(\alpha+M^2+1)}{\Gamma(\alpha+M^2+1/2)} + \frac{\Gamma(\alpha+M^2+3)}{\Gamma(\alpha+M^2+5/2)} - \frac{\Gamma(\alpha+M^2+2)}{\Gamma(\alpha+M^2+3/2)} \right]$$

and

$$(22) \quad \tau_y = 4\omega \left[ \frac{\Gamma(\alpha+M^2+1)}{\Gamma(\alpha+M^2+1/2)} - \frac{2\Gamma(\alpha+M^2+2)}{\Gamma(\alpha+M^2+1/2)} \right] + 8\omega^3 \left[ \frac{4\Gamma(\alpha+M^2+2) - \Gamma(\alpha+M^2+1/3)}{\Gamma(\alpha+M^2+1/2)} - \frac{\Gamma(\alpha+M^2+3)}{\Gamma(\alpha+M^2+5/2)} + \frac{\Gamma(\alpha+M^2+4)}{\Gamma(\alpha+M^2+7/2)} \right].$$

$\tau_x$  and  $\tau_y$  are shown on Figs. 4 and 5 respectively. Owing to a uniformly

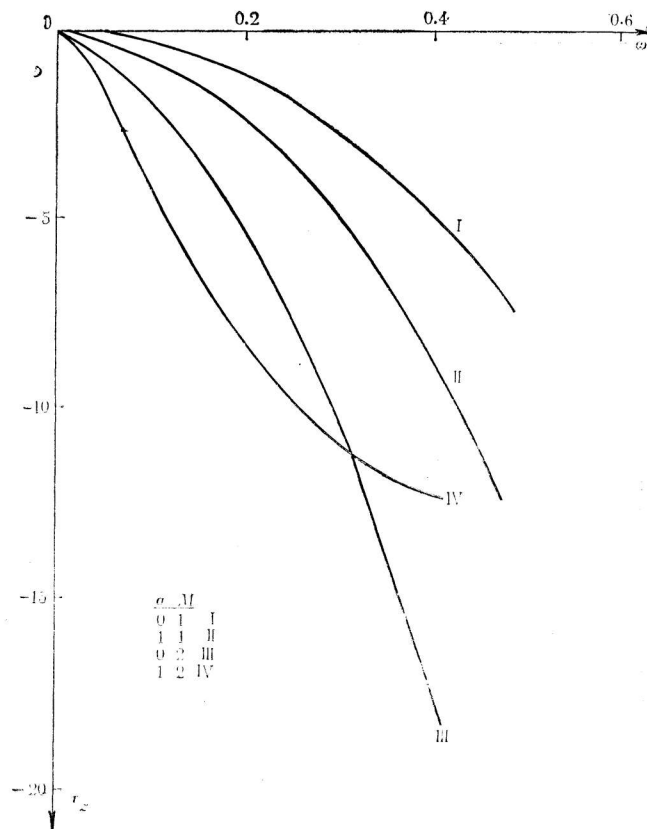


Fig. 5. Transverse skin-friction

accelerated plate,  $\tau_x$  and  $\tau_y$  decrease when  $M$  is constant. But an increase in  $M$  leads to a decrease in  $\tau_x$  and  $\tau_y$ . At small values of  $M$ , an increase in  $\omega$  leads to a rise in  $\tau_x$  whereas at large  $M$ ,  $\tau_x$  decreases.

## References

1. Soundalgekar, V. M. and I. Pop. — Bull. Math., 14(62), 1971, 375—381.
2. Pop, I. — Rendiconti Inst. Lombardi Cl. Sc., 105, 1971, 268—272.

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