

On the Propagation of Thermoelastic Waves in a Semi-Infinite Medium

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It is known from [1], [2], [3] that under the influence of thermal or mechanical disturbance on the boundary of a thermoelastic solid two thermoelastic waves propagate. In the paper [3] the velocities of these waves are determined and it is shown that the jumps of field quantities change exponentially in magnitude.

In this paper an extension of [3] is proposed. Recurrent dependences are obtained for the field quantities (θ , σ , ε , v , q) behind the front of the thermoelastic waves propagating in a semi-infinite medium.

Let us introduce: $\sigma(x, t)$ — stress, $v(x, t)$ — velocity, $\varepsilon(x, t)$ — strain, $T(x, t)$ — absolute temperature, T_0 — uniform reference temperature, $q(x, t)$ — heat flux, ρ — mass density, $t > 0$ — time and $x \geq 0$ — spatial variable. Linear theory is accepted, i. e.

$$\theta(x, t) \ll 1 \text{ and } \varepsilon(x, t) \ll 1,$$

$$\theta(x, t) = \frac{T(x, t) - T(x, 0)}{T(x, 0)}, \quad T(x, 0) \equiv T_0.$$

Let $F = F(x, t)$ is a field quantity which has discontinuity on the wave front $f: x = X(t)$. Then the jump F on f is defined by

$$[F](t) = \lim_{x \rightarrow X^-(t)} F(x, t) - \lim_{x \rightarrow X^+(t)} F(x, t).$$

The following kinematical condition of compatibility takes place

$$(1) \quad \frac{d}{dt} [F](t) = \left[\frac{\partial F}{\partial t} \right](t) + V(t) \left[\frac{\partial F}{\partial x} \right](t),$$

where the discontinuities propagate with velocity $V(t) = X'(t) > 0$.

It is shown in [3] that under some conditions of smoothness the following equations are valid

$$(2) \quad \frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t}$$

$$(3) \quad \frac{\partial q}{\partial x} = 2a_{02} \frac{\partial \theta}{\partial t} + a_{11} \frac{\partial \varepsilon}{\partial t}$$

$$(4) \quad \sigma = a_{11}\theta + 2a_{20}\varepsilon$$

$$(5) \quad q + \tau \frac{\partial q}{\partial t} = -KT_0 \frac{\partial \theta}{\partial x}$$

$$(6) \quad \frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial x}$$

for the field quantities $p = (\theta, \sigma, \varepsilon, v, q)$ and the corresponding jump conditions

$$(7) \quad [\sigma] = -\varrho V[v]$$

$$(8) \quad [q] = -2a_{02}V[\theta] - a_{11}V[\varepsilon]$$

$$(9) \quad [\sigma] = a_{11}[\theta] + 2a_{20}[\varepsilon]$$

$$(10) \quad [q] = \frac{KT_0}{\tau V}[\theta]$$

$$(11) \quad [v] = -V[\varepsilon],$$

where: $a_{20} = \frac{1}{2}(\lambda + 2\mu)$, $a_{11} = -(3\lambda + 2\mu)\alpha T_0$, $a_{02} = -\frac{1}{2}\varrho T_0 C_V$,

λ and μ — Lamé's elastic constants for isothermal deformation,

α — coefficient of thermal expansion, C_V — specific heat at constant volume, K — thermal conductivity and τ — relaxation time.

Let on the boundary $\{x=0\}$ the following boundary condition is accepted

$$(12) \quad \sigma(0, t) = \sigma(t), \quad \theta(0, t) = \theta(t), \quad t \geq 0,$$

where the functions σ and θ are sufficiently smooth.

Using (1) and (2)–(5) at $k \geq 1$ we get

$$(13) \quad \frac{1}{V} \frac{d}{dt} \left[\frac{\partial^{k-1} \sigma}{\partial t^{k-1}} \right] - \frac{1}{V} \left[\frac{\partial^k \sigma}{\partial t^k} \right] = \varrho \left[\frac{\partial^k v}{\partial t^k} \right]$$

$$(14) \quad \frac{1}{V} \frac{d}{dt} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] - \frac{1}{V} \left[\frac{\partial^k q}{\partial t^k} \right] = 2a_{02} \left[\frac{\partial^k \theta}{\partial t^k} \right] + a_{11} \left[\frac{\partial^k \varepsilon}{\partial t^k} \right]$$

$$(15) \quad \left[\frac{\partial^k \sigma}{\partial t^k} \right] = a_{11} \left[\frac{\partial^k \theta}{\partial t^k} \right] + 2a_{20} \left[\frac{\partial^k \varepsilon}{\partial t^k} \right]$$

$$(16) \quad \tau \left[\frac{\partial^k q}{\partial t^k} \right] + \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] = -\frac{KT_0}{V} \frac{d}{dt} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] + \frac{KT_0}{V} \left[\frac{\partial^k \theta}{\partial t^k} \right]$$

$$(17) \quad \left[\frac{\partial^k v}{\partial t^k} \right] = \frac{d}{dt} \left[\frac{\partial^{k-1} v}{\partial t^{k-1}} \right] - V \left[\frac{\partial^k \varepsilon}{\partial t^k} \right].$$

Provided $\sigma(0) \neq 0$ it is shown in [3] that the two wave velocities are obtained from the equation

$$(18) \quad \left(\frac{V}{c} \right)^4 - (b^2 + 1 + \delta) \left(\frac{V}{c} \right)^2 + b^2 = 0,$$

where:

$$c^2 = (\lambda + 2\mu)\varrho$$

$$b^2 = K/\varrho C_V \tau c^2$$

$$\delta = (3\lambda + 2\mu)^2 \alpha^2 T_0 / (\lambda + 2\mu)\varrho C_V.$$

Let $\sigma(0) = 0, \dots, \sigma^{(m-1)}(0) = 0, \sigma^{(m)}(0) \neq 0, m \geq 1$

and $\theta(0) = 0, \dots, \theta^{(l-1)}(0) = 0, \theta^{(l)}(0) \neq 0, l \geq 1$

and $n = \min \{m, l\}$.

We suppose that for each p we have

$$[p]=0, \dots, \left[\frac{\partial^{n-1} p}{\partial t^{n-1}} \right] = 0, \left[\frac{\partial^n p}{\partial t^n} \right] \neq 0.$$

Then the system (13)–(17) takes the form

$$(19) \quad \left[\frac{\partial^n \sigma}{\partial t^n} \right] = -\varrho V \left[\frac{\partial^n v}{\partial t^n} \right]$$

$$(20) \quad \left[\frac{\partial^n q}{\partial t^n} \right] = -2a_{02} V \left[\frac{\partial^n \theta}{\partial t^n} \right] - a_{11} V \left[\frac{\partial^n \varepsilon}{\partial t^n} \right]$$

$$(21) \quad \left[\frac{\partial^n \sigma}{\partial t^n} \right] = a_{11} \left[\frac{\partial^n \theta}{\partial t^n} \right] + 2a_{20} \left[\frac{\partial^n \varepsilon}{\partial t^n} \right]$$

$$(22) \quad \left[\frac{\partial^n q}{\partial t^n} \right] = \frac{KT_0}{\tau V} \left[\frac{\partial^n \theta}{\partial t^n} \right]$$

$$(23) \quad \left[\frac{\partial^n v}{\partial t^n} \right] = -V \left[\frac{\partial^n \varepsilon}{\partial t^n} \right].$$

By combining (19), (21), (23) and (20), (22) the following relations yield

$$\begin{aligned} \left[\frac{\partial^n \theta}{\partial t^n} \right] &= \frac{\varrho V^2 - 2a_{20}}{a_{11}} \left[\frac{\partial^n \varepsilon}{\partial t^n} \right], \\ \left[\frac{\partial^n \theta}{\partial t^n} \right] &= -\frac{\tau V^2 a_{11}}{KT_0 + 2\tau V^2 a_{02}} \left[\frac{\partial^n \varepsilon}{\partial t^n} \right]. \end{aligned}$$

From the above mentioned relations it is obtained

$$(24) \quad \frac{\varrho V^2 - 2a_{20}}{a_{11}} = -\frac{\tau V^2 a_{11}}{KT_0 + 2\tau V^2 a_{02}},$$

that could be written in the form (18), i. e. wave velocities do not depend on the kind of singularity.

It is deduced in [3] that

$$(25) \quad \frac{d}{dt} [\sigma](t) + \frac{\gamma_1}{\tau} [\sigma](t) = 0,$$

$$\text{where: } \gamma_1 = \frac{1}{2} b^2 \delta / \left\{ b^2 - \left(\frac{V}{c} \right)^2 + \delta b^2 \right\}.$$

From (7), (9), (11) we obtain $[\sigma]$ by $[\theta]$ and introducing into (25) it takes the form

$$(26) \quad \frac{d}{dt} [\theta](t) + \frac{\gamma}{\tau} [\theta](t) = 0,$$

$$\text{where: } \gamma = \left[\left(\frac{V}{c} \right)^2 - 1 \right] / 2 \left[\left(\frac{V}{c\sqrt{b}} \right)^4 - 1 \right].$$

Now we shall obtain similar equation for $\left[\frac{\partial^k \theta}{\partial t^k} \right]$ when $k \geq 1$. From (3), (5) it follows

$$(27) \quad \left[\frac{\partial^{k+1} q}{\partial t^k \partial x} \right] - 2a_{02} \left[\frac{\partial^{k+1} \theta}{\partial t^{k+1}} \right] - a_{11} \left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right] = 0$$

$$(28) \quad \tau \left[\frac{\partial^{k+1} q}{\partial t^{k+1}} \right] + \left[\frac{\partial^k q}{\partial t^k} \right] + KT_0 \left[\frac{\partial^{k+1} \theta}{\partial t^k \partial x} \right] = 0$$

and from (1), (27), (28) we obtain

$$(29) \quad \begin{aligned} & \frac{d}{dt} \left[\frac{\partial^k q}{\partial t^k} \right] + \frac{1}{\tau} \left[\frac{\partial^k q}{\partial t^k} \right] + \frac{KT_0}{\tau} \left[\frac{\partial^{k+1} \theta}{\partial t^k \partial x} \right] \\ &= 2a_{02} V \left[\frac{\partial^{k+1} \theta}{\partial t^{k+1}} \right] + a_{11} V \left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right], \\ & \frac{d}{dt} \left[\frac{\partial^k q}{\partial t^k} \right] + \frac{1}{\tau} \left[\frac{\partial^k q}{\partial t^k} \right] + \left(\frac{KT_0}{\tau} + 2a_{02} V^2 \right) \left[\frac{\partial^{k+1} \theta}{\partial t^k \partial x} \right] \\ &= 2a_{02} V \frac{d}{dt} \left[\frac{\partial^k \theta}{\partial t^k} \right] + a_{11} V \left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right]. \end{aligned}$$

From (4) yields

$$\begin{aligned} \left[\frac{\partial^{k+1} \theta}{\partial t^k \partial x} \right] &= \frac{1}{a_{11}} \left[\frac{\partial^{k+1} \sigma}{\partial t^k \partial x} \right] - \frac{2a_{20}}{a_{11}} \left[\frac{\partial^{k+1} \varepsilon}{\partial t^k \partial x} \right] \\ &= \frac{\varrho}{a_{11}} \left[\frac{\partial^{k+1} v}{\partial t^{k+1}} \right] - \frac{2a_{20}}{Va_{11}} \frac{d}{dt} \left[\frac{\partial^k \varepsilon}{\partial t^k} \right] + \frac{2a_{20}}{Va_{11}} \left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right] \\ &= \frac{\varrho}{a_{11}} \frac{d}{dt} \left[\frac{\partial^k v}{\partial t^k} \right] - \frac{\varrho V}{a_{11}} \left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right] - \frac{2a_{20}}{Va_{11}} \frac{d}{dt} \left[\frac{\partial^k \varepsilon}{\partial t^k} \right] + \frac{2a_{20}}{Va_{11}} \left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right]. \end{aligned}$$

Introducing $\left[\frac{\partial^{k+1} \theta}{\partial t^k \partial x} \right]$ into (29) we conclude that the coefficient before the term $\left[\frac{\partial^{k+1} \varepsilon}{\partial t^{k+1}} \right]$ is zero, since V satisfies (24).

From (29), (16) the following relation yields

$$(30) \quad \begin{aligned} & \frac{KT_0}{\tau V} \frac{d}{dt} \left[\frac{\partial^k \theta}{\partial t^k} \right] + \frac{KT_0}{\tau^2 V} \left[\frac{\partial^k \theta}{\partial t^k} \right] - \frac{1}{\tau} \frac{d}{dt} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] - \frac{1}{\tau^2} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] \\ & - \frac{KT_0}{\tau V} \frac{d^2}{dt^2} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] - \frac{KT_0}{\tau^2 V} \frac{d}{dt} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] - 2a_{02} V \frac{d}{dt} \left[\frac{\partial^k \theta}{\partial t^k} \right] \\ & + \left(\frac{KT_0}{\tau} + 2a_{02} V^2 \right) \left(\frac{\varrho}{a_{11}} \frac{d}{dt} \left[\frac{\partial^k v}{\partial t^k} \right] - \frac{2a_{20}}{Va_{11}} \frac{d}{dt} \left[\frac{\partial^k \varepsilon}{\partial t^k} \right] \right) = 0. \end{aligned}$$

The systems (13)–(17) is equivalent to the system

$$(31) \quad \left[\frac{\partial^k \varepsilon}{\partial t^k} \right] = \frac{1}{2a_{20} - \varrho V^2} \left\{ \frac{d}{dt} \left[\frac{\partial^{k-1} \sigma}{\partial t^{k-1}} \right] - \varrho V \frac{d}{dt} \left[\frac{\partial^{k-1} v}{\partial t^{k-1}} \right] - a_{11} \left[\frac{\partial^k \theta}{\partial t^k} \right] \right\}$$

$$(32) \quad \left[\frac{\partial^k v}{\partial t^k} \right] = \frac{1}{2a_{20} - \varrho V^2} \left\{ 2a_{20} \frac{d}{dt} \left[\frac{\partial^{k-1} v}{\partial t^{k-1}} \right] - V \frac{d}{dt} \left[\frac{\partial^{k-1} \sigma}{\partial t^{k-1}} \right] + a_{11} V \left[\frac{\partial^k \theta}{\partial t^k} \right] \right\}$$

$$(33) \quad \left[\frac{\partial^k \sigma}{\partial t^k} \right] = \frac{1}{2a_{20} - \varrho V^2} \left\{ 2a_{20} \frac{d}{dt} \left[\frac{\partial^{k-1} \sigma}{\partial t^{k-1}} \right] - 2a_{20} \varrho V \frac{d}{dt} \left[\frac{\partial^{k-1} v}{\partial t^{k-1}} \right] - a_{11} \varrho V^2 \left[\frac{\partial^k \theta}{\partial t^k} \right] \right\}$$

$$(34) \quad \left[\frac{\partial^k q}{\partial t^k} \right] = -\frac{1}{\tau} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] - \frac{KT_0}{\tau V} \frac{d}{dt} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] + \frac{KT_0}{\tau V} \left[\frac{\partial^k \theta}{\partial t^k} \right]$$

$$(35) \quad \frac{1}{V} \frac{d}{dt} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] - \frac{1}{V} \left[\frac{\partial^k q}{\partial t^k} \right] = 2a_{02} \left[\frac{\partial^k \theta}{\partial t^k} \right] + a_{11} \left[\frac{\partial^k \varepsilon}{\partial t^k} \right].$$

Substitution of (31), (32) into (30) yields

$$\begin{aligned}
 (36) \quad & \left\{ \frac{KT_0}{\tau V} - 2a_{02}V + \left(\frac{KT_0}{\tau} + 2a_{02}V^2 \right) \frac{2a_{20} + \varrho V}{2a_{20} - \varrho V^2} \right\} \frac{d}{dt} \left[\frac{\partial^k \theta}{\partial t^k} \right] \\
 & + \frac{KT_0}{\tau^2 V} \left[\frac{\partial^k \theta}{\partial t^k} \right] = \frac{1}{\tau} \frac{d}{dt} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] + \frac{1}{\tau^2} \left[\frac{\partial^{k-1} q}{\partial t^{k-1}} \right] + \frac{KT_0}{\tau V} \frac{d^2}{dt^2} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] \\
 & + \frac{KT_0}{\tau^2 V} \frac{d}{dt} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] + \left(\frac{KT_0}{\tau} + 2a_{02}V^2 \right) \frac{2a_{20} + \varrho V}{a_{11}(2a_{20} - \varrho V^2)} \frac{d^2}{dt^2} \left[\frac{\partial^{k-1} \theta}{\partial t^{k-1}} \right] \\
 & - \frac{4\varrho a_{20} \left(\frac{KT_0}{\tau} + 2a_{02}V^2 \right)}{a_{11}(2a_{20} - \varrho V^2)} \frac{d^2}{dt^2} \left[\frac{\partial^{k-1} \vartheta}{\partial t^{k-1}} \right].
 \end{aligned}$$

The equation (36) with an accuracy to constant takes the form

$$(37) \quad \frac{d}{dt} \left[\frac{\partial^k \theta}{\partial t^k} \right] (t) + \frac{\gamma}{\tau} \left[\frac{\partial^k \theta}{\partial t^k} \right] (t) = b_k(t),$$

where $b_k(t)$ is a linear combination of the functions

$$\left[\frac{\partial^{k-1} p}{\partial t^{k-1}} \right] (t), \quad \frac{d}{dt} \left[\frac{\partial^{k-1} p}{\partial t^{k-1}} \right] (t) \quad \text{and} \quad \frac{d^2}{dt^2} \left[\frac{\partial^{k-1} p}{\partial t^{k-1}} \right] (t);$$

$$p = \{ \theta, \sigma, \varepsilon, \vartheta, q \}.$$

If the initial condition for the eq. (37) and all derivatives $\left[\frac{\partial^{k-1} p}{\partial t^{k-1}} \right] (t)$ are known then from recurrent dependences (31)–(34) and (37) we can determine $\left[\frac{\partial^k p}{\partial t^k} \right] (t)$.

The solution of the system (2)–(6) at the boundary condition (12) we seek in the form

$$p(x, t) = p_1(x, t) + p_2(x, t); \quad p = \{ \theta, \sigma, \varepsilon, \vartheta, q \},$$

where $p_i(x, t)$ satisfies the system (2)–(6) for $(x, t): x < X_i(t) = V_i t$; $p_i(x, t) \equiv 0$ at $x > X_i(t)$ and on the wave fronts $x = X_i(t)$ the relations (7)–(11) are satisfied at $V = V_i$.

We have for $p_i(x, t)$ the following boundary condition

$$(38) \quad \sigma_1(0, t) + \sigma_2(0, t) = \sigma(t), \quad \theta_1(0, t) + \theta_2(0, t) = \theta(t).$$

Then $p(x, t)$ shall satisfy the system (2)–(6) at the boundary condition (12).

We assume that the functions p_i are sufficiently smooth behind the corresponding wave fronts and they have with their derivatives continuous prolongations on them (on the curves $x = X_i(t)$). Using Taylor expansion we obtain

$$\begin{aligned}
 p_i(x, t) &= \sum_{k=0}^n \frac{(t - X_i^{-1}(x))^k}{k!} \left[\frac{\partial^k p_i}{\partial t^k} \right] (X_i^{-1}(x)) + \mathcal{O}((t - X_i^{-1}(x))^{n+1}) \\
 & \quad t > X_i^{-1}(x)
 \end{aligned}$$

$$\sigma(t) = \sum_{k=0}^n \frac{t^k}{k!} \sigma^{(k)}(0) + \mathcal{O}(t^{n+1})$$

$$\theta(t) = \sum_{k=0}^n \frac{t^k}{k!} \theta^{(k)}(0) + \mathcal{O}(t^{n+1}).$$

Omitting the terms " $\mathcal{O}$ " when " n " is sufficiently large and " t " is small enough we rewrite

$$(39) \quad p_i(x, t) = \sum_{k=0}^n \frac{(t - X_i^{-1}(x))^k}{k!} \left[\frac{\partial^k p_i}{\partial t^k} \right] (X_i^{-1}(x))$$

$$(40) \quad \sigma(t) = \sum_{k=0}^n \frac{t^k}{k!} \sigma^{(k)}(0), \quad \theta(t) = \sum_{k=0}^n \frac{t^k}{k!} \theta^{(k)}(0).$$

Let us write (39), (40) for $p=\sigma$ and $p=\theta$. We set also $x=0$ and we obtain

$$(41) \quad \sigma_i(0, t) = \sum_{k=0}^n \frac{t^k}{k!} \left[\frac{\partial^k \sigma_i}{\partial t^k} \right] (0); \quad \theta_i(0, t) = \sum_{k=0}^n \frac{t^k}{k!} \left[\frac{\partial^k \theta_i}{\partial t^k} \right] (0).$$

Substituting (40), (41) into (38) and taking the coefficients before the same powers of " t " we obtain

$$(42) \quad \left[\frac{\partial^k \sigma_1}{\partial t^k} \right] (0) + \left[\frac{\partial^k \sigma_2}{\partial t^k} \right] (0) = \sigma^{(k)}(0),$$

$$(43) \quad \left[\frac{\partial^k \theta_1}{\partial t^k} \right] (0) + \left[\frac{\partial^k \theta_2}{\partial t^k} \right] (0) = \theta^{(k)}(0).$$

Introducing $V=V_1$ and $V=V_2$ into (33) we rewrite

$$(44) \quad \left[\frac{\partial^k \sigma_1}{\partial t^k} \right] (0) + \frac{a_{11} \varrho V_1^2}{2a_{20} - \varrho V_1^2} \left[\frac{\partial^k \theta_1}{\partial t^k} \right] (0) \\ = \frac{1}{2a_{20} - \varrho V_1^2} \left\{ 2a_{20} \frac{d}{dt} \left[\frac{\partial^{k-1} \sigma_1}{\partial t^{k-1}} \right] (0) - 2a_{20} \varrho V_1 \frac{d}{dt} \left[\frac{\partial^{k-1} v_1}{\partial t^{k-1}} \right] (0) \right\}$$

$$(45) \quad \left[\frac{\partial^k \sigma_2}{\partial t^k} \right] (0) + \frac{a_{11} \varrho V_2^2}{2a_{20} - \varrho V_2^2} \left[\frac{\partial^k \theta_2}{\partial t^k} \right] (0) \\ = \frac{1}{2a_{20} - \varrho V_2^2} \left\{ 2a_{20} \frac{d}{dt} \left[\frac{\partial^{k-1} \sigma_2}{\partial t^{k-1}} \right] (0) - 2a_{20} \varrho V_2 \frac{d}{dt} \left[\frac{\partial^{k-1} v_2}{\partial t^{k-1}} \right] (0) \right\}.$$

From the system (42)–(45) we determine unknown initial conditions $\left[\frac{\partial^k \theta_i}{\partial t^k} \right] (0)$ for the eq. (37). When we know $\left[\frac{\partial^k \theta_i}{\partial t^k} \right] (t)$ from (31)–(34) we can obtain $\left[\frac{\partial^k \varepsilon}{\partial t^k} \right] (t)$, $\left[\frac{\partial^k v}{\partial t^k} \right] (t)$, $\left[\frac{\partial^k \sigma}{\partial t^k} \right] (t)$ and $\left[\frac{\partial^k q}{\partial t^k} \right] (t)$ for the case when $k \geq 1$.

When $k=0$ we use the relations as it is done in [3]. Then the solution $p(x, t) = p_1(x, t) + p_2(x, t)$ will satisfy the system (2)–(6) for x and $t: x \neq X_i(t)$ and

the equations (7), (8) are fulfilled on the wave front. The boundary condition (12) is also satisfied.

The results obtained for a semi-infinite medium are also valid for a layer. These results are more exact when its thickness is smaller (" t " is then a small number). We can determine the relaxation time τ by the experiments in which we can measure the stress σ , deformation ε and temperature θ on both sides of the layer.

References

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