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Servocontrol of Manipulation Robots: II. Experimental Comparison via Numerical Simulation

I. Introduction

Servocontrol under continuous path motion of manipulation robots (MR) is still a challenge for the control engineer, regardless of the many years of research in this field. Among the other reasons, it is due to the nonlinear and bulky equations of motion involving tremendous number of real-time calculations, the mutual influence between the moving links, the model parameter uncertainty, etc.

In the present paper a review and comparative analysis of the most popular servocontrol methods is proposed. In Part I [1] the following methods were presented by respective relations and block schemes:

- (i) feedforward control;
- (ii) classical methods (PD feedback control with or without feedforward dynamic compensation);
- (iii) nonlinear feedback control (direct design and computed torque methods, and their robust and adaptive control versions).

In this second part of the paper the methods considered in [1] are evaluated via numerical simulation using the PSI programme [2] on the example of a two degrees of freedom articulated manipulator operating in a vertical plane. Nevertheless, that the example is relatively simple, its dynamic equation of motion comprises all the three terms — inertial, velocity and gravitational, that makes the results obtained generally representative.

II. Dynamic model of the manipulator

The kinematic scheme of the manipulator is shown in Fig. 1. Assuming for simplicity $l_1=l_2=1\text{ m}$, $m_1=m_2=1\text{ kg}$, $J_1=J_2=1\text{ kg}\cdot\text{m}^2$, the differential equation describing its motion has the following form [3]:

$$\begin{aligned} &6(2+c_2)\ddot{q}_1 + (2+3c_2)\ddot{q}_2 - 3s_2(2\dot{q}_1 + \dot{q}_2)\dot{q}_2 + 30(4c_1+c_{12})=u_1; \\ (1) \quad &(2+3c_2)\ddot{q}_1 + 2\ddot{q}_2 + 3s_2\dot{q}_1^2 + 30c_{12}=u_2, \end{aligned}$$

where the nomenclature of [1] is used and s_i, c_i, c_{ij} stand for $\sin q_i, \cos q_i, \cos(q_i + q_j)$, respectively.

Eq. (1) can be presented in a standard matrix form:

$$(2) \quad A(q)\ddot{q} + b(q, \dot{q}) + g(q) = u,$$

where:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 6(2+c_2) & 2+3c_2 \\ 2+3c_2 & 2 \end{bmatrix};$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 3s_2(2\dot{q}_1 + \dot{q}_2)\dot{q}_2 \\ 3s_2\dot{q}_1^2 \end{bmatrix};$$

$$g = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} 30(4c_1 + c_{12}) \\ 30c_{12} \end{bmatrix}; \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}.$$

III. Desired motion planning

The desired motion is described by a third order spline [4, 5]:

$$(3) \quad q_i^*(t) = a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3,$$

where $a_{ij}, i=\overline{1,2}, j=\overline{0,3}$ are constants, defined by the boundary conditions:

$$q_i^*(0) = q_i^{*0}; \quad q_i^*(t') = q_i^{*f},$$

$$\dot{q}_i^*(0) = \dot{q}_i^{*0}; \quad \dot{q}_i^*(t') = \dot{q}_i^{*f}.$$

Differentiating (3) the desired joint velocities and accelerations are obtained as:

$$(4) \quad \dot{q}_i^*(t) = a_{i1} + 2a_{i2}t + 3a_{i3}t^2;$$

$$(5) \quad \ddot{q}_i^*(t) = 2a_{i2} + 6a_{i3}t.$$

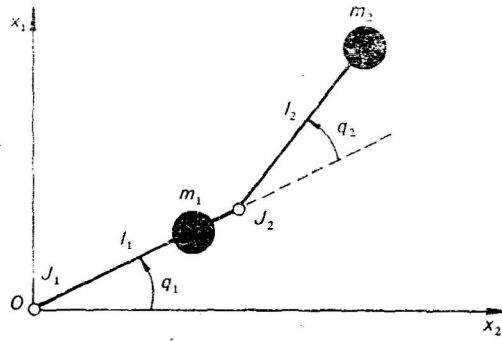


Fig. 1. Kinematic scheme of the manipulator

The coefficients in (3)–(5) are:

$$\begin{aligned}a_{i0} &= q_i^{*0}; \\a_{i1} &= 0; \\a_{i2} &= 3(q_i^{*f} - q_i^{*0})/t_f^2; \\a_{i3} &= -2(q_i^{*f} - q_i^{*0})/t_f^3.\end{aligned}$$

The desired motion (joint position, velocity and acceleration) is shown in Fig. 2.

IV. Experimental results

Usually, the dynamic model (2) is neither completely nor precisely known, and some estimations $\hat{A}$, $\hat{b}$, $\hat{g}$ of A , b , g , respectively, are used as an aid for control synthesis. It is assumed further on, that the matrix $\hat{A}(q)$ and the vectors $\hat{b}(q, \dot{q})$, $\hat{g}(q)$ are constant and equal to their mean values defined along the desired motion. Due to its clear physical sense, this assumption is rather convenient for experimental evaluation of different servoing algorithms. It gives a universal way to reducing the volume of the real-time computations and simplifying the controller's hardware—therefore, it can find wide application.

In the particular case the mean values of $q(t)$ and $\dot{q}(t)$ are:

$$\begin{aligned}q_1 &\approx 0.6550 \text{ rad}; \quad \dot{q}_1 \approx 0.60 \text{ s}^{-1}; \\q_2 &\approx 0.3475 \text{ rad}; \quad \dot{q}_2 \approx 0.25 \text{ s}^{-1}\end{aligned}$$

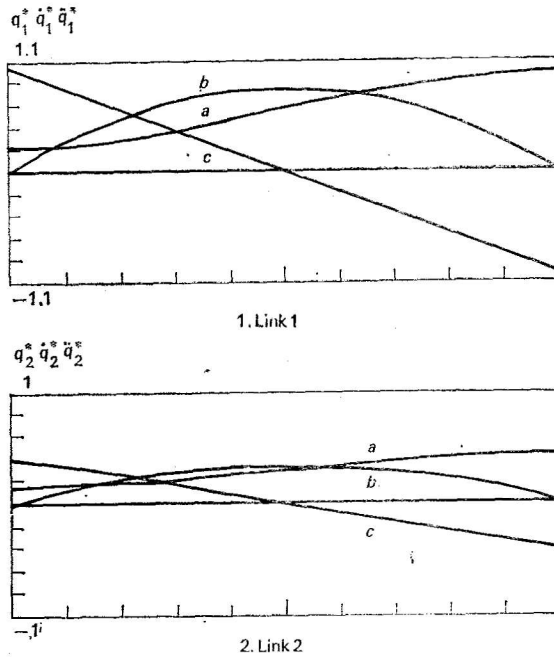


Fig. 2. The desired motion in joint coordinates
1. Link 1 ($q_1^{*0}=0.26 \text{ rad}$; $q_1^{*f}=1.05 \text{ rad}$)
2. Link 2 ($q_2^{*0}=0.175 \text{ rad}$; $q_2^{*f}=0.52 \text{ rad}$)
a — position; b — velocity; c — acceleration

and:

$$\hat{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{bmatrix} = \begin{bmatrix} 17.64 & 4.82 \\ 4.82 & 2.00 \end{bmatrix};$$

$$\hat{b} = \begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \end{bmatrix} = \begin{bmatrix} -0.328 \\ 0.370 \end{bmatrix}; \quad \hat{g} = \begin{bmatrix} \hat{g}_1 \\ \hat{g}_2 \end{bmatrix} = \begin{bmatrix} 111.33 \\ 16.12 \end{bmatrix}.$$

Simulation results with the methods considered in [1] are shown below.

IV. 1. Classical methods

Here two methods are evaluated based on PD feedback control.

IV. 1. 1. PD control. The joint position errors are shown in Fig. 3, where by a÷e the

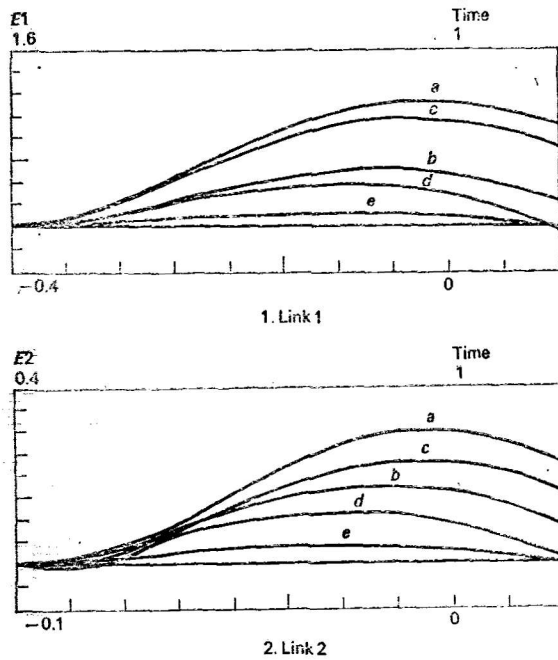


Fig. 3. Position errors under various PD controls
($k_{pi}=256$; $k_{di}=32$, $i=1, 2$)
1. Link 1
2. Link 2

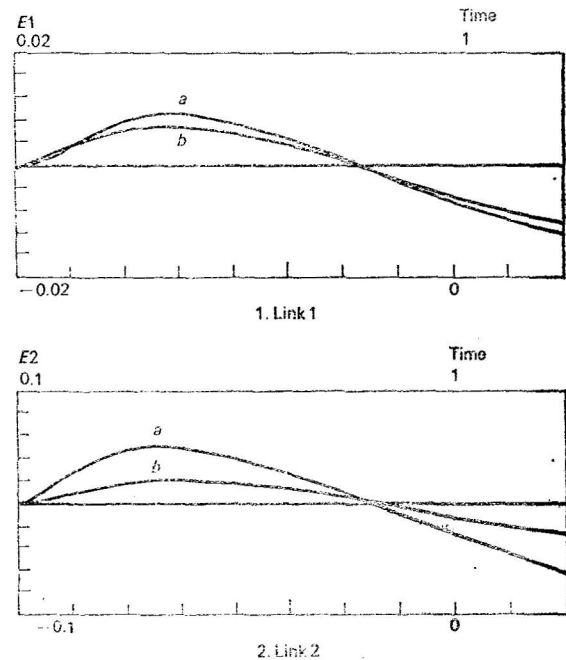


Fig. 4. Position errors under PD control with compensation
($k_{pi}=256$; $k_{di}=32$, $i=1, 2$)
a — reduced compensation;
b — complete compensation
1. Link 1
2. Link 2

various controls (5.a)÷(5.e) of [1] are denoted. It is easily seen that the results are better when MR dynamics is considered more completely — the control (5.e), based on the exact dynamic model, provides the best motion characteristics. Besides this, the significance of the gravitational forces should be also emphasised.

IV. 1.2. PD control plus complete or reduced feedforward dynamic compensation. The results for both methods are shown in Fig. 4. Naturally, the position errors are smaller in the first case.

IV. 2. Nonlinear feedback control

These methods are evaluated for different values of the position and velocity feedback gains.

IV.2.1. Direct design method. The results are shown in Fig. 5 for four values of $\lambda_1 = \lambda_2$ (Table 1). As seen from the figure, there are significant position errors and the system becomes unstable for large values of λ_1, λ_2 .

IV.2.2. Computed torque method. The results are shown in Fig. 6 for a set of different values of the feedback gain matrices K_p, K_d (listed in Table 2), providing critical

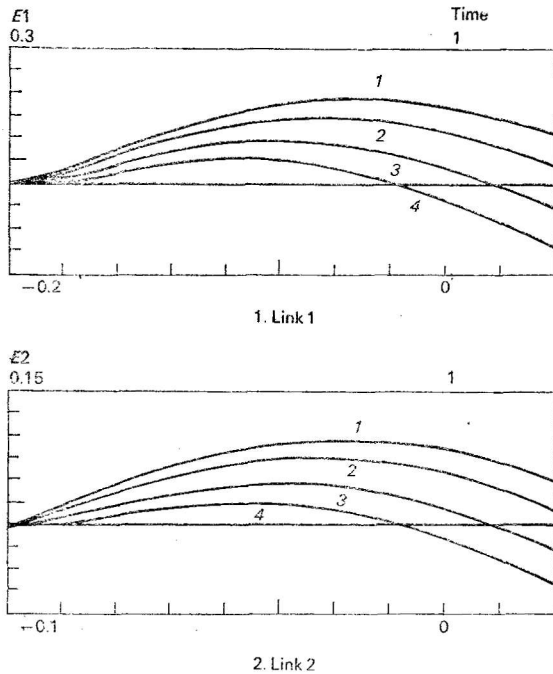


Fig. 5. Position errors under direct design control
($k_{pi}=259; k_{di}=32, i=\overline{1,2}$)
1. Link 1
2. Link 2

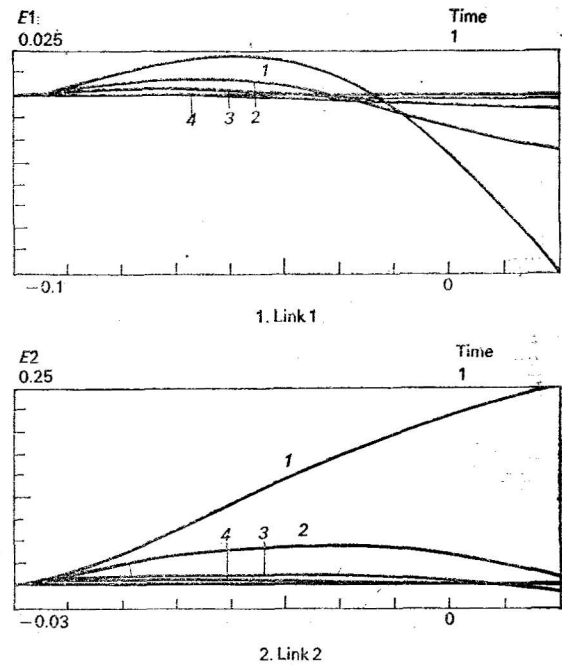


Fig. 6. Position errors under computed torque control
1. Link 1
2. Link 2

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damping [1]. The position errors are significantly reduced relatively to the direct design method, but the system is rather sensitive to model uncertainties and parameter variations.

IV.2.3. α -computed torque method. In Fig. 7 results are shown for $k_{p1}=k_{p2}=256$ and $k_{d1}=k_{d2}=32$. Following [6], increasing α should almost completely compensate the influence of the model uncertainties. The experiment shows, however, that α can not be assigned arbitrarily large values — in this particular case the system becomes

Table 1
The set of poles $\lambda_1=\lambda_2$

No	1	2	3	4
$\lambda_1=\lambda_2$	240	256	280	300

Table 2
The set of feedback gains

No	1	2	3	4
$k_{p1}=k_{p2}$	16	64	256	1024
$k_{d1}=k_{d2}$	8	16	32	64

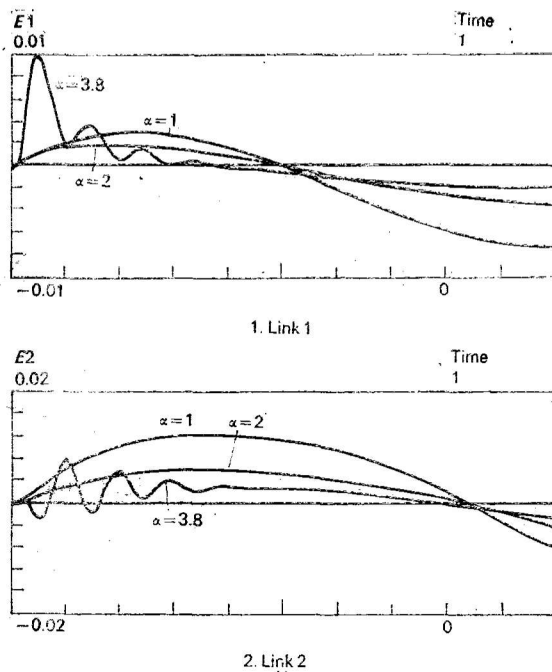


Fig. 7. Position errors under α -computed torque control
($k_{p1}=256$; $k_{d1}=32$, $i=1, 2$)
1. Link 1
2. Link 2

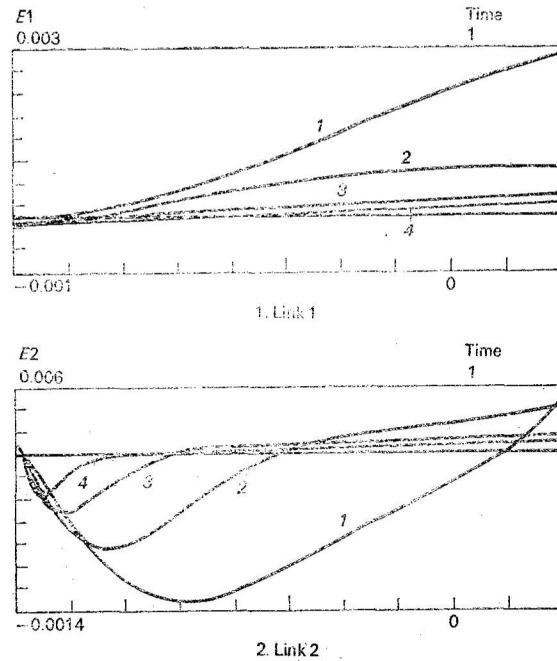


Fig. 8. Position errors under model referenced adaptive control
1. Link 1
2. Link 2

unstable for $\alpha > 3.8$ (of course, this limit depends on the degree of model uncertainty).

IV.2.4. Model referenced adaptive control. The position errors for both joints are shown in Fig. 8, and comparison with the computed torque method is presented in Fig. 9 for the set of feedback gains from Table 2. It is seen that the adaptive control gives much better results than all other methods discussed, and the extended closed loop system is robust due to the model uncertainties compensation in real time.

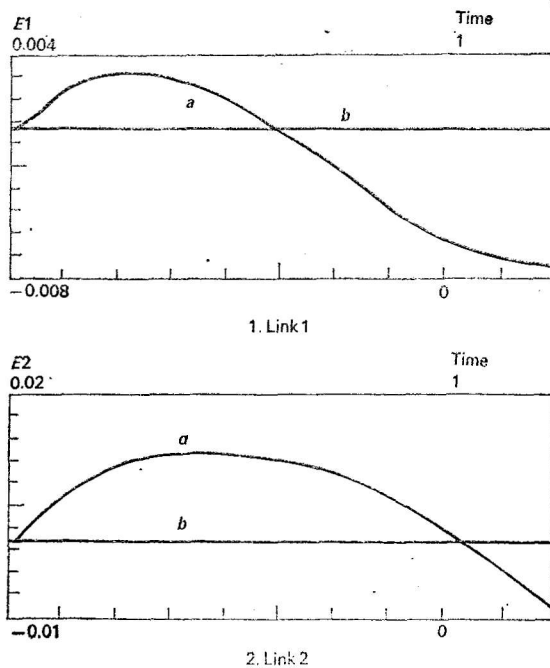


Fig. 9. Position errors under computed torque control (a) and model referenced adaptive control (b)

1. Link 1
2. Link 2

V. Conclusions

The analysis of the experimental numerical simulation results shows the following
(a) the more precise and complete is the MR dynamics considered for control synthesis, the better is the accuracy of the motion;

(b) the nonlinear feedback control methods are both general and efficient, and the best results are obtained for the model referenced adaptive control algorithm;

(c) increasing the feedback gains improves the accuracy of the motion, but there are limits over which the system stability is corrupted; these limits depend on the degree of uncertainty in the dynamic model of MR.

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