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A Consistent Approach to the Nonlinear Analysis of Shells*

I. Introduction

1.1. Preliminary remarks. The rigorous demands of contemporary engineering design have been the motivation for a great deal of activity in the nonlinear finite elements analysis of shells. Significant progress has been made in many aspects of the overall problem but measured on the desirable features for practical analyses with still relatively limited success. The vast variety of alternative schemes (e. g. [1]) reveals the inability of existing elemental formulation to satisfy a series of requirements.

1.2. "Optional" formulation. The following features contribute to an "optional" formulation: (I) the basic theory should be consistent with continuum mechanics concepts; (II) especially, in nonlinear problems the underlying theory should incorporate consistent simplifications appropriate to reduce the required computational effort; (III) the discrete approximation must possess the attributes which ensure convergence in the limit and (IV) must be consistent with the continuum theory; (V) each step of the formulation should be founded on sound arguments without recourse to special manipulation schemes; (VI) the formulation should simulate in a correct manner all rigid and deformational modes and avoid the occurrence of zero energy modes; (VII) superfluous energy caused by certain modes ("locking") should be avoided; (VIII) the element formulation should be simple, reliable and economical and extension to nonlinear problems should be straightforward; (IX) the element should deal with multiple junctions and stiffeners and be insensitive to mesh distortion; (X) finally, if some of the aforementioned requirements can not be satisfied, one should be aware of the limitations of the formulation.

1.3. The purpose of this paper is to present some concepts which provide a consistent approach to the nonlinear analysis of shells and which offer a great deal of promise when measured on all of the abovementioned requirements. These concepts deal with (I) the appropriate choice of reference systems for the formulation of the underlying theory for the shell; (II) a general shell theory which can be used for the effective treatment of large rotation, small strain problems; (III) modified versions of the Hu-Washizu variational principle; (IV) choice of appropriate interpolation functions

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for the field variables, and (V) construction of simple quadrilateral and triangular elements for the nonlinear finite element analysis of shells based on C^0 and C^1 continuity requirements. Due to space limitations only the essential ideas will be presented here.

II. Basic Theory

2. 1. Linear shell theory. One basic feature needed in order to achieve a simple element formulation is to depart from the Kirchhoff-Love Hypothesis and to start with a theory which includes the effect of transverse shear strains. As reference systems for the definition of the strains we use coordinate systems which describe in a more natural way the given geometry of the shell.

2.1.1. For the quadrilateral element we use convected coordinate system. Here the extensional ($\epsilon_{\alpha\beta}$), flexural ($K_{\alpha\beta}$) and transverse shear strains ($\gamma_{\alpha\beta}$) are defined in terms of the tangent base vectors ($\vec{a}_\alpha$), the normal to the undeformed surface ($\vec{n}_n$) and the displacement vector ($\vec{w}$) of the mid-surface and rotation vector ($\vec{\Phi}$), respectively [2]:

$$(1) \quad 2\epsilon_{\alpha\beta} = \vec{a}_\alpha \cdot \vec{w}_{,\beta} + \vec{a}_\beta \cdot \vec{w}_{,\alpha};$$

$$(2) \quad 2K_{\alpha\beta} = \vec{a} \cdot \vec{\Phi}_{,\beta} + \vec{a}_\beta \cdot \vec{\Phi}_{,\alpha} - b_\alpha^\mu \vec{a}_\mu \cdot \vec{w}_{,\beta} - b_\beta^\mu \vec{a}_\mu \cdot \vec{w}_{,\alpha};$$

$$(3) \quad 2\gamma_{\alpha 3} = \vec{n} \cdot \vec{w}_{,\alpha} + \vec{a}_\alpha \cdot \vec{\Phi}.$$

2.1.2. In the case of triangular elements geometrical bias can be avoided if we abandon the Cartesian coordinate system and refer all strains and stresses to the directions of the edges. Towards this aim we define [3] three membrane (ϵ_i) and three flexural strains (k_p) as the projection of the derivatives of the displacement and rotation vector on the tangent (e_i) and normal (e^i) directions of the edges. We also define three shear strains (γ_i) as the projection of the tangent to the θ^3 -line (not necessarily normal to the deformed surface) on the direction of the edge

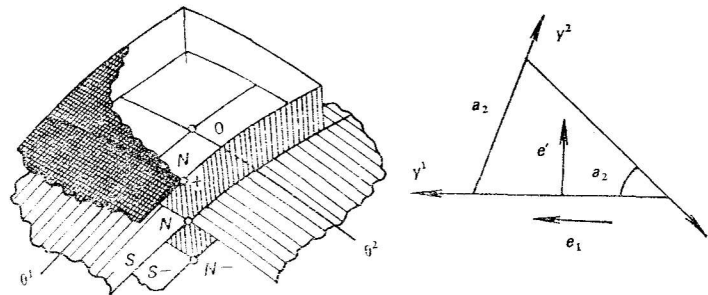


Fig. 1. Reference systems

$$(4), (5) \quad \varepsilon_i = e_i \frac{\partial w}{\partial y^i}; \quad k_i = e^i \frac{\partial \Phi}{\partial y^i}; \quad i = 1, 2, 3;$$

$$(6) \quad \gamma_i = e_i \cdot N.$$

The use of three tangential components of the shear strain vector may look unnatural, since the usual definition of shear strains (with respect to a Cartesian coordinate system) yields only pairs of shear components. But it should be pointed out that in the continuum problem these three components are not independent but are related to each other by the compatibility condition

$$(7) \quad \sin \alpha_1 \gamma_1 + \sin \alpha_2 \gamma_2 + \sin \alpha_3 \gamma_3 = 0.$$

The use of three shear components offers also advantages with respect to the discretization since the approximation contains three shear deformational modes. Special case is required in deriving the expression for the shear strain energy [4].

2.2. Variational principle. Any discrete approximation of the displacement or rotation vector is related to certain deformational modes which must be prohibited by a correct amount of energy, if we want to avoid phenomena such as locking and zero energy modes [2], [5]. In this context the choice of an appropriate variational principle as the basis of the element formulation is essential.

2.2.1. From a practical point of view, the discrete approximation should be formulated in terms of displacements and characterized by a symmetrical and positive definite matrix. These properties are attributes of the principle of virtual work. But the simple example of a beam under pure bending or the case of a linear triangle show that any consistent approximation of the displacements (i. e. the a priori satisfaction of the strain-displacement relationships) required by the principle of virtual work increases the potential. In the beam example we have shear strains under pure bending [2]. In the case of a linear triangle we obtain [5] for the transverse shear the following expression in terms of the constant curvatures and twist ($\bar{K}_{xx}, \bar{K}_{xy}$) and shears ($\bar{\gamma}_{3x}, \tau$) at the centroid:

$$(8) \quad \gamma_{3x} = \bar{\gamma}_{3x} - \tau y - \bar{k}_{xx} x - \bar{k}_{xy} y.$$

We observe that a consistent approximation includes the underlined terms (the curvature and twist) which are inhibited by bending energy. The flaw in the model is not the approximation of the displacement but the correct expression of the strain-displacement relationships. These simple examples show that it is not enough to make assumptions for the displacements only, but the strains or/and stresses should be also approximated.

2.2.2. Variational principle of Hu-Washizu. One way to satisfy many of the requirements mentioned in the introduction is to employ the functional of the Hu-Washizu. In this case, the strain energy U is suitably modified to include the appropriate strain-displacement relationships

$$(9) \quad F = \int_A U(\varepsilon_{\alpha\beta}, k_{\alpha\beta}, \gamma_{\alpha 3}) - \int_A n^{\alpha\beta} (\varepsilon_{\alpha\beta} - E_{\alpha\beta}) - \int_A m^{\alpha\beta} (k_{\alpha\beta} - \tilde{K}_{\alpha\beta}) - \int_A q^\alpha (\gamma_{\alpha 3} - \Gamma_{\alpha 3}) - \dots$$

Here the symbols $E_{\alpha\beta}$, $k_{\alpha\beta}$ and $\Gamma_{\alpha 3}$ denote the corresponding differential operators on the displacement and rotation vectors. The previous experience has shown that the

functional of Hu-Washizu provides a powerful tool in the derivation of consistent and simple element formulations. It allows to handle stresses, strains and displacements as independent variables, so that better approximations of stresses and strains are admissible, albeit not pointwise compatible. The use of Hu-Washizu principle as a starting point for the discretization offers a series of advantages: (I) the variational principle provides consistent relationships between all discrete variables; (II) the independent approximation of strains and stresses allows the straightforward representation of constant and higher order deformation modes. The homogeneous strains are augmented by specific terms that are introduced to accommodate deformational modes of higher order; (III) The omission of superfluous higher order terms reduces the internal energy, so that the element is less stiff than elements derived from the principle of minimum potential energy; (IV) The evaluation of the coefficients of the stiffness matrix requires much simpler operations than those of comparable displacement models; (V) A displacement formulation can be achieved on the element level without inversions. This leads to symmetric and positive definite matrices with all the desirable advantages; (VI) The effects of extensional and flexural behaviour are represented by comparable approximations; (VII) There is no recourse to special manipulation schemes.

III. Identification of Deformational Modes

3.1. Preliminary remarks. An essential step towards a reliable and effective implementation is the identification and proper description of the deformational modes associated with the discrete approximation of the displacement and rotation vector. Correct identification of the deformational modes leads to element formulations which are free from zero energy modes, i. e. there exist no deformational modes which are not accompanied by energy.

3.2. Linear quadrilateral elements. In the case of the linear quadrilateral, the bilinear approximation of the displacement and rotation vector yields 20 degrees of freedom (d. o. f.) which correspond to 6 rigid body motions and 14 deformational modes. In the degenerate case of a plate these 14 modes are expressed in the following form.

$$(10) \quad w_\alpha = \underline{w}_\alpha + \underline{w}_{\alpha\beta} \xi^\beta + \underline{\varepsilon}_{\alpha\beta} \xi^\beta \dots;$$

$$(11) \quad w_\beta = \underline{w}_\beta + \underline{w}_{\beta\alpha} \xi^\alpha + \dots$$

$$(12) \quad \varphi^\mu e_{\alpha\mu} = \underline{\gamma}_\alpha - \underline{w}_{\beta\alpha} - \underline{k}_{\alpha\beta} \xi^\beta + \dots$$

Here the six underlined terms are constants and represent the six degrees of small rigid motion. The other terms represent homogeneous extensional ($\underline{\varepsilon}_{\alpha\beta}$), flexural ($\underline{k}_{\alpha\beta}$) and transverse shear strains ($\underline{\gamma}_\alpha$). There are six additional modes which represent higher order modes and correspond to the bilinear terms in displacements and rotations

$$(13), (14) \quad w_\alpha = \tilde{\varepsilon}_\alpha \xi^1 \xi^2; \varphi^\mu e_{\alpha\mu} = -\tilde{k}_\alpha \xi^1 \xi^2.$$

The last two modes correspond to warping and torsion:

$$(15), (16) \quad w_\beta = \tilde{\gamma} \xi^1 \xi^2; \varphi^\alpha = -\tilde{t} \xi^\alpha.$$

The higher order modes are shown in Fig. 2.

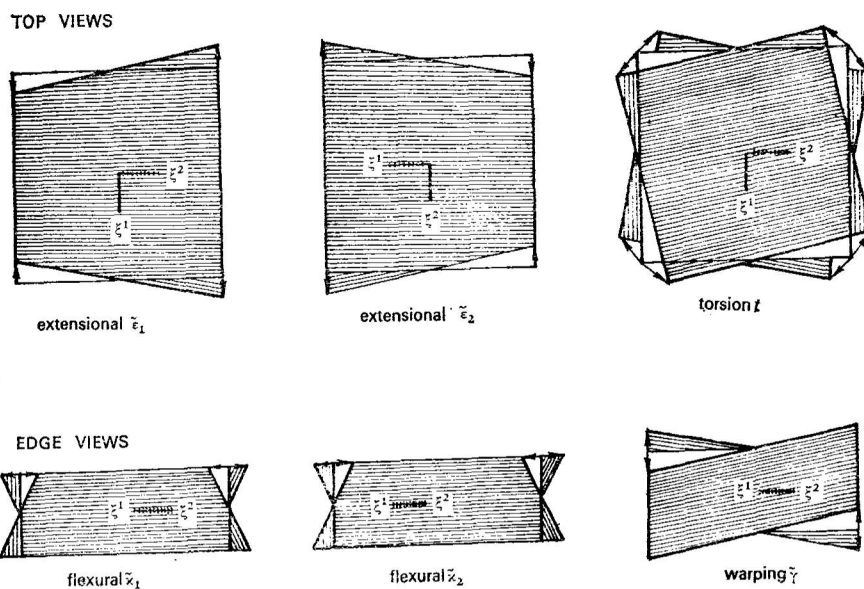


Fig. 2. Quadrilateral elements; higher order modes

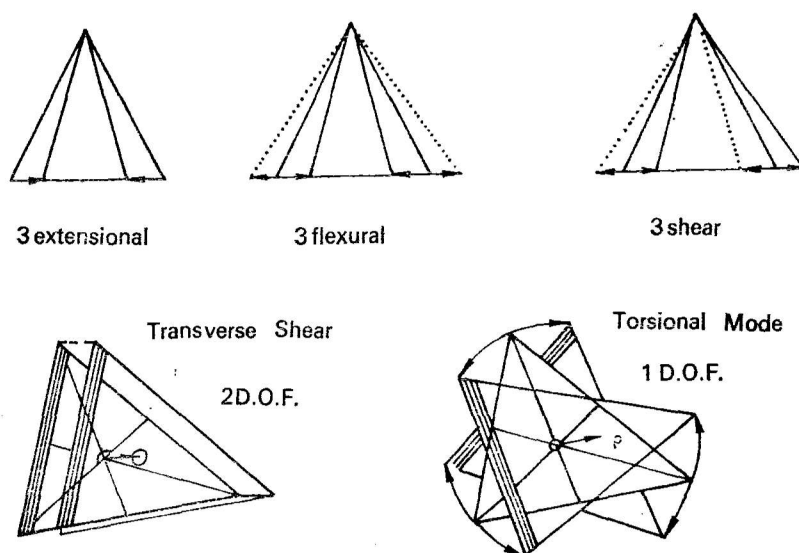


Fig. 3. Triangular elements: deformational modes

3.3. Linear triangular element. In the case of the linear C^0 triangle the 15 d. o. f. include the 6 rigid motions. The remaining 9 d. o. f. correspond to 3 constant extensional and 3 flexural modes. From the three shear modes shown in Fig. 3 two are constant and one is of higher order: the torsion mode. The torsion mode causes no bending and the shear due to torsion increases with the distance from the centroid. As mentioned in chapter 2.1.2., the underlying theory contains three shear components instead of the usual two components. Through the use of the concept of three natural directions, the components of shear include both the constant and the higher order terms. As an additional advantage, since the shear strain energy is expressed in terms of three shear components, we can accommodate shear in a more direct way by choosing three independent approximations for the shear strains.

IV. Discretization

4.1. Preliminary remarks. A crucial step in the formulation of the proposed elements is the introduction of the simplest admissible approximations for each field. The use of the generalized principle of Hu-Washizu requires the independent approximation of displacements, strains and stresses. The displacement vector is approximated by a bilinear form. The approximation of the strains and stresses are also to be as simple as possible, yet consistent with established criteria such as the so-called "limitation theorem" or the "patch test".

4.2. Strain assumptions for the quadrilateral elements. For the quadrilateral the assumptions for the strains take the form:

$$(17), (18) \quad \varepsilon_{11} = \bar{\varepsilon}_{11} + \varepsilon_{11}\xi^2; \quad \varepsilon_{22} = \bar{\varepsilon}_{22} + \varepsilon_{22}\xi^1;$$

$$(19) \quad \varepsilon_{12} = \bar{\varepsilon}_{12} + \varepsilon_{12}\xi^1 + \varepsilon_{12}\xi^2;$$

$$(20), (21) \quad k_{11} = \bar{k}_{11} + k_{11}\xi^2; \quad k_{22} = \bar{k}_{22} + k_{22}\xi^1;$$

$$(22) \quad k_{12} = \bar{k}_{12} + k_{12}\xi^1 + k_{12}\xi^2;$$

$$(23), (24) \quad \gamma_1 = \bar{\gamma}_1 + \gamma_1\xi^2; \quad \gamma_2 = \bar{\gamma}_2 + \gamma_2\xi^1.$$

The terms with a bar represent the constant states of strain and must be always present. The additional terms represent the higher modes shown in Fig. 2. It can be proved that it is enough to have constant shears and linear strains or constant strains and linear shear strains. If we retain all the additional terms then the element will become stiffer and will yield superfluous energy. In the displacement approach these terms occur together. This is the reason why elements based on the principle of virtual work yield stiffer solutions. The example of the pinched problem (Fig. 4) demonstrates that the proposed formulation, although very simple, gave good results compared with existing formulations. Furthermore, the element is suitable to both thick and thin cases.

4.3. Strain assumptions for the triangular element. The situation with the triangle is more severe. Due to the irregular shape a strong coupling between bending and shear occurs. Especially here, the use of the Hu-Washizu principle proves to be a powerful tool for selection of appropriate strain assumptions. In this case we proceed in the following way. The approximations for the curvature are constant. The approximations for the shear strains should avoid locking, yet inhibit all three shear deformational modes. Towards this aim we assume the following form.

$$(25) \quad \gamma = c(1 + \alpha \theta^i).$$

The motivation for choosing this form has been presented elsewhere [3], [5] and will not be repeated here. In these assumptions "c" denotes the tangential components of shear strains at the midpoints of each side. To estimate the factor "α" we may proceed as follows. We require that for an actual bending field Φ^* the formulation should not yield any shear energy, or that the corresponding γ^* should be zero. Now, the use of the Hu-Washizu principle allows us to satisfy the requirement in a consistent way: it turns out that the assumed stress field q_i (or δq_i) should be orthogonal to the shear strains (Γ) resulting from the actual bending field (Φ_i)

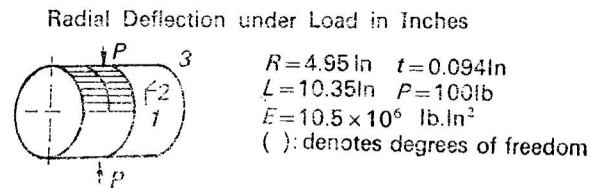


Fig. 4. Quadrilateral elements: pinched problem

Mesh In octant	Cantin	Ashwell, Sabir	Thomas Gallagher	Bogner et al.	Cantin, Clough	Dawe	Present
1×1		0.104(20)				0.106	
1×4		0.111(50)	0.111(67)	0.109(120)			0.098(32)
1×6							0.105(48)
1×7					0.999(96)		
1×8			0.112(131)				0.108(64)
1×9					0.106(120)		
2×2	0.93(54)	0.110(45)				0.112(96)	
2×4							0.099(53)
2×6							0.106(79)
2×9					0.107(180)		
4×4	0.113(150)	0.113(125)				0.114(374)	
6×6	0.144(294)	0.114(245)					

Mesh	$t = 0.094 \text{ in (thick)}$ $P = 100 \text{ (b)}$				$t = 0.1548 \text{ in (thin)}$ $P = .1 \text{ (b)}$	
	Degrees of freedom	Model 1	Model 2	Model 3	Model 1	Model 2
1×2	16	.06221	.06228	—	.01382	.01382
1×4	32	.09757	.09803	—	.02135	.02139
1×8	64	.1078	.1083	—	.02364	.02367
2×2	27	.06301	.06318	.0676	.01383	.01383
2×8	105	.1089	.1092	.1156	.02366	.02367
2×16	209	.1117	.1120	.1188	.02427	.02428

$$(26), (27) \quad \int_A \delta q_i \cdot \Gamma(\vec{\Phi}_i^*) = 0; \quad \delta \bar{q}_i \int_A (1 + \alpha \theta^i) \cdot \Gamma(\vec{\Phi}_i^*) = 0.$$

From the orthogonality condition (27) we can estimate the coefficient “ α ” of the equation (25). This approximation avoids locking and yields a mean value for the shear, a fact which is important in case of thick plates and shells. Now the question which arises is: to which rotation field should the assumed forces be orthogonal? In order to deal with simple expressions the rotation field shown in Fig. 5 and which corresponds to “cylindrical bending” is assumed. It turns out that in this case “ α ” equals minus 2. A more general formulation which makes the element more insensitive to mesh distortions can be obtained if the relative rotations are decomposed into a part which is due to a constant state of shear, a part caused by torsion and a part which corresponds to bending. In this case the coefficient “ α ” is a function of the geometry of the triangle.

4.3.1. The test example of the pinched cylinder (see Fig. 6) demonstrates the insensitivity of the proposed formulation to mesh distortions. Furthermore, the element is suitable for both thick and thin cases. The alternative formulation “M2” of Fig. 7 with constant assumptions for the shear strains improves results significantly in the case of thick plates.

4.3.2. An alternative formulation leads to an element, especially suitable for thick shells, and which by imposing certain constraints yields an element with only 6 d. o. f. Towards this aim we assume the shear strain ($\vec{\gamma}$) to be equal to the actual shear strain under a state of pure shear. The constant shear strain ($\vec{\gamma}$) is augmented by a higher order term, the torsional mode “ τ ”

$$(28) \quad \vec{\gamma} = \vec{\gamma} + \tau(\vec{n} \times \vec{\rho}).$$

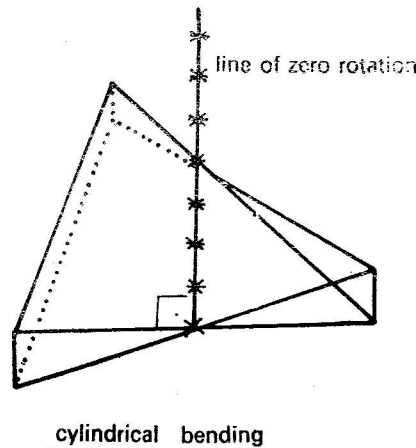


Fig. 5. Cylindrical bending

Enforcement of 3 discrete Kirchhoff constraints leads to a simple triangle with only 6 d. o. f. Since the element contains only tangential rotations at the midpoints of the edges, it is especially suited for problems with multiple junctions and corners. These simple elements can be combined to form macroelements [7]. Compatible beam elements can also be constructed in order to deal with shell problems with stiffeners. The pinched problem demonstrates that the element shows a good convergence behaviour and is relatively insensitive to mesh configurations.

V. Elastoplastic Analysis

5. 1. Preliminary remarks. The various concepts presented previously yield simple but reliable element formulations. These properties are essential in nonlinear problems where incremental procedures require repetition computations and additional storage. Extension of the presented formulations to elastoplastic analysis problems is straightforward. Again the basis of the formulation is the Hu-Washizu principle wherein each of the functions is replaced by the corresponding increment

$$(29) \quad \int_A \frac{1}{2} C_T \underline{\dot{\epsilon}} \underline{\dot{\epsilon}} - \dots q [\underline{\gamma} - G(\underline{w}, \underline{\phi})] \dots$$

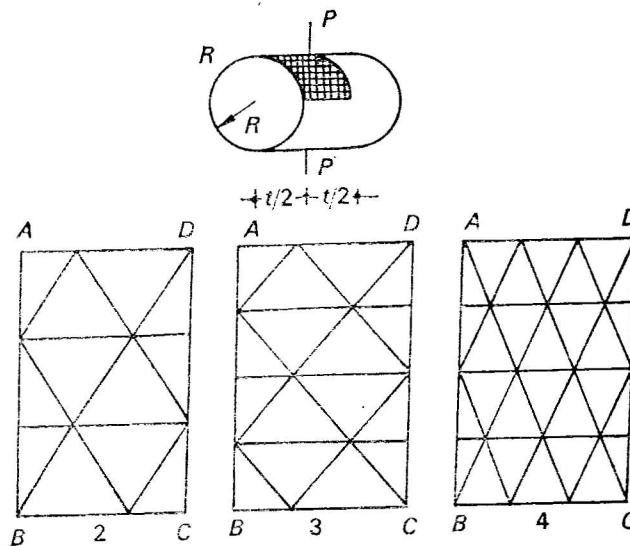


Fig. 6. Triangular elements: pinched problem

Displacement under D			
Mesh	D.O.F.	thick	thin
2	44	.0905	0.02006
3	59	.0986	0.02180
4	95	.0987	0.02180
Cantin	291	.1139	—
Ashwell & Sabir	—	—	0.02439

The tangent modulus C_T is the sum of the elastic modulus S_E and a term $-B_{PF}$ which accounts for plastic flow. The moduli are based on von Mises yield criterion and provide for hardening. The theory of plasticity is applied to layers.

5. 2. Discretization. With respect to the strain/stress assumptions we introduce an important modification which achieves further simplifications: The linear variations of strains and stresses are replaced by piecewise constant approximations which are just sufficient to accommodate the various deformational modes. We can formulate different models, e. g. constant extensional and flexural strains and piecewise constant shear strains. Since stresses are constant within subregions, homogeneous yielding occurs within such regions, which then provides a consistent mechanism for a stepwise progression of yielding through the entire element. Consequently, plastic deformations are more accurately represented by relatively fewer elements. The reevaluation of the stiffness matrix at successive increments of load requires no additional integrations over the area of the element. In this respect the element is simpler than comparable "displacement models". The evaluation of internal forces is also very simple and consequently the correction of unbalances is readily achieved via the iterative process. Finally, we obtain good estimates for the stresses. The piecewise constant values agree with values in finer meshes (see Fig. 10). In the case of triangular elements we can

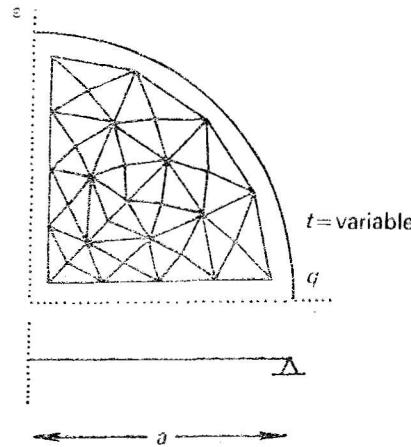


Fig. 7. Simply supported circular plate

$t/2a$	.200	.100	.0375	.0100	.00375	.0010
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M 1 % error	15.3	2.97	-1.11	-1.89	-2.04	-2.55
M 2 % error	1.23	-0.99	-1.77	-1.98	-2.20	-2.86

$$M1 : q_1 = \bar{q}_1 (1 - 2\theta^2) ; \gamma_1 = \bar{\gamma}_1 (1 - 2\theta^2)$$

$$M2 : q_1 = \bar{q}_1 (1 - 2\theta^2) \quad \gamma_1 = \bar{\gamma}_1$$

apply the concept of piecewise constant regions by combining four 6 — d. o. f. triangles to a macroelement [7].

VI. Geometrically Nonlinear Analysis

6.1. Preliminary remarks. In the following, the essential steps of a formulation are presented which are especially suited to shell problems with large rotations but small strains. The theory departs from the usual total or updated Lagrangian formulations and takes into account the decomposition of motion into a rigid body motion and a pure deformation.

6.2. Three-dimensional continuum. It is well known that in the 3-dimensional theory a motion can be decomposed into a rigid rotation which carries the initial tangent vectors $\vec{g}_i$ to an intermediate system $\vec{g}'_i$ and a stretch (with components C'_i) which carries the triad $\vec{g}'_i$ to the current (deformed) system $\vec{G}_i$:

$$(30), (31) \quad \vec{g}'_i = \vec{r}_i \vec{g}_i; \quad \vec{G}_i = C'_i \cdot \vec{g}'_i.$$

The component of stretch $C_{ij} = C_{ji}$ is related to the component of engineering strain by the relationship:

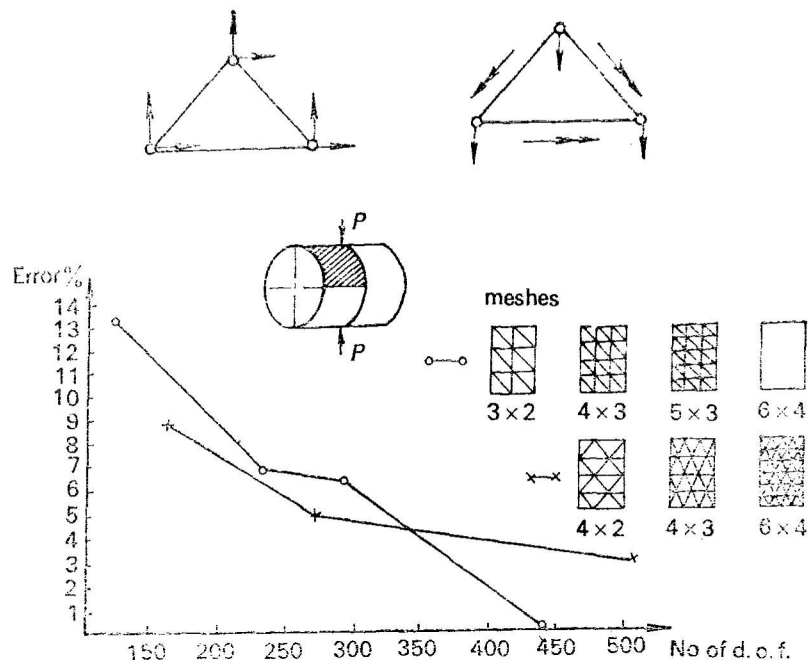


Fig. 8. Triangular elements with discrete Kirchhoff constraints

$$(32) \quad h_{ij} = C_{ij} - \vec{g}_i \cdot \vec{g}_j.$$

The rotation r'_i is the rotation of the principle lines.

6.3. Shell theory. The rotation of the principle lines is the natural choice in three dimensions, both mathematically and physically. However, when transverse shear occurs in a shell, the rotation (of principle lines) carries the tangent vectors out of the reference surface. Consequently, a different, more natural rotation is employed to describe the motion of the shell. We rotate the initial triad $\vec{a}_j$ to a triad $\vec{b}_j$ such that $\vec{b}_\alpha$ are tangent and $\vec{b}_3$ normal to the deformed surface. The orientation of $\vec{b}_\alpha$ is such that the stretch of the surface is given by a symmetrical tensor:

$$(33), (34) \quad C_{\alpha\beta} = \vec{b}_\alpha \cdot \vec{A}_\beta = \vec{b}_\beta \cdot \vec{A}_\alpha; \quad \vec{b}_3 \cdot \vec{A}_\alpha = 0.$$

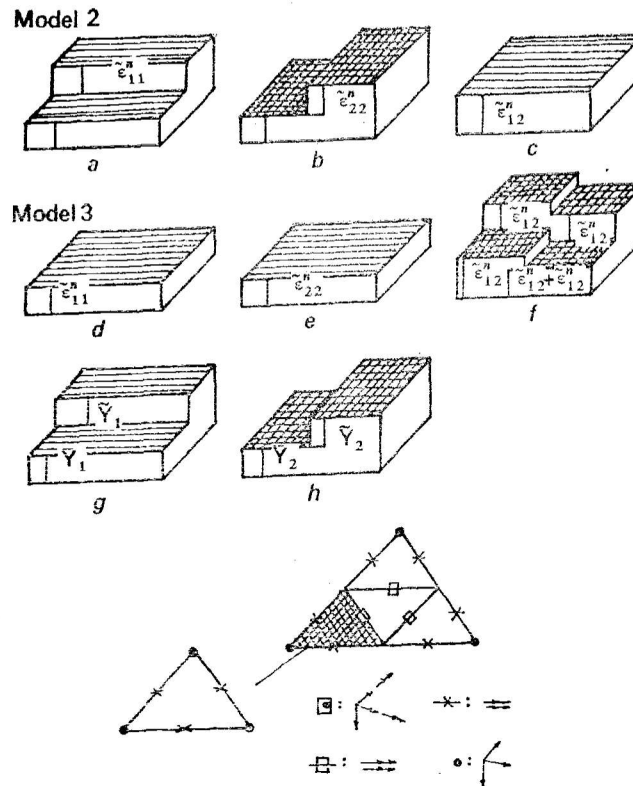


Fig. 9. Elastoplastic analysis

Using this basis a general theory of shells can be derived which takes into account the effects of finite rotations, large strains and transverse shear. The corresponding Hu-Washizu principle:

$$(35) \int W_a - N^{\alpha\beta} [h_{\alpha\beta} - \vec{b}_\beta \cdot (\vec{A}_\alpha - b_\alpha)] - M^{\alpha\beta} [K_{\alpha\beta} - \vec{b}_\beta \cdot (\vec{A}_{3,\alpha} + b_\alpha^\mu \vec{b}_\mu)] + N^{\alpha 3} [\vec{N} \cdot A_\alpha] + \dots$$

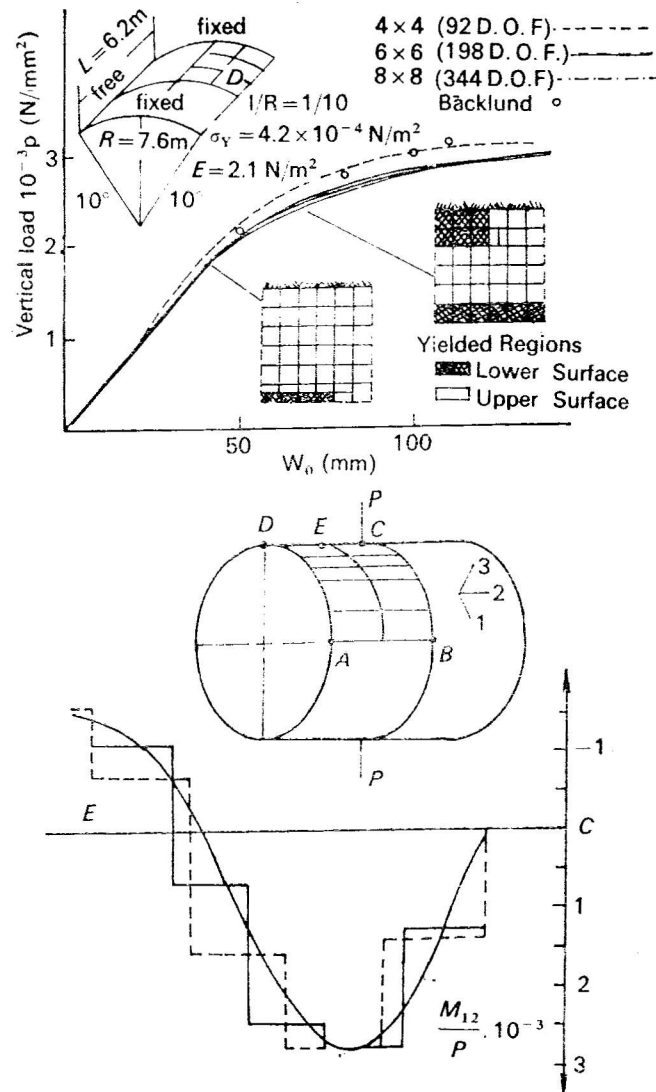


Fig. 10. Piecewise constant concept; stresses

can serve as a basis for finite element approximations.

6.4. Simplified incremental approach. In most shell problems strains remain small, although rotations become large. If strains are small then the relative rotations of lines become small. This idea applies to the infinitesimal element of the continuous body. However, the same concepts can be applied to a small finite region, but the accuracy depends on the dimensions of the region. In other words, if strains are small and the elements are small enough, then the stiffness matrix of the element can be formulated using a linear theory. The essential nonlinearities arise in the assembly. These ideas serve to reduce significantly the computational effort required during an analysis using "incremental" methods.

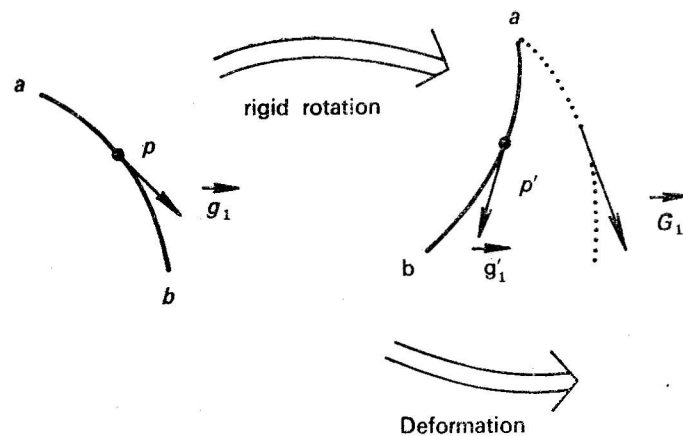


Fig. 11. Decomposition of motion

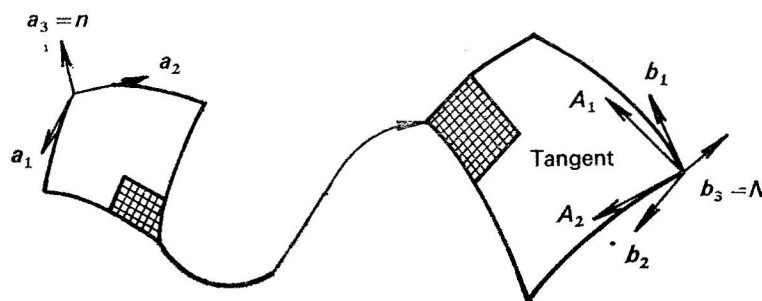


Fig. 12. Natural reference basis for shell

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