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A Generalized Design Approach and a Common Synthesis Method for Mechanisms with Rotating Input Link

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1. Introduction

Commonly, the designer begins the mechanism design procedure in selecting the mechanism's type (structure), and then he determines the invariable parameters of the selected mechanism diagram in accordance with its prescribed characteristics. Such a mechanism synthesis algorithm could lead to an optimum solution only by virtue of the designer's rich experience. However, a more reliable way is to synthesize several types of mechanisms, to select the perspective variants, and to determine the optimum one in conformity with all design objectives. In this case, one has to apply several synthesis methods conformed to the diverse types of mechanisms.

With the aim to facilitate the designer's task in designing different types of single-loop functional mechanisms with a rotating input link, an inversed sequence of the steps of their synthesis and a common method for their synthesis is proposed: determination of some mutual kinematical invariants, dimensioning of the different mechanisms diagrams, determination of the perspective variants of the particular types of mechanisms in conformity with the synthesis conditions, and the indentification of the optimal by type and parameters mechanism, in conformity with all design objectives.

2. Kinematical equivalence of the mechanisms

Let us consider four types of elementary kinematically equivalent mechanisms: a centrode mechanism, a cam mechanism, a band mechanism, and a linkage. Two generalized kinematic diagrams of these mechanisms with a rotating input link and with three mutual kinematical invariants: the relative instantaneous center of rotation P, the absolute instantaneous center of rotation Q and the collineation axis q, are shown in Fig. 1. Indeed, the mechanisms shown in Fig. 1 (a) have a rectlinear motion of the output link, and these shown in Fig. 1 (b) — a rotary motion. The kinematical equivalence between the centrode mechanism having the centrodes z_{12} and z_{21}, the cam mechanism having the cam profile k and the band mechanism having the cam profile e_A, is in a finite range of the generalized coordinate φ, whereas for

the linkage, drawn by dashed lines, the kinematical equivalence with the remaining mechanisms is of the second order only for a given value of φ. Obviously, point A is the center of curvature of the cam profile of the cam mechanism, the last point of contact between the band and the cam profile of the band mechanism, and the conditional kinematic pair of the linkage. The straight line BP determines the position of the normal to the cam profile of the cam mechanism, the band of the band mechanism and the conditional coupler of the linkage. Furthermore, the curve e_A is an evolute of the curve k, γ being the pressure angle of the cam mechanism, the band mechanism and the linkage.

The proposed common method for the synthesis of all four types of transfer mechanisms is based on well-known theorems. By the help of the theorems by Willis [1] and Aronhold-Kennedy [2, 3] one determines the position of the instantaneous center of rotation P — the point of contact between the conjugated centrodes (cams) of the centrode mechanism, the position of the link with general plane motion of the band mechanism and the linkage, and the normal BP of the cam profile and its equidistants of the cam mechanism. The theorems by Bobillier [4] and by Freudenstein [5] in conjunction with the theorem by Aronhold-Kennedy, are used to determine the respective points A of these mechanisms.

The above mentioned theorems are used for the synthesis of band mechanisms [6] and cam mechanisms [7], but they permit to set up a generalized mathematical model for the synthesis also of centrode mechanisms and linkages in a finite range of φ.

3. Preliminary selection of the types of mechanisms

The possibility to synthesize on principle suitable for the practice centrode mechanisms, cam mechanisms, band mechanisms and linkages, is established by taking into consideration their possibilities to realize different displacement functions and by assessment of the influence of cusp points (if any) in the displacement function and the transfer functions on the cam profile of the contour mechanisms.

In fact, the cam mechanisms realize arbitrary displacement functions. Usually, the output link changes its sense of motion at unidirectional rotation of the cam, for that reason the laws 1, 2 and 4, depicted in Fig. 2, are realized merely in a limited range of φ. The linkages are realizing well the laws 1, 2 and 3, in particular when this is done approximately and in a limited range of φ. The band mechanisms and the centrode mechanisms provided with links in the form of frictional centrode cams, circular or non-circular gears, centrode cams encircled by a band, realize the first two laws and their possibilities are exhausted thereby.

The influence of the cusp points in the displacement function and the transfer functions on the cam profile of the contour mechanisms is illustrated in Fig. 3. Usually, displacement functions with cusp points are avoided, since this results in a so called "hard stroke" in the mechanisms [8]. The presence of a cusp point in the displacement function leads to a cusp point of the cam profile k (comprising the arcs k_1 and k_2) of the cam mechanism. In case that consecutively the first and the second transfer functions possess cusp points, such points would have the cendrode mechanism and the band mechanism, respectively. Commonly, ruptures and cusp points are inadmissible in

the cam profiles of the contour mechanisms. If the first transfer function has a cusp point, the most appropriate functional mechanism would be the cam mechanism. If only the second transfer function has a cusp point, the cam mechanism and the centrode mechanism would be suitable. By a cusp point on the third and the higher order transfer functions, all the three types of contour mechanisms would be expedient. Strictly speaking, the linkages generate only smooth functions. Obviously, one could draw the following substantial conclusion: the technological approximations of cam profiles having circular arcs or other arcs with mutual osculation of the first order result in a finite jump in the second transfer function for the cam mechanisms, in the third transfer function for the centrode mechanisms, and in the fourth transfer function for the band mechanisms.

4. Common synthesis method

Indeed, the ascertainment of the possibility to synthesize well fitted for the practice mechanisms of different types calls for a subsequent selection, which is feasible only after their kinematic synthesis. Creating a common synthesis method for single loop functional mechanisms of different types is possible by virtue of the kinematical equivalency, determined up to the second order by the theorems of Willis, Aronhold-Kennedy, Bobillier and Freudenstein.

In order to facilitate the presentation and the application of the synthesis method, we have shown in Fig. 4 the necessary minimum of analytic relations pertinent to each one of the designated mechanisms. The left hand side column relates to mechanisms comprising a slider, as illustrated in Fig. 1 (a), and the right hand side column concerns mechanisms having a rocker, as shown in Fig. 1 (b), wherein

$S=S(\varphi)$, $\psi=\psi(\varphi)$ are displacement functions
$S'=S'(\varphi)$, $i=\psi'(\varphi)$ are first transfer functions
$S''=S''(\varphi)$, $i'=\psi''(\varphi)$ are second transfer functions
k_1 is the angular coefficient of the straight line BC (Fig. 1 (b))
x_P is the abscissa of P
e is the offset of the mechanism (Fig. 1 (a))
R is the length of the rocker of the cam, band and bar mechanisms (Fig. 1 (b))
L is the length of the frame
x_{B_i}, y_{B_i} are coordinates of the points on the cam profile and its equidistants of the cam mechanism, lying on a common normal
r_1 is the radius of the roler of the cam mechanism
r_i is the distance between the cam profile and its equidistants
k_2 is the angular coefficient of the straight line BP
k_3 is the angular coefficient of the collineation axis q
x_Q, y_Q are coordinates of Q
k_4 is the angular coefficient of the straight line OQ
x_A, y_A are coordinates of point A
$(r_P)_{12}, \varphi_{12}$ are polar coordinates of points defining the input centrode z_{12}
$(X_P)_{12}(Y_P)_{12}$ are coordinates of points defining the output centrode z_{21}
$(r_P)_{21}, \psi_{21}$ are polar coordinates of points defining the output centrode z_{21}
X_{Bi}, Y_{Bi} are coordinates of points in a plane XOY fixed to the input link, these points defining the cam profile of the cam mechanism and its equidistants

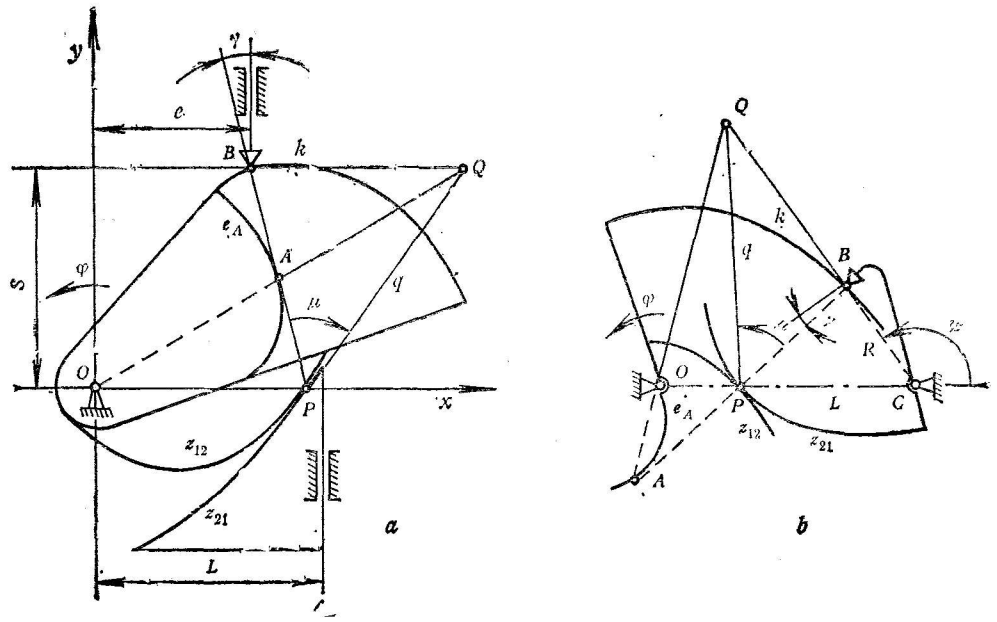


Fig. 1

X_A, Y_A are coordinates of points in the plane XOY, defining the cam profile e_A of the band mechanism

X_{A^*}, Y_{A^*} are coordinates of point A^* in the XOY between the input link and the coupler of the linkage

n is the number of displacements (steps) of φ.

The positions of the instantaneous center of rotation P in the plane of the input link and the output link determine the respective centrodes of the relative motion — profiles of the centrode mechanism's cams. The positions of the output link (of point B) in the plane XOY of the input link determine by points, for each combination of the parameters S_0, e in Fig. 1 (a) and ψ_0, R in Fig. 1 (b) varied in limits imposed by the limiting condition $\gamma \leq \gamma_{adm}$, a profile k of the cam of the cam mechanism. Upon a distance r_i on the straight lines BP lie points B_i defining equidistants of the profile k. The collineation axes q are disposed under respective angles μ with regard to these straight lines and through respective instantaneous centers of rotation P. Each collineation axis q intersects a respective straight line passing through point B and its center of rotation in the instantaneous center of rotation Q. Two respective straight lines BP and OQ intersect in a point A. The locus of points A in the plane of the input link determines the evolute e_A of the cam profile k and the profile of the band mechanism's cam. The optimal, with respect to precision, linkage corresponds to a cam mechanism having a cam profile which deviates slightly from a circular arc, whereas the profile evolute contracts to the highest degree. Then, a minimum value of the mean arithmetic deviation

$$\Delta a = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(X_{Ai} + X_{A^*})^2 + (Y_{Ai} - Y_{A^*})^2} \tag{1}$$

of points $A_i(X_{Ai}, Y_{Ai})$ from point $A^*(X_{A^*}, Y_{A^*})$ is obtained, wherein the point A^* is specified as the center of gravity of the evolute e_A, provided that its

line density is in accordance with the dislocation density of its n in number points obtained by a n-fold alteration of φ. One determines the difference $\Delta b = b_{max} - b_{min}$, for which b_{max} and b_{min} are extreme values of the distance

$$b = \sqrt{(X_B - X_{A^*})^2 + (Y_B - Y_{A^*})^2} \tag{2}$$

from the points B of the cam profile to the center of gravity A^* of the evolute e_A. The half of this difference, comprising the maximum deviation of a circumference of the cam profile having a radius

$$b = 0{,}5(b_{max} + b_{min}) \tag{3}$$

measured on respective normals of the circumference, could be compared with the possible technological deviations of the cam or of the coupler of the bar mechanism.

A linkage, or a cam mechanism having a circular cam, can be used for the realization of a prescribed displacement function, if the deviation of the displacement function of these mechanisms from the prescribed one does not exceed a present admissible value. For the mechanisms having a slider as output ink this deviation is

$$\Delta S = \frac{\Delta b}{2}\sqrt{1 + \mathrm{tg}^2\gamma} \tag{4}$$

or for the mechanisms a rocker of a length R as output link

$$\Delta\psi = \frac{\Delta b}{2R}\sqrt{1 + \mathrm{tg}^2\gamma} \tag{5}$$

it is possible to use a circular cam or a linkage, respectively, to solve the set problem. In such a case there is nothing for it but to determine the offset of the circular cam or the length a of the input link of the linkage, respectively,

$$a = \sqrt{X_{A^*}^2 + Y_{A^*}^2} \tag{6}$$

and the angle

$$\alpha = \mathrm{arctg}\frac{Y_{A^*}}{X_{A^*}} \tag{7}$$

which determines its initial position. The length of the coupler is equal to the radius b of the circular cam defined by the expression (3).

The ultimate aim of this design stage is to determine the perspective alternatives of the different types of mechanisms surpassing substantially the remainder. This is done in evaluating the degree by which the additional conditions bound only with the results of the mechanisms synthesis are satisfied: lengths of the links, values of the pressure angle, technological effectiveness of the kinematic pairs and others. In the case that one alternative of the synthesized mechanisms surpasses sustantially the remainder, the evaluation of the mechanism's type is accomplished. Otherwise, the specification of the mechanism's type should be done in the next stage of structural design in which the satisfaction of all additional conditions of the design (9) related to the mass characteristics of the mechanisms and their marketing should be taken into consideration.

Example

The proposed generalized design approach and common synthesis method is applied to a functional mechanism (Fig. 5), which is built-in into a system for the automatic control of a technological process. The mechanism is designed to transform a rotary motion in a rectilinear motion in accordance

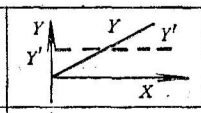
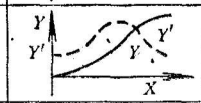
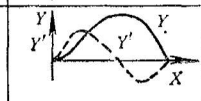
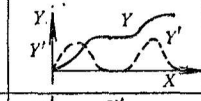
displacement function Y first transfer function Y'		MECHANISM TYPES			
		cam mechanisms	linkages	band mechanisms	centrode mechanism
1		×	×	×	×
2		×	×	×	×
3		×	×		
4		×			
5		×			

Fig. 2

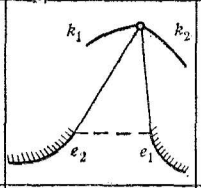
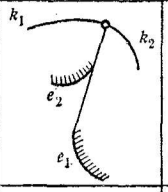
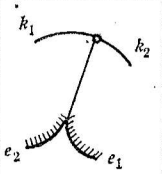
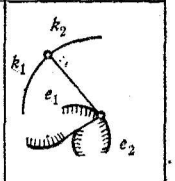
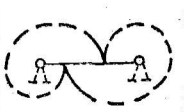
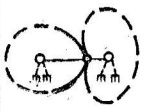
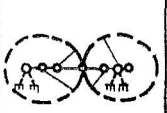
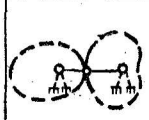
cam profile	CUSP POINT			
	in the displacement function	in the first transfer funktion	in the second transfer function	in the third transfer function
of the cam mechanism—k (involute) of the band mechanism—e (evolute)				
of the centrooe· mechanism				

Fig. 3

Mechanisms with Rotating Input Link		
No	with slider (Fig. 1a)	with rocker (Fig. 1b)
1	———	$k_1=\operatorname{tg}\ \psi$
2	$x_p=S'$	$x_p=\dfrac{Li}{i-1}$
3	$K_2=\dfrac{S}{e-S'}$	$k_2=\dfrac{R\sin\psi}{L+R\cos\psi-x_p}$
4	$X_{Bi}=e+\dfrac{(r_i-r_1)\operatorname{sgn}k_2}{\sqrt{1+k_2^2}}$	$x_{Bi}=L+R\cos\psi+\dfrac{(r_i-r_1)\operatorname{sgn}k_2}{\sqrt{1+k_2^2}}$
5	$y=k_2(x_{Bi}-x_p)$	
6	$\operatorname{tg}\gamma=-\dfrac{1}{k_2}$	$\operatorname{tg}\gamma=\dfrac{1+k_1k_2}{k_1-k_2}$
7	$\operatorname{tg}M=\dfrac{S'}{S''}$	$\operatorname{tg}M=\dfrac{i(1-i)}{i'}$
8	$k_3=\dfrac{k_2+\operatorname{tg}M}{1-k_2\operatorname{tg}M}$	
9	$x_Q=x_p+\dfrac{S}{k_3}$; $y_Q=S$	$x_Q=\dfrac{k_1L-k_3x_p}{k_1-k_3}$; $y_Q=k_1(x_Q-L)$
10	$k_4=\dfrac{y_Q}{x_Q}$	
11	$x_A=\dfrac{k_2x_p}{k_2-k_4}$; $y_A=k_4x_A$	
12	$(r_p)_{12}=S'(\varphi)$; $\varphi_{12}=-\varphi$ $\begin{cases}(x_p)_{21}=L-S'\cos\varphi\\(y_p)_{21}=-\int S'(\varphi)\,d\varphi+C=-s(\varphi)\end{cases}$	$(r_p)_{12}=\dfrac{Li(\varphi)}{i(\varphi)-1}$ $\begin{cases}(r_p)_{21}=L-(r_p)_1=f(\varphi)\\\varphi_{21}=-\int i(\varphi)\,d\varphi+C=-\psi(\varphi)\end{cases}$
13	$X_{Bi}=x_{Bi}\cos\varphi+y_{Bi}\sin\varphi$; $y_{Bi}=-x_{Bi}\sin\varphi+y_{Bi}\cos\varphi$	
14	$x_A=x_A\cos\varphi+y_A\sin\varphi$; $y_A=-x_A\sin\varphi+Y_A\cos\varphi$	
15	$x_A^*=\dfrac{1}{n}\sum_{i=1}^{n}x_A$; $y_A^*=\dfrac{1}{n}\sum_{i=1}^{n}$	

Fig. 4

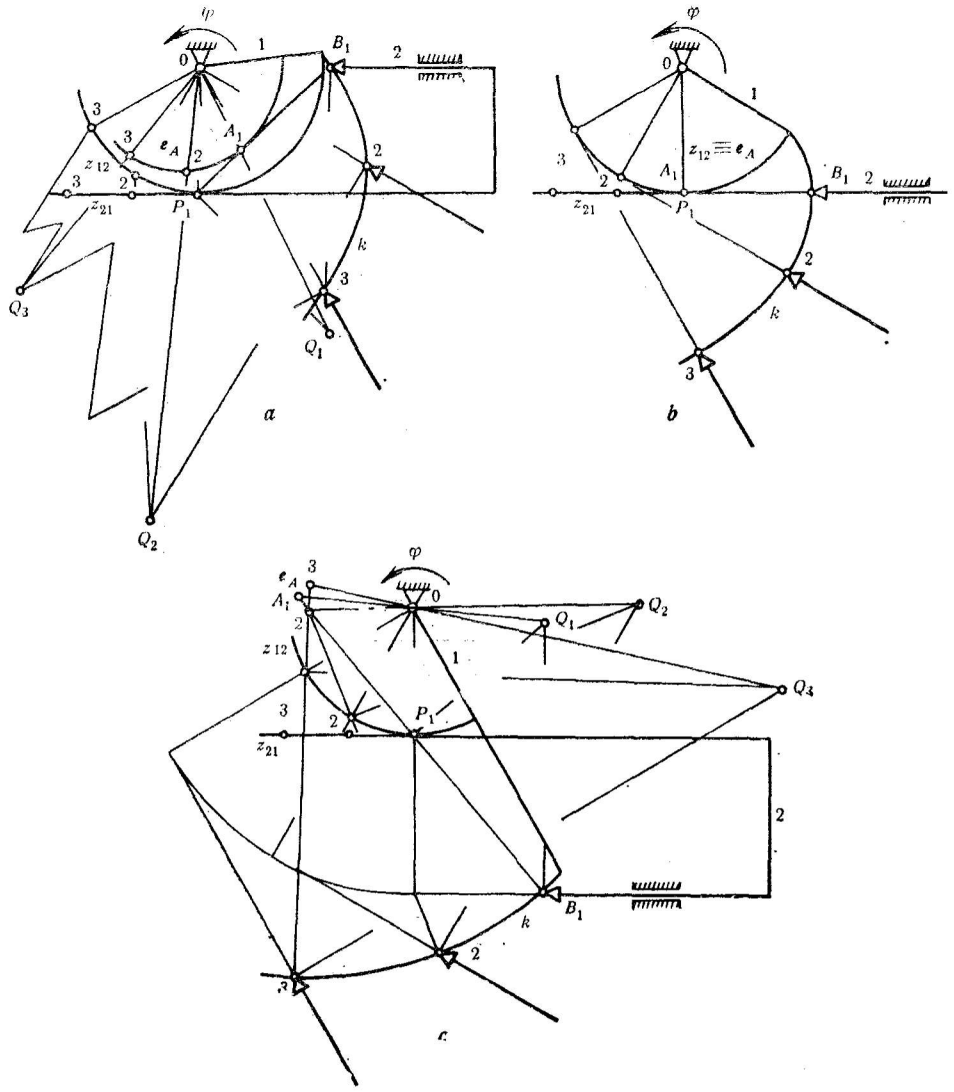


Fig. 5

with the prescribed function $S=23\left(1+\frac{\varphi^\circ}{60}\right)$ at the limiting conditions $\varphi[0, 60°]$, $\gamma \leq 50°$ and an admissible error $\delta S=\frac{\Delta S}{h}=0.08\,\%$ (h is the stroke of the slider). In order to determine the perspective alternatives of the different types of mechanisms, an objective function Z is introduced as the sum of the estimates of two criteria: the criterion $k_\gamma=|\cos\gamma|_{\max}$ for the force transmission on the end link and the criterion k_τ for the technological effectiveness of the cylindrical work surfaces of the kinematic pairs which realise the connection between the input and the output links. The maximum values of the introduced objective function are considered as optimal values. In conformity with

the particular manufacturing conditions the valules of k_τ are assumed as follows:

cylindrical work surfaces		values of k_τ	note
circular	without offset	1.0	For a cylindrical work surface with a band or a toothing of a circular cylindrical work surface k_τ should be decreased by 0.2
	with offset	0.9	
non-circular	Archimedean spiral	0.7	
	involute	0.7	
	arbitrary	0.5	

While using the expressions listed in the left hand side column in Fig. 4, a set of mechanisms is synthesized and the perspective alternatives are selected (see Fig. 5), which are obtained for three values of the offset:

mechanism	$e=0$	$e=S'$	$e=50$ mm
cam	1.4	1.7	1.54
band	1.0	1.8	
centrode	1.74	1.74	1.74
linkage	the precision condition is not meet		1.67

Perspective are: the centrode mechanism ($Z=1.74$) with conjugated centrodes which are invariant in deviating e, these being considered in the present case as a circular gear and a gear rack; the cam mechanism ($Z=1.7$) and the band mechanism ($Z=1.8$) for $e=S'$ and the linkage ($Z=1.67$) for $e=50$ mm and a percentage error $SS=0.074\,\%$.

It is most likely that the designer, making use of the proposed procedure, would direct his attention only to the gear and the band mechanisms. However, this alternative could be found impractical: a cam mechanism provided with a cam shaped as an Archimedean spiral for $e=0$, or as a circular involute for $e=S'$, would possibly meet still better the additional conditions, related to the mass characteristics of the mechanisms and their marketing priorities. Taking into consideration these conditions, it turned out that the linkage is the most suitable in this case.

The application of the proposed generalized design approach and common synthesis method to planar functional mechanisms leads positively to the optimum solution without waste of time and superfluous mental work.

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