

Boundary Layer Growth on a Rotating Circular Cylinder in an Infinite Fluid

Z. Zapryanov, I. Lambova

1. Introduction

As early as the 60s Wang studies a flow past a circular cylinder which is started impulsively from rest (1). Both the effect of the cylinder surface curvature and the interaction between the boundary layer and the outer inviscid flow are accounted for in accordance with the boundary layer theory. In the solution to the Navier-Stokes equations the method of inner and outer expansions is made use of (2). Slavchev gives a generalized treatment of the unsteady motion of a circular cylinder with a velocity defined as a certain power of time (3). The boundary layer growth on a spinning sphere in an infinite viscous fluid is to be found in Zapryanov (4).

This paper deals with the same kind of problems. It is concerned with the rotatory motion of a circular cylinder with a velocity proportional to the flow velocity. The case is of utmost practical importance.

2. Statement of the problem

An infinite incompressible viscous fluid with kinematic viscosity ν and density ρ flows past an infinite circular cylinder in a direction perpendicular to its axis. The flow velocity varies in the manner

$$v'(t) = U_{\infty} \left(\frac{t'}{T_0} \right)^{\alpha}; \quad v(t) = \frac{v'(t)}{U_{\infty}} = t^{\alpha}$$

where t' is a dimensional time, T_0 and U_{∞} are a characteristic time and a characteristic velocity, respectively, to be defined exactly below, and $\alpha \geq 0$ is a non-dimensional constant. The cylinder rotates with an angular velocity

$$\Omega'(t) = \beta \frac{U_{\infty}}{L} \left(\frac{t'}{T_0} \right)^{\alpha}; \quad \Omega(t) = \frac{\Omega'(t) \cdot L}{U_{\infty}} = \beta t^{\alpha}$$

where β is a non-dimensional parameter and L is the radius of the cylinder.

The flow field is completely characterized by two non-dimensional parameters, namely, a Reynolds number $R = U_{\infty} L / \nu$ and a Strouhal number $S = L / U_{\infty} T_0$.

The following simplifying assumptions are made about the system:

- (i) large Reynolds and Strouhal numbers, that is a slightly viscous fluid;
- (ii) small times compared to the initial detachment time.

Thus we can define $\varepsilon = \frac{1}{s} = \frac{1}{vR}$ where ε is a small number and v' is a constant of order one, and accordingly $U_\infty = \frac{1}{\varepsilon v'} \frac{v}{L}$ and $T_0 = \frac{1}{v'} \frac{v}{U_\infty^2}$.

The normalized Navier-Stokes equations and the continuity equation then take the form

$$(1) \quad \frac{\partial \bar{v}}{\partial t} + \frac{1}{S} (\bar{v} \cdot \nabla) \bar{v} = -\nabla p - \frac{1}{RS} \nabla \times (\nabla \times \bar{v})$$

$$(2) \quad \nabla \cdot \bar{v} = 0.$$

The pressure is normalized by $\rho U_\infty^2 S$.

The function p introduced by (1) is arbitrary for multi-connected regions. To obtain a single-valued pressure field, we set

$$\oint [(\partial p / \partial x) dx + (\partial p / \partial y) dy] = 0$$

which is to be fulfilled for any closed curved line streaming past an obstacle.

We shall apply a cylindrical coordinate system (r, θ) . The normalized Navier-Stokes equations and the continuity equation become

$$(3) \quad \frac{\partial u}{\partial t} + \varepsilon \left[u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} \right] = -\frac{\partial p}{\partial r} + v' \varepsilon^2 \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} - \frac{u}{r^2} \right]$$

$$(4) \quad \frac{\partial v}{\partial t} + \varepsilon \left[u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + \frac{uv}{r} \right] = -\frac{1}{r} \frac{\partial p}{\partial \theta} + v' \varepsilon^2 \left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} - \frac{v}{r^2} \right]$$

$$(5) \quad \frac{\partial}{\partial r} (ru) + \frac{\partial v}{\partial \theta} = 0$$

where

$$(6) \quad u = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad v = -\frac{\partial \psi}{\partial r}.$$

The cylinder surface can be given as $r=1$. For the flow field we find $r \geq 1$, $0 \leq \theta \leq 2\pi$.

The boundary conditions are

$$(7) \quad \Psi = 0 \quad \text{for } t < 0$$

$$(8) \quad \Psi = t^\alpha \left(1 - \frac{1}{r^2} \right) r \sin \theta + \Psi^\beta \quad \text{as } t \rightarrow 0^+$$

where

$$(9) \quad \Psi^\beta = -\frac{\Gamma}{2\pi} \ln r$$

$$(10) \quad \Psi \sim t^\alpha r \sin \theta \quad \text{as } r \rightarrow \infty, t \geq 0.$$

$$(11) \quad \psi = 0, \quad \frac{\partial \psi}{\partial r} = \beta t^\alpha \quad \text{as } r = 1, t \geq 0.$$

The β -indexed terms explicitate the solution for a rotating cylinder. The condition for a single-valued pressure field has the form

$$(12) \quad \int_0^{2\pi} \frac{\partial p}{\partial \theta} d\theta = 0.$$

3. Inner and outer expansions

The variables in both regions are expressed in series of the small parameter ε . The outer flow behaves as an ideal fluid and the outer stream function satisfies the Laplace equation. At the moment of starting the flow is fully potential. The initial condition (8) is assumed to be the zeroth approximation of the outer flow stream function. The expansion coefficients in the boundary layer depend on the stretched variable $\frac{r-1}{\varepsilon}$. Substituting the inner expansions in (3) and (4) and comparing like powers of ε we obtain the equations for the zeroth, the first, etc. approximations. The inner and the outer solutions are matched by the intermediate variable $\tilde{r} = \frac{1}{2\sqrt{v't}} \frac{r-1}{\varepsilon}$ to give the wanting conditions which together with the inviscation condition (11) form the boundary conditions.

4. Zeroth approximation

The condition for a single-valued pressure field requires the contour circulation to be zero. Therefore, for the outer expansion we find

$$(13) \quad \Psi_0(r, \theta, t) = t^\alpha \left(1 - \frac{1}{r^2}\right) r \sin \theta$$

and for the inner one

$$(14) \quad v_0 = t^\alpha \sin \theta [-2 + 2^{2\alpha+1} \Gamma(\alpha+1) g_\alpha(\tilde{r})] + v_0^\beta$$

$$\text{where } g_\alpha(\tilde{r}) = \frac{2}{\sqrt{\pi} \Gamma(2\alpha+1)} \int_{\tilde{r}}^{\infty} (\gamma - \tilde{r})^{2\alpha} e^{-\gamma^2} d\gamma$$

$$(15) \quad v_0^\beta = \beta t^\alpha [-2^{2\alpha} \Gamma(\alpha+1) g_\alpha(\tilde{r})]$$

$$(16) \quad \Psi_0 = \sqrt{v'} t^{\alpha+1/2} \sin \theta \left[-\frac{2\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} + 4\tilde{r} + 2^{2\alpha+2} \Gamma(\alpha+1) g_{\alpha+1/2}(\tilde{r}) \right] + \Psi_0^\beta$$

$$(17) \quad \Psi_0^\beta = \beta \sqrt{v'} t^{\alpha+1/2} \left[\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} - 2^{2\alpha+1} \Gamma(\alpha+1) g_{\alpha+1/2}(\tilde{r}) \right].$$

5. First approximation

If we satisfy the condition for a single-valued pressure field

$$(18) \quad \int_0^{2\pi} \frac{\partial p_1}{\partial \theta} d\theta = 0$$

for the outer flow we find

$$(19) \quad \Psi_1(r, \theta, t) = \sqrt{v'} t^{\alpha+1/2} \frac{\sin \theta}{r} \left[-\frac{2\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} \right] + \Psi_1^\beta(r, \theta, t)$$

where

$$(20) \quad \Psi_1^\beta(r, \theta, t) = \beta \sqrt{v'} t^{\alpha+1/2} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)}$$

and for the inner one

$$(21) \quad v_1(\tilde{r}, \theta, t) = \sqrt{v'} t^{\alpha+1/2} \sin \theta \left[-\frac{2\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} + 4\tilde{r} - 2^{2\alpha} \Gamma(\alpha+1) g_{\alpha-1/2}(\tilde{r}) \right. \\ \left. + 2^{2\alpha+2} (\alpha+3/2) \Gamma(\alpha+1) g_{\alpha+1/2}(\tilde{r}) \right] + t^{2\alpha+1} \sin \theta \cos \theta \left\{ 2^{2\alpha+1} \frac{1}{\alpha+2} \Gamma(\alpha+1) g_{\alpha-1}(\tilde{r}) \right. \\ \left. - 2^{2\alpha+1} \frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+5/2)} g_{\alpha-1/2}(\tilde{r}) - 2^{2\alpha+3} \frac{\alpha-1}{\alpha+1} \Gamma(\alpha+1) g_{\alpha}(\tilde{r}) + 2^{4\alpha+3} \Gamma^2(\alpha+1) g_{\alpha+1/2}^2(\tilde{r}) \right. \\ \left. - 2^{4\alpha+3} \Gamma^2(\alpha+1) g_{\alpha}(\tilde{r}) g_{\alpha+1}(\tilde{r}) + \left[\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2) \Gamma(\alpha+5/2)} - \frac{\Gamma^2(\alpha+1)}{2\Gamma^2(\alpha+3/2)} \right. \right. \\ \left. \left. - \frac{6-\alpha}{2(\alpha+1)(\alpha+2)} \right] 2^{4\alpha+4} \Gamma(2\alpha+2) g_{2\alpha+1}(\tilde{r}) \right\} + v_1^\beta(\tilde{r}, \theta, t)$$

where

$$(22) \quad v_1^\beta(\tilde{r}, \theta, t) = \beta \sqrt{v'} t^{\alpha+1/2} [2^{2\alpha-1} \Gamma(\alpha+1) g_{\alpha-1/2}(\tilde{r}) - 2^{2\alpha+1} (\alpha+1/2) \Gamma(\alpha+1) g_{\alpha+1/2}(\tilde{r}) \\ + \beta t^{2\alpha+1} \cos \theta \left\{ -2^{2\alpha} \frac{1}{\alpha+2} \Gamma(\alpha+1) g_{\alpha-1}(\tilde{r}) + 2^{2\alpha} \frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+5/2)} g_{\alpha-1/2}(\tilde{r}) \right. \\ \left. + 2^{2\alpha+2} \frac{\alpha-1/2}{\alpha+1} \Gamma(\alpha+1) g_{\alpha}(\tilde{r}) - 2^{4\alpha+2} \Gamma^2(\alpha+1) g_{\alpha+1/2}^2(\tilde{r}) + 2^{4\alpha+2} \Gamma^2(\alpha+1) g_{\alpha}(\tilde{r}) g_{\alpha+1}(\tilde{r}) \right] \\ \left. - \left[\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2) \Gamma(\alpha+5/2)} - \frac{\Gamma^2(\alpha+1)}{2\Gamma^2(\alpha+3/2)} - \frac{2-3\alpha}{2(\alpha+1)(\alpha+2)} \right] 2^{4\alpha+3} \Gamma(2\alpha+2) g_{2\alpha+1}(\tilde{r}) \right\}$$

$$(23) \quad \psi_1(\tilde{r}, \theta, t) = v' t^{2\alpha+1} \sin \theta \left[-\frac{1}{\alpha+1} + \frac{4\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} \tilde{r} - 4\tilde{r}^2 - 2^{2\alpha+1} \Gamma(\alpha+1) g_{\alpha}(\tilde{r}) \right. \\ \left. + 2^{2\alpha+3} (\alpha+3/2) \Gamma(\alpha+1) g_{\alpha+1}(\tilde{r}) \right] + \sqrt{v'} t^{2\alpha+3/2} \sin \theta \cos \theta \left\{ -\left[\frac{4\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2) \Gamma(\alpha+5/2)} \right. \right. \\ \left. \left. - \frac{2\Gamma^2(\alpha+1)}{\Gamma^2(\alpha+3/2)} - \frac{2(6-\alpha)}{(\alpha+1)(\alpha+2)} \right] \frac{\Gamma(2\alpha+2)}{\Gamma(2\alpha+3/2)} - \frac{2(7\alpha+11)}{(\alpha+1)(\alpha+2)} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+5/2)} \right. \\ \left. - 2^{2\alpha} \frac{1}{\alpha+2} \Gamma(\alpha+1) g_{\alpha-1/2}(\tilde{r}) - 2^{2\alpha+2} \frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+5/2)} g_{\alpha}(\tilde{r}) - 2^{2\alpha+4} \frac{\alpha-1}{\alpha+1} \Gamma(\alpha+1) g_{\alpha+1/2} \right. \\ \left. - 2^{4\alpha+4} \Gamma^2(\alpha+1) g_{\alpha+1/2}(\tilde{r}) g_{\alpha+1}(\tilde{r}) - 2^{4\alpha+5} \Gamma^2(\alpha+1) \int_0^{\tilde{r}} g_{\alpha+1/2}^2(\tilde{r}) d\tilde{r} \right. \\ \left. + \left[\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2) \Gamma(\alpha+5/2)} - \frac{\Gamma^2(\alpha+1)}{2\Gamma^2(\alpha+3/2)} - \frac{6-\alpha}{2(\alpha+1)(\alpha+2)} \right] 2^{4\alpha+5} \Gamma(2\alpha+2) g_{2\alpha+3/2}(\tilde{r}) \right\} + \psi_1^\beta(\tilde{r}, \theta, t)$$

where

$$(24) \quad \psi_1^\beta(\tilde{r}, \theta, t) = \beta v' t^{\alpha+1} \left[-\frac{1}{2(\alpha+1)} + 2^{2\alpha} \Gamma(\alpha+1) g_{\alpha}(\tilde{r}) - 2^{2\alpha+2} (\alpha+1/2) \Gamma(\alpha+1) g_{\alpha+1}(\tilde{r}) \right] \\ + \sqrt{v'} t^{2\alpha+3/2} \cos \theta \left\{ \left[\frac{2\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2) \Gamma(\alpha+5/2)} - \frac{\Gamma^2(\alpha+1)}{\Gamma^2(\alpha+3/2)} - \frac{2-3\alpha}{(\alpha+1)(\alpha+2)} \right] \frac{\Gamma(2\alpha+2)}{\Gamma(2\alpha+5/2)} \right.$$

$$\begin{aligned}
& -\frac{2\alpha^2-5}{(\alpha+1)(\alpha+2)} \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+5/2)} - 2^{2\alpha+1} \frac{1}{\alpha+2} \Gamma(\alpha+1) g_{\alpha-1/2}(\tilde{r}) + 2^{2\alpha+1} \frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+5/2)} g_{\alpha}(\tilde{r}) \\
& + 2^{2\alpha+3} \frac{\alpha-1/2}{\alpha+1} \Gamma(\alpha+1) g_{\alpha+1/2}(\tilde{r}) + 2^{4\alpha+3} \Gamma^2(\alpha+1) g_{\alpha+1/2}(\tilde{r}) g_{\alpha+1}(\tilde{r}) \\
& + 2^{4\alpha+4} \Gamma^2(\alpha+1) \int_0^{\tilde{r}} g_{\alpha+1/2}^2(\tilde{r}) d\tilde{r} - \left[\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2) \Gamma(\alpha+5/2)} - \frac{\Gamma^2(\alpha+1)}{2\Gamma^2(\alpha+3/2)} \right. \\
& \left. - \frac{2-3\alpha}{2(\alpha+1)(\alpha+2)} \right] 2^{4\alpha+4} \Gamma(2\alpha+2) g_{2\alpha+3/2}(\tilde{r}) \}.
\end{aligned}$$

6. Uniformly valid solution

A uniformly valid solution for the stream function accurate to be found by adding the zeroth and the first outer approximations and the zeroth inner approximation and subtracting the matched terms.

7. Flow properties

The appearance of a zero skin friction point marks the onset of the flow reversal. The main purpose of the following parametric study is to find the

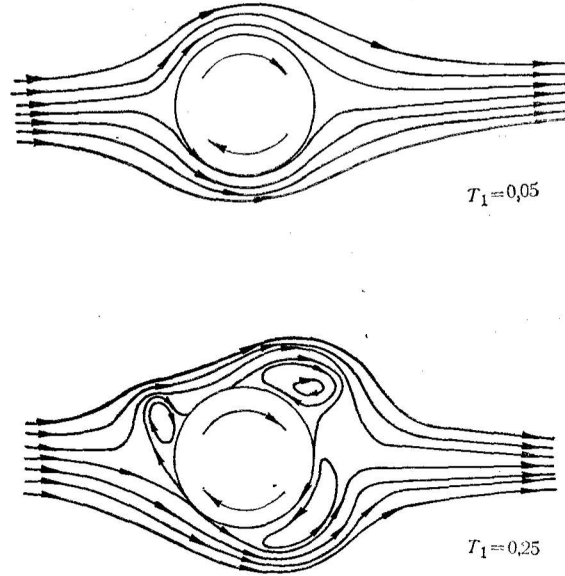


Fig. 1. Streamline pattern for a uniformly accelerated flow past a rotating circular cylinder at $\beta=1$ and $R=100$ before and after detachment

effect of the acceleration parameter α , the rotation parameter β and the Reynolds number R on the detachment points. Setting the shear stress at the body to be zero we have

$$(25) \quad \frac{\partial v_0}{\partial r} \Big|_{\tilde{r}=0} + \varepsilon \frac{\partial V_1}{\partial r} \Big|_{\tilde{r}=0} - \varepsilon 2 \sqrt{v't} v_0 \Big|_{\tilde{r}=0} = 0.$$

After some algebra we obtain

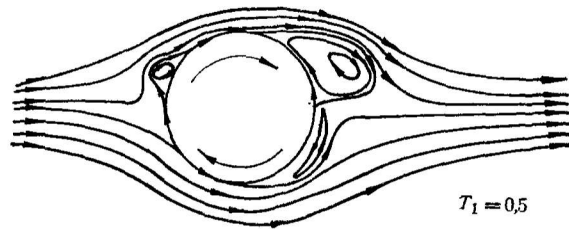
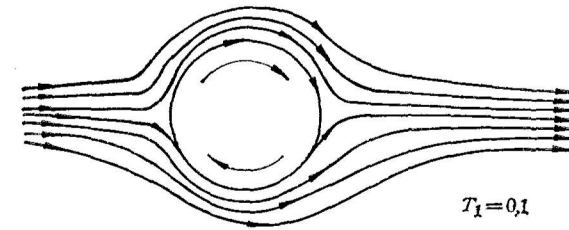


Fig. 2. Streamline pattern for an impulsive flow past a rotating circular cylinder at $\beta=0.5$ and $R=100$ before and after detachment

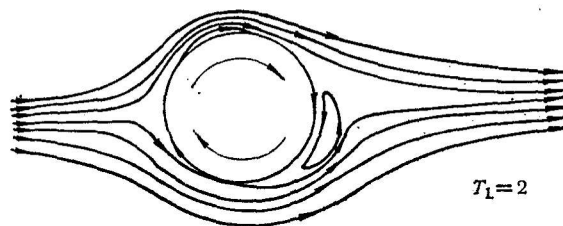
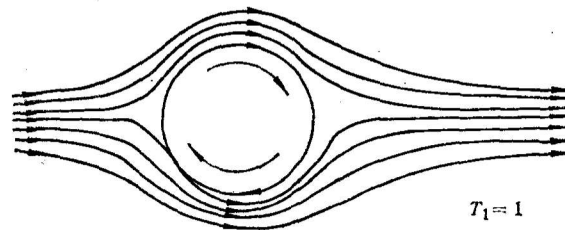


Fig. 3. Streamline pattern for an impulsive flow past a rotating circular cylinder at $\beta=5$ and $R=100$ before and after detachment

$$\begin{aligned}
(26) \quad (v'R)^a \cos \theta & \left\{ \sin \theta \left[\left(\frac{4(6-a)}{(a+1)(a+2)} - \frac{8\Gamma^2(a+1)}{\Gamma(a+1/2)\Gamma(a+5/2)} + \frac{4\Gamma^2(a+1)}{\Gamma^2(a+3/2)} \right) \frac{\Gamma(2a+2)}{\Gamma(2a+3/2)} \right. \right. \\
& + \left. \left(\frac{4(3a-4)}{(a+1)(a+2)} + \frac{4(a-3/2)}{(a+1/2)(a+3/2)} \right) \frac{\Gamma(a+1)}{\Gamma(a+1/2)} \right] + \beta \left[\frac{2(2-3a)}{(a+1)(a+2)} - \frac{4\Gamma^2(a+1)}{\Gamma(a+1/2)\Gamma(a+5/2)} \right. \right. \\
& + \left. \left. \frac{2\Gamma^2(a+1)}{\Gamma^2(a+3/2)} \right) \frac{\Gamma(2a+2)}{\Gamma(2a+3/2)} - \left(\frac{10a}{(a+1)(a+2)} + \frac{2(a-3/2)}{(a+1/2)(a+3/2)} \right) \frac{\Gamma(a+1)}{\Gamma(a+1/2)} \right] \right\} T_1^{a+1} \\
& - \frac{1}{\sqrt{R}} (2 \sin \theta - 3\beta) \sqrt{T_1} - \sin \theta \frac{4\Gamma(a+1)}{\Gamma(a+1/2)} + \beta \frac{2\Gamma(a+1)}{\Gamma(a+1/2)} = 0.
\end{aligned}$$

Here $T_1 = \frac{\text{actual time}}{L/U_\infty}$.

It should be emphasized that the coefficient β stands for the ratio of the rotation velocity of the cylinder to the translation velocity of the flow at infinity.

Figs. 1, 2 and 3 show the streamline patterns for a uniformly accelerated and an impulsive flows past a rotating cylinder at various β before and after the boundary layer detachment. At $\beta=0$ the cylinder is at rest and the flow pattern is symmetrical (see /3/). In the case of a rotating cylinder the structure of the flow changes significantly and the symmetry is distorted. There are two trailing-edge vortices and one leading-edge vortex for values of the angular velocity smaller or equal to those of the flow velocity, and only one small vortex for values of the angular exceeding considerably those of the flow velocity.

Eq. (2), as plotted in Fig. 4 and 5 at $R=100$ and various a and β , illustrates the movement of the detachment points for an impulsive and a uniformly accelerated flows.

The minimal time $T_{1 \min}$ is the time when boundary layer detachment first occurs. Fig. 6 indicates its distribution according to β : T_1 decreases at $0 \leq \beta < 1$ and increases at $\beta > 1$. The fact is significant for boundary layer control. Dependence on a is inversely proportional, that is flow acceleration cuts down the initial detachment time.

The non-dimensional parameter R points to the viscous effect. Slightly viscous fluids require early detachment as it is shown in Fig. 7.

At $\beta=0$ and $R \rightarrow \infty$ the problem posed here is equivalent to that of Wang (1) and juxtaposition reveals identical first-order solutions to $T_{1 \min}$ in both cases.

The hydrodynamic force interaction is defined by the torque and the force which the cylinder exerts upon the fluid. For the force components in the direction of x and y to the first order we have

$$(27) \quad F_x = - \int_0^{2\pi} [p_{rr} \cos(i_r, x) + p_{r\theta} \cos(i_\theta, x)] d\theta$$

$$(28) \quad F_y = - \int_0^{2\pi} [p_{rr} \cos(i_r, y) + p_{r\theta} \cos(i_\theta, y)] d\theta$$

and by recasting them we obtain

$$(29) \quad F_x = \int_0^{2\pi} (p_0 + \varepsilon p_1) \cos \theta d\theta + \varepsilon \frac{1}{2} \sqrt{\frac{v}{\varepsilon}} \int_0^{2\pi} \frac{\partial v_0}{\partial r} \Big|_{\tilde{r}=0} \sin \theta d\theta$$

$$(30) \quad F_y = \int_0^{2\pi} (p_0 + \varepsilon p_1) \cos \theta \, d\theta - \varepsilon \frac{1}{2} \sqrt{\frac{v'}{t}} \int_0^{2\pi} \frac{\partial v_0}{\partial r} \Big|_{r=0} \cos \theta \, d\theta.$$

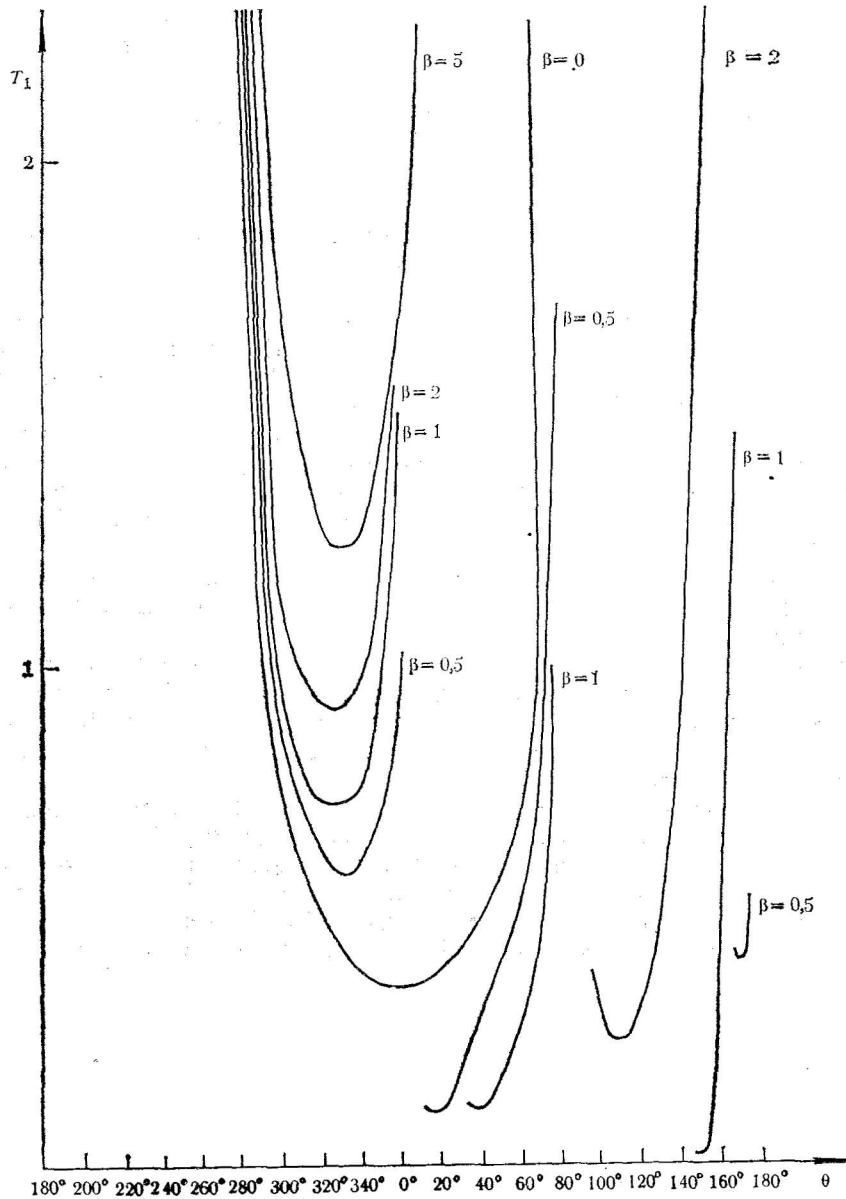


Fig. 4. Movement of the detachment points for an impulsive flow past a rotating circular cylinder at $R=100$ and various β

The force F_y is zero accurate to $O(\varepsilon^2)$.
The cylinder torque M can be given by

$$(31) \quad M = -\varepsilon \int_0^{2\pi} \frac{1}{2} \sqrt{\frac{v}{t}} \frac{\partial v_0}{\partial r} \Big|_{\tilde{r}=0} d\theta.$$

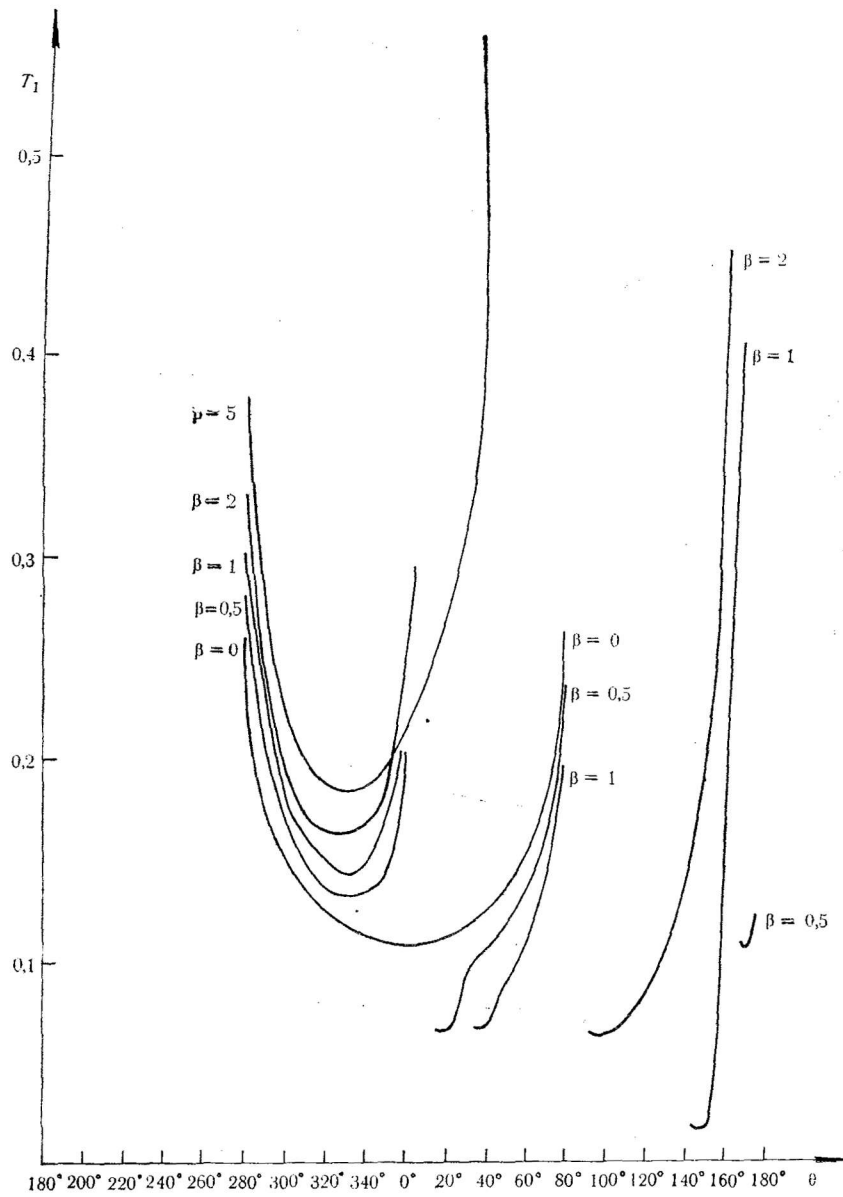


Fig. 5. Movement of the detachment points for a uniformly accelerated flow past a rotating circular cylinder at $R=100$ and various β

The torque is tested on the acceleration parameter, the Reynolds number R and the time, and evidence is recorded in Fig. 8, 9 and 10, respectively. It is quite natural that M is a decreasing function of T_1 for an impulsive flow and an increasing one for a uniformly accelerated flow.

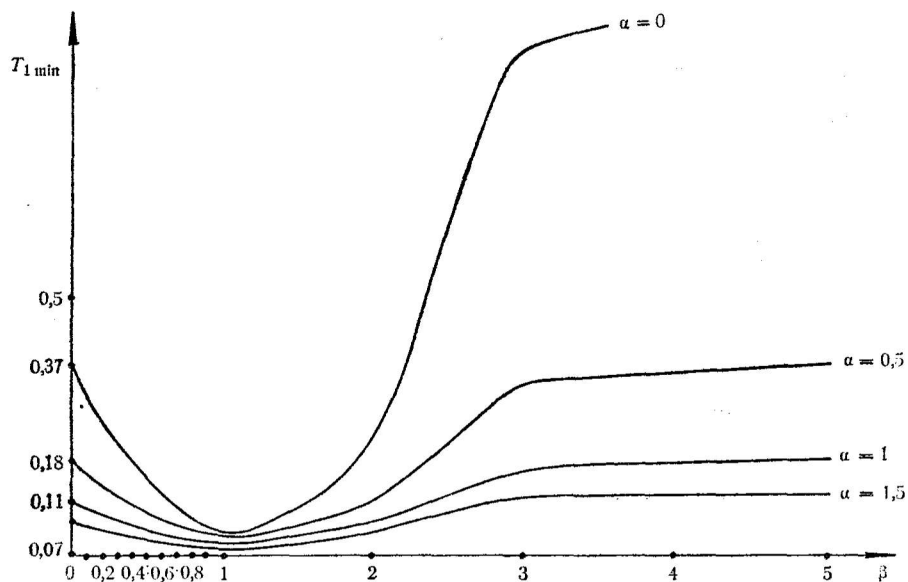


Fig. 6. Initial detachment time as a function of β at various α

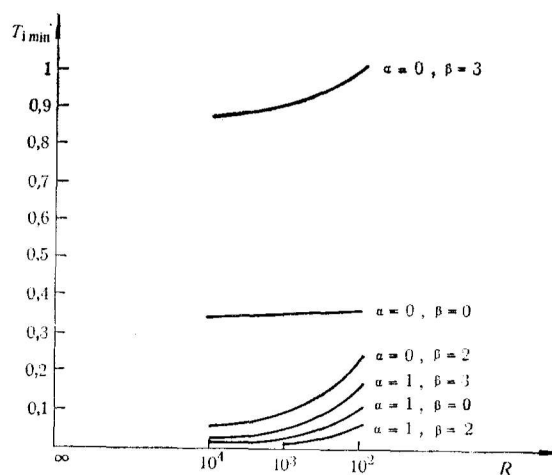


Fig. 7. Initial detachment time as a function of the Reynolds number R at various α and β

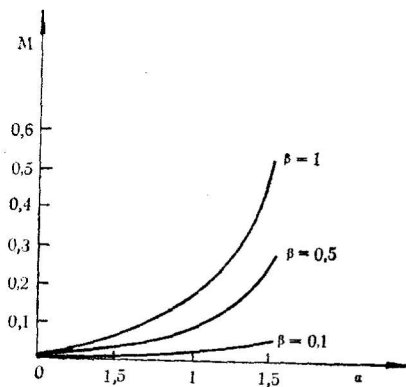


Fig. 8. Dependence of the torque on the acceleration parameter α at $T_1=0.05$ and various β

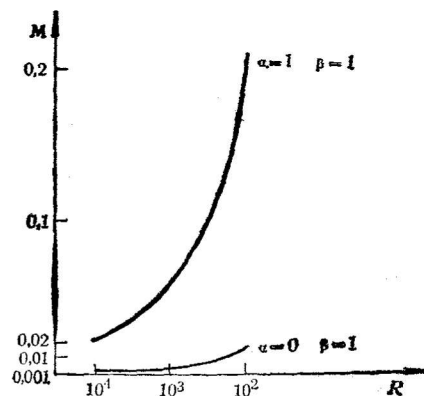


Fig. 9. Dependence of the torque on the Reynolds number R at $T_1=0.1$ and various α and β

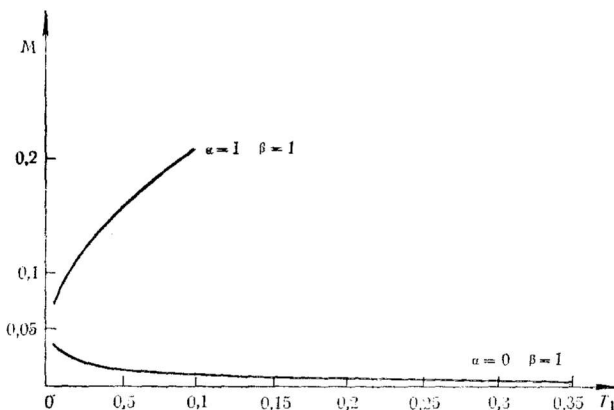


Fig. 10. Dependence of the torque on the time T_1 at $R=100$ and various α and β

8. Conclusion

The analytic solution to the full Navier-Stokes equations for the flow field on a body started from rest is at present beyond our capabilities. That is why we choose an iterative procedure. The method of inner and outer expansions seems most suitable in this case.

The boundary layer growth on a rotating circular cylinder in a viscous infinite fluid is studied analytically for large Reynolds and Strouhal numbers and small times. The rotation of the cylinder causes asymmetry in the flow pattern and, consequently, in the eddy formation pattern. The fluid viscosity as well as the flow acceleration influence the fluid detachment from the cylinder as it was envisaged. The dependence of the minimal time of boundary layer detachment on the rotation-to-the-flow-velocity ratio allows for boundary layer control.

References

1. Wang, C. Y. The flow past a circular cylinder which is started impulsively from rest, J. Math. Phys., 46, 195 (1967).
2. Van Dyke, M. Perturbation Methods in Fluid Mechanics, 1964
3. Slavchev, S. Boundary layer growth on a circular cylinder, Theor. and Appl. Mech., Year VI, No. 4, 49 (1975).
4. Zapryanov, Z. Boundary layer growth on a spinning sphere, ZAMM, 57, 41 (1977).

Received 5. 06. 1985