

Description of concrete within the framework of the kinematic theory of damages

P. Henryk

Concrete is a good example of a brittle material in which the development of microcracks and failure of structure are of greatest importance in both designing and predicting the longevity of the construction elements. A simultaneous deformation and failure of structure create numerous difficulties in constructing a formal model which would comprehensively describe real properties of concrete. Hence the attempts at describing those properties in terms of the theory of elasticity, the theory of plasticity, the mechanics of damages, or through the mechanics of a body with internal parameters. It seems, however, that the problem of a formal model for concrete remains still unsolved. In the present paper an attempt has been made to build a linear (in the geometric sense) model within the framework of the kinematic theory of damages (I).

1. An outline of a non-linear model

Splitting of the real motion into two fictitious stages (Fig. 1) has been a basis for the discussion of a non-linear model of a body with damages which are being propagated in the process of deformation. The first stage constitutes the cause of the process in which the body is submitted to virgin deformation (a lack of damage) $B_r \rightarrow B_i^*$. The reaction of the material to the virgin state of strain (it has been submitted to) is demonstrated, on the one hand, by the stresses, a so-called "dynamic response" and, on the other hand, by cracking, the effect of which is a loosening of the structure, viz. "a structural response"; in other words an additional motion denoting the second stage $B_i^* \rightarrow B_i$. It is essential that the second stage takes place without an increase of stresses (Fig. 1), where $F = \Phi \cdot F_*$ while F_* is gradient of virgin deformation, and F — gradient of real deformation, and Φ — gradient of structure loosening phase. Two tensors of stresses are introduced: T_* which characterized internal forces per unit of virgin surface da_* and T which describes the same forces but per unit of real surface da . Therefore the condition

$$(1.1) \quad T da = T_* da_* \rightarrow T = J_\Phi^{-1} \Phi^T T_*$$

must be fulfilled.

An analysis of the change of a surface element resulting from loosening of the structure allows to introduce a tensor object characterizing the state of damages in the material

$$(1.2) \quad \Psi^{dj} = 1 - J_\Phi^{-1} \Phi^T T.$$

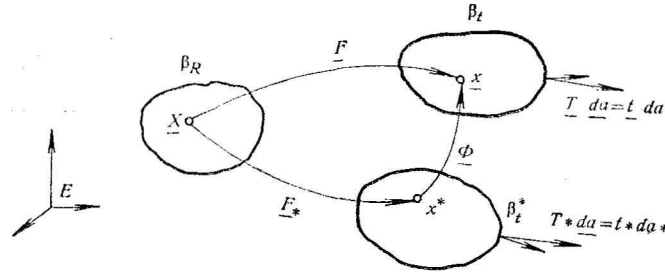


Fig. 1

If the linear model is considered geometrically then

$$(1.3) \quad H = H_* + H_\phi,$$

where H , H_* and H_ϕ are gradients of the particular phases of deformation. The geometric relations will then take the form

$$(1.4a) \quad 2\varepsilon = H + H^T = 2\varepsilon_* + 2\varepsilon_\phi,$$

where

$$(1.4b) \quad 2\varepsilon_* = H_* + H_*^T, \quad 2\varepsilon_\phi = H_\phi + H_\phi^T.$$

Taking into account that

$$(1.5) \quad J_* \cong 1 + \text{tr} H_*, \quad J_\phi \cong 1 + \text{tr} H_\phi,$$

the gradient of deformations can be determined in the equation

$$(1.6) \quad \Psi = \text{tr} H_\phi 1 - H_\phi^T.$$

An object thus introduced characterizes the size as well as the direction (including the rotation) of a relative change of the surface vector due to cracking. Let us adopt the tensors of damages defined through symmetrical part Ψ as a measure of the state of damages in a body

$$(1.7) \quad \Omega = 0.5(\Psi + \Psi^T) = \text{tr} H_\phi 1 - 0.5(H_\phi + H_\phi^T) = \text{tr} \varepsilon_\phi 1 - \varepsilon_\phi.$$

The antisymmetrical part characterizes the rotation resulting from the development of damages

$$(1.8) \quad \omega_\phi = 0.5(\Psi - \Psi^T) = 0.5(H_\phi - H_\phi^T).$$

According to the assumed principles of linearization of the state of deformation, the differences between the tensors of stresses disappear $T = T_* = \sigma$.

Thus the discussed approximation results in a geometrically linear model of a body which means that the geometrical relations are independent from each other. Naturally, ε_* and ε_ϕ continue to be physically dependent objects through the brittle properties of the body, that is a constitutive equation defining a structural response of material.

2. Constitutive relations

A definition of material in the proposed model must comprise not only the physical relations describing properties of a body from the point of view of deformation, but it should also contain the relations characterizing the process of appearing and growing of the damages.

Some experiments indicate that cracking is an anisotropic process in which the privileged directions are determined by both the state of deformation and structure of the body. It is convenient to express them as versors of the main (virgin) directions

of the strain state. The directions thus chosen describe the effect mentioned above and, additionally, define the material anisotropy (the privileged directions in the material can be expressed through the main directions ε_*).

The behaviour of concrete is different when under loading from that under unloading; moreover, the property intensifies at each next cycle of loading-unloading. There also exists a strong correlation between the response of concrete and the mode of loading. At a constant velocity of displacements there appears a non-stable phase (a decrease of stresses at an increase of strains) which is not discernible during loading at a constant velocity of stresses. Thus it follows that because of its brittle properties concrete possesses a memory. Let us then assume that "a structural response" of the concrete is determined by rate type equations with the versors of the main directions ε_* among the arguments. At this point equations of the first degree are the simplest and the most convenient for an experimental verification. If moreover we assume that in the model under consideration concrete is a homogenous material then the physical relations to be looked for in the form of functional

$$(2.1) \quad F_1[\dot{\Omega}, \Omega, \dot{\varepsilon}_*, \varepsilon_*, n^i] = 0,$$

where n^i are main directions of ε_* and $i=1, 2, 3$.

When analysing properties of the material it is convenient to have among the arguments also some objects characterizing the strains in the real motion $\varepsilon, \dot{\varepsilon}$. Applying the geometrical relations (1.4) and especially the functional (2.1), it is possible to write

$$(2.2) \quad F_2[\dot{\Omega}, \Omega, \dot{\varepsilon}_*, \varepsilon_*, \dot{\varepsilon}, \varepsilon, n^i] = 0.$$

Let us further assume that the equations (2.2) can be solved for the velocity of the tensor of damage $\dot{\Omega}$ and that the velocity depends on the state of damages Ω only through the state of strains, both virgin and real

$$(2.3) \quad \dot{\Omega} = F[\dot{\varepsilon}_*, \varepsilon_*, \dot{\varepsilon}, \varepsilon, n^i].$$

Because of the principle of the material objectivity, F must be an isotropic tensor function of its arguments. Representation for any function of this type is (2).

$$(2.4) \quad \dot{\Omega} = \alpha^k G_k,$$

where α^k are scalar functions of non-reducible set of invariants while the generators G_k are tensor objects consisting of a combination of arguments (2.3).

The received form of the constitutive relation defines the brittle properties of concrete only in a very general sense. Concrete is such a material for which the destruction of the structure is of decohesive type. The growth of the microsurface takes place in the main directions of the strain state. Among various possibilities inherent in the equations (2.4) this property is described by the equation

$$(2.5) \quad \dot{\Omega} = \sum_{k=1}^3 f_1^k[\dot{\varepsilon}^j, \varepsilon^j, \dot{\varepsilon}_*^j, \varepsilon_*^j] 1 + f_2^k[\dot{\varepsilon}^j, \varepsilon^j, \dot{\varepsilon}_*^j, \varepsilon_*^j] n^k \otimes n^k,$$

where f_1^k, f_2^k are functions expressing anisotropy of both the structure and the process. If we next assume that concrete is an isotropic material then $f_1^k = f_1$ and $f_2^k = f_2$ for $k=1, 2, 3$.

An experimental study carried out on concrete elements under tension and compression indicate to different effects of the damage propagation in both cases, while positive main strains always have a decisive influence on the process. An observation of the morphology of microcracks occurring in the concrete under loading (3) shows that most of the microsurface is created in the direction of the positive strains but there also exist such ones which develop in other directions as well. In case of

tension they evoke the effect of loosening of the structure also in the directions of the negative strains. In order to describe the presented space effects, functions f_1 and f_2 have been modified by means of Heaviside's function while the equations (2.5) have taken the form

$$(2.6) \quad \dot{\Omega} = \sum_{k=1}^3 g_1^* \kappa(\varepsilon_*^k) [1 - n_*^k \otimes n_*^k] + g_2^* \kappa(\varepsilon_*^k) n_*^k \otimes n_*^k - m^* g_3^* \tilde{\kappa}(-\varepsilon_*^k) n_*^k \otimes n_*^k + g_4^* \kappa(-\varepsilon_*^k) [1 - n_*^k \otimes n_*^k],$$

where $m = \kappa \left[\sum_{i=1}^3 \kappa(\varepsilon_*^i) \right]$, ε_*^k = eigenvalues ε_* , functions g_l for $l=1, 2, 3, 4$ have been assumed in the form

$$(2.7) \quad g_l = f_l(\varepsilon^k) \cdot g_l(\dot{\varepsilon}^k) \varepsilon_*^k + f_l'(\varepsilon^k) \cdot g_l'(\dot{\varepsilon}^k) \dot{\varepsilon}_*^k$$

k — is an indicator corresponding with the indicator in equation (2.6) and $\varepsilon^k, \dot{\varepsilon}^k$ — eigenvalues $\dot{\varepsilon}, \varepsilon$.

In order to facilitate an analysis of the material thus defined let us omit some less significant effects represented by functions g_2^*, g_4^* and let us consider an attempt to submit a concrete element to tension and compression. For tension we shall have

$$(2.8) \quad \dot{\omega}_1 = \dot{\omega}_2 = f_1(\varepsilon^3) g_1(\dot{\varepsilon}^3) \varepsilon_*^3 + f_1'(\varepsilon^3) g_1'(\dot{\varepsilon}^3) \dot{\varepsilon}_*^3 - f_3(\varepsilon^1) g_3(\dot{\varepsilon}^1) \varepsilon_*^1 - f_3'(\varepsilon^1) g_3'(\dot{\varepsilon}^1) \dot{\varepsilon}_*^1,$$

$\dot{\omega}_3 = 0, \dot{\omega}_i$ — eigenvalues $\dot{\Omega}$,

while for compression

$$(2.9) \quad \begin{aligned} \dot{\omega}_1 = \dot{\omega}_2 &= f_1(\varepsilon^1) g_1(\dot{\varepsilon}^1) \varepsilon_*^1 + f_1'(\varepsilon^1) g_1'(\dot{\varepsilon}^1) \dot{\varepsilon}_*^1 \\ \dot{\omega}_3 &= 2[f_1(\varepsilon^1) g_1(\dot{\varepsilon}^1) \varepsilon_*^1 + f_1'(\varepsilon^1) g_1'(\dot{\varepsilon}^1) \dot{\varepsilon}_*^1] \\ &\quad - f_3(\varepsilon^3) g_3(\dot{\varepsilon}^3) \varepsilon_*^3 - f_3'(\varepsilon^3) g_3'(\dot{\varepsilon}^3) \dot{\varepsilon}_*^3. \end{aligned}$$

Let us assume that concrete is an elastic — brittle material for which the dynamic response is defined by Hook's relations. Let the element undergo compression at a constant velocity $\varepsilon^3 = -vt$. On the grounds of (1.4a), (1.7), (2.9) we shall obtain

$$(2.10) \quad \begin{aligned} f_3(-vt) g_3(-v) \varepsilon_*^3 + f_3'(-vt) g_3'(-v) \dot{\varepsilon}_*^3 + 2\dot{\varepsilon}_*^3 &= -2v \\ f_1(\varepsilon^1) g_1(\dot{\varepsilon}^1) \varepsilon_*^1 + f_1'(\varepsilon^1) g_1'(\dot{\varepsilon}^1) \dot{\varepsilon}_*^1 - 2\dot{\varepsilon}_*^3 + \dot{\varepsilon}_*^1 &= \varepsilon^1. \end{aligned}$$

The first of the equations permits to determine ε_*^3 , and, allowing for $\sigma^3 = \varepsilon_*^3 E$, it helps to find curve $\sigma^3 = \varepsilon^3$. The second equation determines real strains ε^1 . In case of tension $\varepsilon^3 = vt$ virgin strains ε_*^3 ought to be determined from the equation

$$(2.11) \quad f_1(vt) g(v) \varepsilon_*^3 + f_1'(vt) g'(v) \dot{\varepsilon}_*^3 + \varepsilon_*^3 = v - R(\varepsilon_*^3, \dot{\varepsilon}_*^3),$$

where

$$R(\varepsilon_*^3, \dot{\varepsilon}_*^3) = v \varepsilon_*^3 f_3(-v \varepsilon_*^3) g_3(-v \dot{\varepsilon}_*^3) + v \dot{\varepsilon}_*^3 f_3'(-v \varepsilon_*^3) g_3'(-v \dot{\varepsilon}_*^3).$$

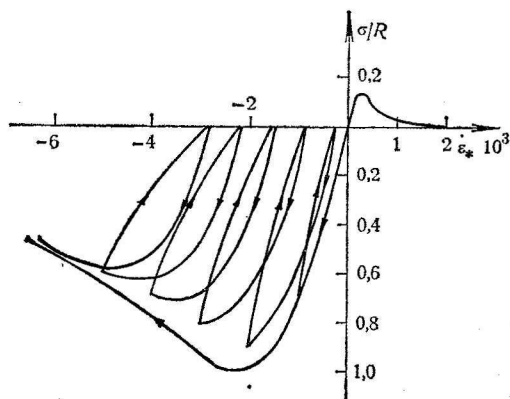


Fig. 2

$$f_1(\varepsilon) = \varepsilon^{1.3}$$

$$p_1(\varepsilon) = 3 \cdot 10^3 \text{ for } \varepsilon > 0$$

$$g_1'(\varepsilon) = \kappa(-\varepsilon)$$

$$f_3(\varepsilon) = \varepsilon^{0.75}$$

$$p_3(\varepsilon) = \begin{cases} 64 \cdot 10^4 & \text{for } \varepsilon > 0 \\ -16,1 \cdot 10^4 & \text{for } \varepsilon < 0 \end{cases}$$

$$f_3'(\varepsilon) = (800 \varepsilon)^2 + 800 \varepsilon$$

$$g_3'(\varepsilon) = \kappa(\varepsilon)$$

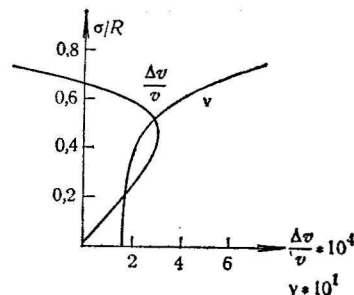


Fig. 3

According to (2.8) there will be $\varepsilon^1 = \varepsilon^1 = +\nu \varepsilon^3$ for perpendicular strains (ν — Poisson's ratio). Some studies indicate that in a certain range of velocity (short times of loading) it can be assumed that concrete is not sensitive to velocity. Such an effect can be obtained if it is assumed that

$$(2.12) \quad g_k(\varepsilon^i) = \varepsilon^i p_k(\varepsilon^i), \quad g_k', \quad p_k = \text{const.}$$

In reality g_k' and g_k are any positive functions and in order to be able to describe important properties, viz. the loop in the loading — unloading process, g_k' and p_k ought to be approximated by constant values, different for negative and positive arguments (see Fig. 2). Other functions have been taken to be root functions. Material constants occurring in those functions have been chosen with the help of other studies (4; 5). In Fig. 2 there have been presented curves $\sigma^3 \sim \varepsilon^3$ for the discussed model; under both compression and tension the non-stable phase is visible. For compression, moreover, the process of repeated loading and unloading has been shown. The diagrams (Fig. 3), illustrating the change of both real dilatation and Poisson's coefficient, confirm the experiments as well.

The presented results constitute a particular case of the possibilities the model yields for description of a number of properties of concrete; it appears particularly useful under conditions of long-lasting processes for an analysis of the longevity of constructions.

References

1. Grabacki, J. XXV th — Polish Solid Mech. Conf. (1984).
2. Wang, C. C. Part 1,2. Arch. Rat. Mech. Anal., 3, 36 (1970).
3. Stroeve, P. Ph. D. Thesis, Delft, 1973.
4. Kowalczyk, R., W. Dilger. IPPT PAN 44/1971, Warszawa.
5. Hughes, B. P., G. P. Chapman, RILEM Bull, 30 (1966).

Received 25. XI. 1985