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FLUID MECHANICS

MAGNETO SOUND IN PLASMA AND NONLINEAR OPTICS ON MODIFIED TRUNCATED EXPANSION METHODS FOR THE ANALYTICAL SOLITON OF FIFTH ORDER (1+1) DIMENSIONAL TRIPLE NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS IN MATHEMATICAL PHYSICS AND MECHANICS

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ABSTRACT: In this manuscript, two modified truncated expansion methods (i.e MTEM-I and MTEM-II) are built up to obtain analytical solution (i.e so called traveling wave solutions) for triple (1+1) dimensional nonlinear partial differential equations (NLPDEs)such as Sawada-Kotera equation (SKE), generalized Korteweg-de Vries equation (GKdVE) and Kaup-Kuperschmidt equation (KKE), which have been widely used in mathematical physics. The present topic minimizes the complex nature and non-integrable characteristics to obtain solution of NLPDEs. To demonstrate the influence of the parameters, 3D plots are surfed for triple NLPDEs. certification.

KEY WORDS: Modified truncated expansion methods, Triple nonlinear partial differential equations, Mathematical Physics, Magneto Sound in Plasma, Nonlinear optics.

1 INTRODUCTION

NLPDEs are highly helpful in a variety of domains, including fluid dynamics, water surface gravity waves, electromagnetic radiations, and ion acoustic waves in plasma. Several techniques are adopted to search the analytical solitons to NLPDEs including methods as found in [1–4]. The propagation of waves and sound in flat surface such as capillary-gravity and shallow water and magneto sound in plasma [5] are based on modelling of GKdVE, as described in equation (1.2) in Mathematical Physics respectively. The integrable fifth-order KdVE and the necessary conditions for the nonlinearity and dispersion parameters are investigated by Hirota method [6]. Later [2] invented stationary wave solutions of binary waves for fifth-order KdVE as results of Kudryashov-expansion and sine–cosine function techniques. In order to obtain

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two kinds of approximate solutions a simple standard truncated expansion approach (SSTEA) has been suggested for fifth-order GKdVE [7]. The Bäcklund transformation by new GKdVE is elaborated in [8] to extract the bilinear form as well as structural wave phenomenon of the governing model on account of gravity field in shallow water to carry out the propagation of long waves. The exponential stability results by suitable, Lyapunov functionals on KdVE with time-dependent delay on the boundary is very much attracted in [9]. The suitable transformation is taken in [10] to reduce the GKdVE to a quadratic ordinary differential equation by implementing the new version trial equation approach for finding new solitary wave solutions.To estimate the movement of water waves the sech method is established to explore novel analytical traveling wave solutions for a nonlinear KdV in [11]. The exp-function and modified exp-function methods are implemented in [12] to explore solitary, peakon, periodic, cuspon and kink wave solution of KdVE. Authors [13] applied the natural decomposition method for solving the time-fractional coupled KdVE. Numerical soliton KdVE in infinite dimension using steepest descent method is found in [14]. The numerical solution of KdVE by [15] is based on trapezoidal and implicit midpoint method via Pade technique.

The various mathematical physics concepts like quantum, optics and travelling wave solution in shallow water are mainly focused on modelling of SKE. The Hirota bilinear techniques [16, 17] and modified auxiliary equation technique (MAET) and the extended direct algebraic technique (EDAT)in [18] are adopted for different analytical solitary solitons of fifth order SKE. Further, Hirota direct method [19] is investigated to exploit the soliton solutions like, single soliton, double soliton and triple soliton solutions of the integral SKE. The independent transformation by [20] is implemented for multidimensional KdV-Sawada-Kotera-Ramani equation (KdVSKRE). The Exp-function technique has been offered to find analytical solutions of generalized SKE in [21]. The tanh- coth method is imposed on SKE to achieve explicit exact soliton [22]. Yang transform (YT) with the Adomian decomposition method (ADM) and homotopy perturbation technique (HPT) are used on the seventh-order time-fractional Sawada–Kotera–Ito problem (TFSKIP) to obtain the solution in [23].

The KKE is implemented in physical sciences in different approaches like plasma physics, nonlinear optics and fluid dynamics. The analytical solution of traveling wave of KKE is found by using the Fan sub-equation method [24]. The nonlinear evolution equation particularly time fractional KKE is solved in [25] to obtain new general forms of analytical solutions by employing improved and extended G'/G technique that would applicable in mathematical modeling of nonlinear form. The analytical solution of KKE is extracted by implementing double $(G'/G,\ 1/G)$ expansion approach in [26]. The authors in [27] have shown their interest towards

homotopy perturbation transform method (HPTM) and Yang transform decomposition method (YTDM), to solve the fractional nonlinear seventh-order KKE through Caputo operator. The homotopy method is addressed in [28, 29] to investigate the numerical solution of KKE. Further, the one method is synthesized in [30, 31] to get analytical soliton of (1+1)-Dimensional KKE. In addition to these, several computational approaches may be employed to approximate soliton solutions, including the modified auxiliary function approach [32], the natural decomposition method with Atangana–Baleanu derivative in Caputo manner and Caputo–Fabrizio [33]. There are several methods [34]–[53] are very much associated with this study.

As fifth order NLPDEs are host the contributor towards genuine properties in several scientific and technical applications with lot of physical characteristics [1–6], motivated by these in this manuscript we have employed two types of MTEM (i.e MTEM-I and MTEM-II) to obtain the analytical solutions of the three fifth order NLPDEs in mathematical Physics. This encourages us to study the analytical solution by implementing MATHEMATICA software.

The fifth order (1+1) dimensional SKE is represented in equation (1)

$$u_t + 45u^2u_x + 15u_xu_{xx} + 15uu_{xxx} + u_{xxxxx} = 0\,. \qquad \text{(refer [29])} \tag{1}$$

Generalized fifth order Kdv equation (GKdvE) is:

$$u_t + \mu u_{xxx} + 6uu_x + vu_{xxxxx} = 0\,. \qquad \text{(refer [7])} \tag{2}$$

The fifth order Kaup-Kuperschimdt equation (KKE) is:

$$-u_t + 5u^2u_x + \frac{25}{2}u_xu_{xx} + 5uu_{xxx} + u_{xxxxx} = 0\,. \qquad \text{(refer [24])} \tag{3}$$

The plan lay out of this paper is attracted in different sections. Section 2 bears theoretical illustrations of MTEM-I. Sections 3 and 4 tackle the application of MTEM-I for SKE and KdVE respectively. The METM-II approach is suggested in Section 5 and implementation of this technique towards KKE is reflected in Section 6. The concluding remarks are appended in Section 7.

2 Modified Truncation Expansion Method-I (MTEM-I)

Let us take METM-I approach for the following NLPDE as mentioned below

$$H\left(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots\right) = 0\,, \tag{4}$$

where H is a polynomial in unknown function $u(x,t)$ with its nonlinear terms and higher order partial derivatives.

The traveling wave variable is represented in equation (1) as:

$$u\,(x,t) = y(r) = y\,(kx + wt)\ , \tag{5}$$

where, k and w are are the wave length and frequency permit us to reduce equation (4) to an ODE as given in equation (6) as mentioned below:

$$Q(y,\, y',\, y''',\, y^{(\mathrm{v})}\,, \ldots) = 0\,. \tag{6}$$

(a) With the placement of equation (7) in equation (6), we can able to collect the singularity of dominant terms

$$y\,(r) = r^{-m}\,. \tag{7}$$

We select at least two terms with the lowest degree among all terms in equation (6) and the highest value of m is the pole and represented by integer M. If M is a fraction or a negative integer, we obey the following transformation:

Associating the transformations $y(r) = \varphi^{v/s}(r)$ and $y(r) = \varphi^{M}(r)$ when M is fraction and negative integer respectively and reselect M again from the new equation.

(b) Let us take the analytical solution of equation (6) in the form

$$y(r) = a_0 + a_1 Q(r) + a_2 Q^2(r) + \cdots + a_n Q^n(r)\,, \tag{8}$$

where $r = kx + wt$ and a_i $(i = 1, 2, \ldots, n)$ are n constant to be calculated and

$$Q(r) = (1 + e^r)^{-1}\,. \tag{9}$$

(c) Applying MATHEMATICA, we find first, second and higher order derivatives of $y(r)$ as follows:

$$\begin{aligned} y(r) &= a_0 + a_1 Q(r) + a_2 Q^2(r) + \cdots + a_n Q^n(r)\,,\\ y'(r) &= a_1 Q(r) + (a_1 + 2a_2) Q^2(r) + 2a_2 Q^3(r)\,,\\ y''(r) &= a_1 Q(r) + (3a_1 + 4a_2) Q^2(r) + (2a_1 + 10a_2) Q^3(r) + 6a_2 Q^4(r)\,. \end{aligned} \tag{10}$$

(d) Equation (4) is modified by substituting equations (8)–(10). Then, using the function $Q(r)$, we gather all terms with the same power and set the expressions equal to zero. As a consequence, simplifying the system of equations the unknown parameters are determined This approach is easily adaptable to polynomial differential equations of any order.

Based on this MTEM-I, the analytical solutions of SKE and KdV equations are obtained in Section 3

3 Application of MTEM-I for Sawada-Kotera Equation

The modified truncated expansion method (MTEM-I) for the two variables in general form of NLPDE can be represented by

$$H\left(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots\right) = 0\,.$$

In general form, let $u\left(x,t\right) = y\left(kx + wt\right)$, it can be converted into non-linear ordinary differential equation (ODE).

$$wy' + 45ky^2y' + 15k^3y'y'' + k^5y^{(v)} = 0\,. \tag{11}$$

Let us take

$$y(z) = a_0 + a_1q + a_2q^2\,. \tag{12}$$

Associating equation (12) in equation (11) and comparing the coefficients of Q^i, where $i = 1, 2, \ldots, 7$, the following algebraic equations (13)–(19) are obtained:

$$-wa_1 - 45ka_0^2a_1 - 15k^3a_0a_1 - k^5a_1 = 0\,, \tag{13}$$

$$\begin{aligned} w(a_1 - 2a_2) + 45ka_0^2(a_1 - 2a_2) - 90ka_0a_1^2 - 30k^3a_1^2 \\ + 15k^3a_0(7a_1 - 8a_2) + k^5(31a_1 - 32a_2) = 0\,, \end{aligned} \tag{14}$$

$$\begin{aligned} 2wa_2 + 90ka_0^2a_2 + 90ka_0a_1(a_1 - 2a_2) - 45ka_1(a_1^2 + 2a_0a_2) \\ - 15k^3a_1(4a_2 - 3a_1) + 15k^3a_1(a_1 - 2a_2) + 15k^3a_0(38a_2 - 12a_1) \\ + 15k^3a_1(7a_1 - 8a_2) - 15k^3a_1a_2 + k^5(422a_2 - 180a_1) = 0\,, \end{aligned} \tag{15}$$

$$\begin{aligned} 180ka_0a_1a_2 + 45k(a_1^2 + 2a_0a_2)(a_1 - 2a_2) - 90ka_1^2a_2 \\ - 15k^3a_1(2a_1 - 10a_2) + 15k^3(a_1 - 2a_2)(4a_2 - 3a_1) + 30k^3a_2a_1 \\ + 15k^3a_0(6a_1 - 54a_2) + 15k^3a_1(38a_2 - 12a_1) \\ + 15k^3a_2(7a_1 - 8a_2) + k^5(390a_1 - 1710a_2) = 0\,, \end{aligned} \tag{16}$$

$$\begin{aligned} 90ka_2(a_1^2 + 2a_0a_2) + 90ka_1a_2(a_1 - 2a_2) - 45ka_1a_2^2 - 90k^3a_1a_2 \\ + 15k^3(a_1 - 2a_2)(2a_1 - 10a_2) + 30k^3a_2(4a_2 - 3a_1) \\ + 360k^3a_0a_2 + 15k^3a_1(6a_1 - 54a_2) + 15k^3a_2(38a_2 - 12a_1) \\ + k^5(3000a_2 - 360a_1) = 0\,, \end{aligned} \tag{17}$$

$$\begin{aligned} 180ka_1a_2^2 + 45ka_2^2(a_1 - 2a_2) + 90k^3a_2(a_1 - 2a_2) + 30k^3a_2(2a_1 - 10a_2) \\ + 360k^3a_1a_2 + 15k^3a_2(6a_1 - 54a_2) + k^5(120a_1 - 2400a_2) = 0\,, \end{aligned} \tag{18}$$

$$90ka_2^3 + 540k^3a_2^2 + 720k^5a_2 = 0\,. \tag{19}$$

With the help of MATHEMATICA, the set of equations (13)–(19) are solved and the following different cases arise.

CASE 1:

$$a_0 = \frac{k^2}{3}\,, \quad a_1 = 4k^2\,, \quad a_2 = -4k^2\,, \quad w = -k^5\,,$$

where $r = (kx + wt) = k(x - k^4 t)$

$$y(r) = \frac{k^2}{3}(1 + 12q - 12q^2)\,,$$

where $q = (1 + e^r)^{-1}$.

Taking $k = 1.5$, the 3D graph is shown in Fig. 1.

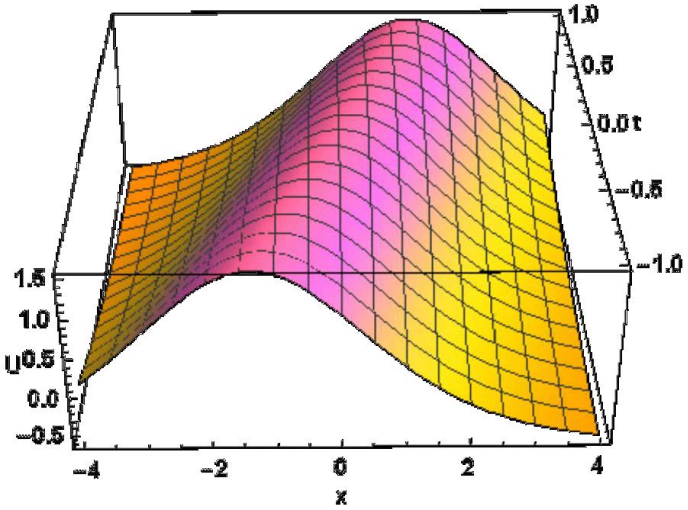

Fig. 1. Analytical solution representation of Sawada-Kotera for Case 1 with $-4 \leq x \leq 4$ and $-1 \leq t \leq 1$.

CASE 2:

$$a_0 = k^2\,, \quad a_1 = 2k^2\,, \quad a_2 = -2k^2\,, \quad w = -61k^5\,,$$

where $r = (kx + wt)$ and

$$y\,(r) = k^2\left(1 + 2q - 2q^2\right),$$

where $q = 1 + e^{r-1}$.

Taking $k = 0.5$, the 3D graph is shown in Fig. 2

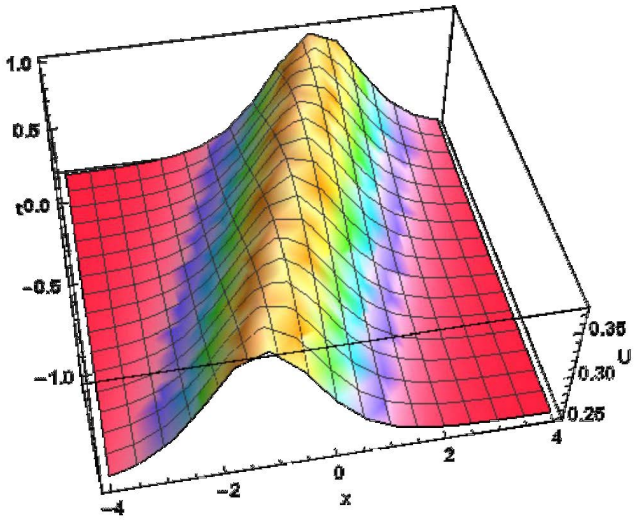

Fig. 2. Analytical solution representation of Sawada-Kotera for Case 2 with $-4 \leq x \leq 4$ and $-1 \leq t \leq 1$.

CASE 3:

$$a_1 = 2k^2\,, \quad a_2 = -2k^2\,, \quad a_0 = 1\,, \quad w = -\left(k^5 + 15k^3a_0 + 45ka_0^2\right),$$
$$r = (kx + wt) = kx - (k^5 + 15k^3a_0 + 45ka_0^2)t \quad \text{and} \quad y(r) = 1 + 2k^2(q - q^2)$$

where $q = 1 + e^{r-1}$.

Taking $k = 0.5$, the 3D graph for analytical solution is shown in Fig. 3

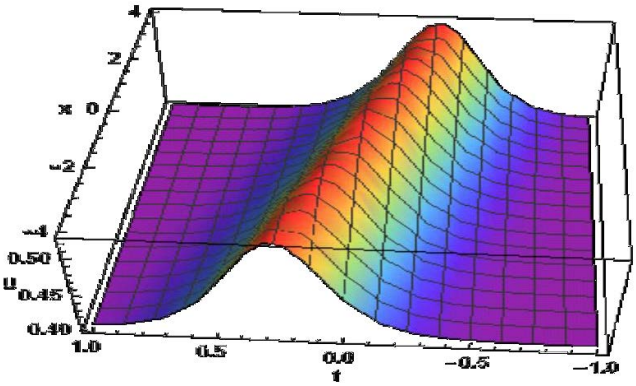

Fig. 3. Analytical solution representation of Sawada-Kotera for Case 3 with $-4 \leq x \leq 4$ and $-1 \leq t \leq 1$.

4 Application of MTEM-I to Fifth Order Kdv Equation

In general form, let $u(x,t) = y(kx + wt)$, where $w \neq 0$ and equation (2) takes the nonlinear differential equation as given in equation (20) as below:

$$wy' + \mu k^3 y''' + 6kyy' + vk^5 y^{(v)} = 0\,. \tag{20}$$

Let us take

$$y\,(r) = a_0 + a_1 Q + a_2 Q^2\,. \tag{21}$$

Associating equation (21) in equation (20) and comparing the coefficients of Q^i, where $i = 1, 2, \ldots, 7$ the following algebraic equations are obtained:

$$\begin{aligned}
Q &: -wa_1 - \mu k^3 a_1 - 6ka_0 a_1 - vk^5 a_1 = 0\,,\\
Q^2 &: w(a_1 - 2a_2) + \mu k^3(7a_1 - 8a_2) + 6ka_0(a_1 - 2a_2) - 6ka_1^2\\
&\quad + vk^5(31a_1 - 32a_2) = 0\,,\\
Q^3 &: 2wa_2 + \mu k^3(38a_2 - 12a_1) + 12ka_0 a_2 + 6ka_1(a_1 - 2a_2) - 6ka_1 a_2\\
&\quad + vk^5(422a_2 - 180a_1) = 0\,,\\
Q^4 &: \mu k^3(6a_1 - 54a_2) + 12ka_1 a_2 + 6ka_2(a_1 - 2a_2) + vk^5(390a_1 - 1710a_2)\,,\\
Q^5 &: vk^5(3000a_2 - 360a_1) = 0\,,\\
Q^6 &: vk^5(120a_1 - 2400a_2) = 0\,,\\
Q^7 &: 720vk^5 a_2 = 0\,.
\end{aligned}$$

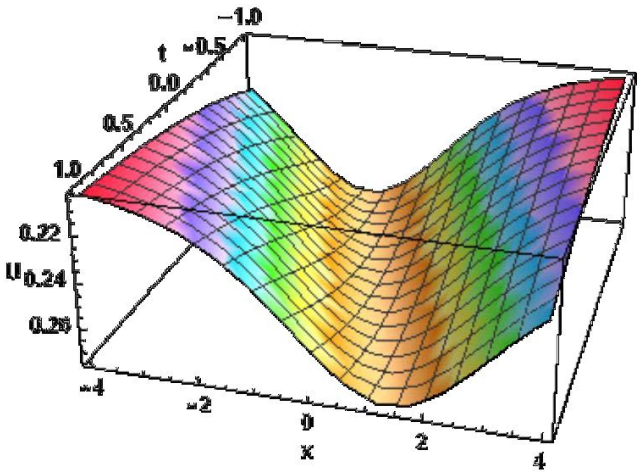

Fig. 4. Analytical solution representation of Fifth Order Kdv equation with $-4 \leq x \leq 4$ and $-1 \leq t \leq 1$.

CASE 1:

$$a_2 \to -a_1\,, \quad w \to \frac{1}{2}k\,(12a_0 + a_1)\,, \quad \mu \to \frac{a_1}{2k^2}\,, \quad v \to 0\,,$$
$$a_0 = 0.2\,, \quad a_1 = 0.3$$
$$r = kx - wt = kx - \frac{kt}{2}\,(12a_0 + a_1)\,,$$
$$y\,(r) = a_0 + a_1\left(q - q^2\right),$$

where $q = 1 + e^{r-1}$.

Taking $k = 0.8$, the 3D graph is shown in Fig. 4.

5 TYPE-II MODIFIED TRUNCATION EXPANSION METHOD-II (MTEM-II)

Let us suggest METM-II approach for the following NLPDE as given below:

$$H\,(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) = 0\,. \tag{22}$$

Here $u(xt)$ with its nonlinear terms and higher order derivatives are embedded in H.

The traveling wave variable is represented in equation (3) as:

$$u\,(x,t) = y(r) = y\,(kx - wt)\,, \tag{23}$$

where k and w are the wave length and frequency permit us to reduce equation (22) to an ODE as given in equation (24)

$$Q\left(y, y', y''', y^{(\mathrm{v})}, \ldots\right) = 0\,. \tag{24}$$

(a) With the placement of equation (25) in equation (24), we can able to collect the singularity of dominant terms

$$y\,(r) = r^{-m}\,. \tag{25}$$

We select the at least two terms with the lowest degree among all terms in equation (24) and the highest value of m is the pole and represented by integer M. If M is a fraction or a negative integer, we obey the following transformation:

Associating the trans formations $y(r) = \varphi^{v/s}(r)$ and $y\,(r) = \varphi^{M}(r)$ when M is fraction and negative integer respectively and reselect M again from the new equation.

(b) Let us take the analytical solution of equation (24) in the form

$$y(r) = a_0 + a_1 Q(r) + a_2 Q^2(r) + \cdots a_n Q^n(r)\,, \tag{26}$$

where $r = kx - wt$ and a_i $(i = 1, 2, \ldots, n)$ are n constant to be calculated and

$$Q(r) = (1 + e^r)^{-1}. \tag{27}$$

(c) Applying MATHEMATICA, we find first, second and higher order derivatives of $y(r)$ as follows:

$$\begin{aligned} y(r) &= a_0 + a_1 Q(r) + a_2 Q^2(r) + \cdots + a_n Q^n(r), \\ y'(r) &= -a_1 Q(r) + (a_1 - 2a_2) Q^2(r) + 2a_2 Q^3(r), \\ y''(r) &= a_1 Q(r) + (4a_2 - 3a_1) Q^2(r) + (2a1 - 10a_2) Q^3(r) + 6a_2 Q^4(r). \end{aligned} \tag{28}$$

(d) Equation (20) is modified by substituting equations (26)–(28). Then, using the function Q(r), we gather all terms with the same power and set the expressions equal to zero. As a consequence, the evaluation of unknown parameters can be investigated by simplifying the system of equations. Solving this system yields the values of the unknown parameters. This approach is easily adaptable to polynomial differential equations of any order.

Based on this MTEM-II, the analytical solutions of KKE equations are obtained in Section 5.

6 Application MTEM-II to Kaup-Kupershimdt Equation

In this section, the procedure for MTEM-II in Section 5 is implemented for KKE which is represented in equation (3). Applying the procedure **(a)-(d)** of algorithm of Section 5 and using travelling wave variable $u(x,t) = y(r)$, where $r = kx - wt$ in equation (3), it can be converted into nonlinear ordinary differential equation (NLODE) of the form

$$wy'(z) + 5ky^2(z)y'(z) + \frac{25}{2}k^3y'(z)y''(z) + 5k^3y(z)y'''(z) + k^5y^{(\mathrm{v})}(z) = 0. \tag{29}$$

Taking on integration by parts and substitution method in equation (22), it takes the form

$$wy(z) + \frac{5}{3}ky^3(z) + \frac{15}{4}k^3y'(z)^2 + 5k^3y(z)y''(z) + k^5y^{(\mathrm{iv})}(z) + c_1 = 0. \tag{30}$$

Let us take

$$y(r) = a_0 + a_1 Q + a_2 Q^2. \tag{31}$$

Associating equation (31) in equation (30) and comparing the coefficients of Q^i, where $i = 1, 2, \ldots, 7$, the following algebraic equations are obtained:

$$a_0 w + \frac{5}{3}ka_0^3 + c_1 = 0,$$

$$
\begin{aligned}
&a_1 w + 5k a_0^2 a_1 + 5k^3 a_0 a_1 + k^5 a_1 = 0\,,\\
&a_2 w + 5k(a_0 a_1^2 + a_0^2 a_2) + \frac{15}{4}k^3 a_1^2 + 5k^3(4a_0 a_2 - 3a_0 a_1 + a_1^2)\\
&\qquad + k^5(16a_2 - 15a_1) = 0\,,\\
&\frac{5}{3}k(a_1^3 + 6a_0 a_1 a_2) - \frac{15}{4}k^3\left(2a_1^2 - 4a_1 a_2\right) + 5k^3(2a_0 a_1 - 10a_0 a_2\\
&\qquad + 5a_1 a_2 - 3a_1^2) + k^5\left(50a_1 - 130a_2\right) = 0\,,\\
&5k(a_1^2 a_2 + a_0 a_2^2) + \frac{15}{4}k^3\left((a_1 - a_2)^2 - 4a_1 a_2\right)\\
&\qquad + 5k^3(6a_0 a_2 + 2a_1^2 - 10a_1 a_2 + 4a_2^2 - 3a_1 a_2) + k^5(330a_2 - 60a_1) = 0\,,\\
&5k a_1 a_2^2 + 15k^3(a_1 a_2 - 2a_2^2) + 5k^3(8a_1 a_2 - 10a_2^2) + k^5(24a_1 - 336a_2) = 0\,,\\
&\frac{5}{3}k a_2^3 + 15k^3 a_2^2 + 30k^3 a_2^2 + 120k^5 a_2 = 0\,.
\end{aligned}
$$

CASE 1:

$$
a_0 = -\frac{k^2}{4}\,,\quad a_1 = 3k^2\,,\quad a_2 = -3k^2\,,\quad c_1 = \frac{k^7}{96}\,,\quad w = -\frac{k^5}{16}\,,
$$

$$
r = kx - wt = kx + \frac{k^5}{16}t\,,
$$

$$
y\,(z) = -\frac{k^2}{4} + 3k^2\left(Q - Q^2\right)\,,
$$

where $Q = (1 + e^r)^{-1}$.

Taking $k = 0.8$, the analytical solution of 3D is surfed and shown in Fig 5.

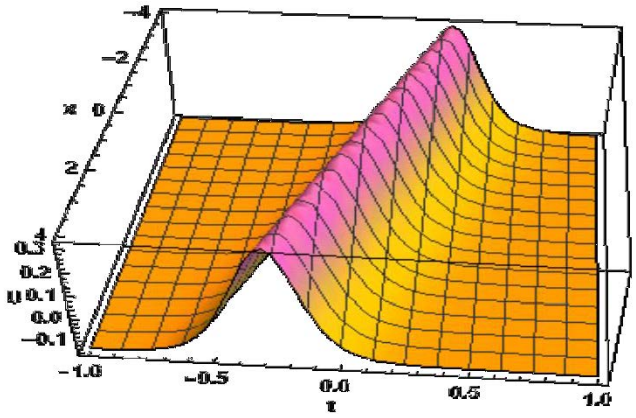

Fig. 5. Analytical solution of Kaup-Kupershmidt equation for Case 1 with $-4 \le x \le 4$ and $-1 \le t \le 1$.

CASE 2:

$$a_0 = -2k^2\,, \quad a_1 = 24k^2\,, \quad a_2 = -24k^2\,, \quad c_1 = -\frac{26}{3}k^7\,, \quad w = -11k^5\,,$$

$$z = kx + 11k^5 t\,,$$

$$y\,(r) = -2k^2 + 24k^2\left(q - q^2\right),$$

where, $q = 1 + e^{r-1}$

Taking $k = 1$, the analytical solution of 3D graph is shown in Fig. 6.

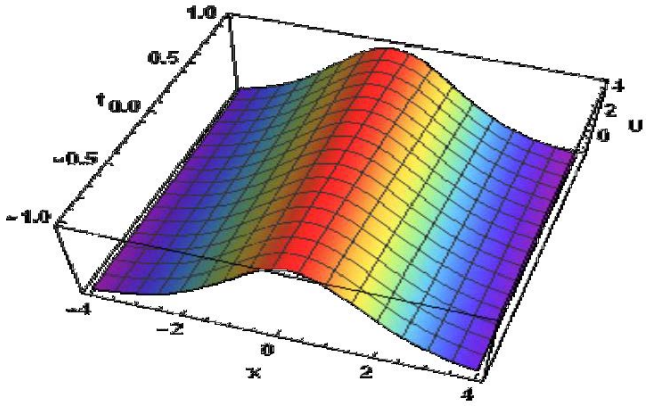

Fig. 6. Analytical solution representation of Kaup-Kupershmidt equation for Case 2 with $-4 \leq x \leq 4$ and $-1 \leq t \leq 1$..

7 CONCLUSIONS

In this manuscript, the SKE, KdvE and KKE are analyzed to obtain new analytical traveling wave solutions. The MTEM-1 and MTEM-1I techniques are employed to obtain several types of solutions to the problems under consideration. The analytical solutions of solitons are surfed on different justified parameters to strengthen the quality of present work. In future work, the analytical methods MTME-I and MTME- II in the present manuscript can be implemented to other types of NLPDEs for computing the solitons of a suitable model that would be applicable in Physic sciences.

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