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SOLID MECHANICS

FRICTION VARIATOR WITH MINIMAL SLIDING BETWEEN THE WORKING SURFACES WHEN ADJUSTING THE TRANSMISSION RATIO

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ABSTRACT: The subject of the present study is the synthesis of a friction Variator with theoretically zero geometric sliding between the working surfaces when changing the transmission ratio, i.e. the adjustment takes place with centroid rolling.

The working surfaces are obtained by rotating the centroid pairs around in axes, provided that their point of intersection always lies on the common tangent of the two contours. With the help of elements of differential geometry the firs order system equations have been derived and solved in an analytical form, whose integrals uniquely define the centroid pairs. The possibilities for changing the transmission ratio of the innovative kinematic and structural schemes of the friction Variator have been analyzed. Its practical applicability in Industry has been proved.

KEY WORDS: fricton variator, transmisson ratio, minimal siding.

1 INTRODUCTION

Step less transmission systems, which received citizenship with name variators, are very widespread in engineering practice thanks to recent great achievements in the field of new friction materials and innovative design solutions.

Step less speed convertors use different principles of motion transformation – input/output, such as mechanical [1–4], electro-mechanical [5], hydraulic [6], magnetic [7] etc. It is well known that any transformation of energy from one form to another is accompanied by losses. Therefore, the most promising are those in which the energy flow is uniform, for example – from mechanical to mechanical.

In modern industry (transport [8], textile, metallurgical etc.), the most widespread are the mechanical variators, conditionally divided intofriction [1, 3], impulse [9, 10] and those with continuous transformation of the movement [4] and [11].

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Friction variators are characterized by the simplicity of construction, high reliability, durability etc., but along with that – wear, and heat due to the presence of relative sliding between friction surfaces, well known as a geometrical sliding, during the change of the transmission ratio.

This problem has been solved to some extent with Mazaro variators [12], the pulse one [9], those with intermediate spheres [11] and the variator withIncrease of Capacity and minimum slippage during steering [13], as well as the toroidal variators [14].

The aim of the present study is to create a methodology for the synthesis of rotational friction surfaces and innovative structural scheme for variator with theoretical zero geometric sliding during gear ratio change. Achieving this effect significantly increases the quality indicators of this type of transmission mechanism.

By using elements of differential and analytics geometry [15, 16], as well as theory of noncircular gears [17], the contours of the centroid pair were obtained in an analytical form, which when rotated around selected axes form the working surfaces. The orientations of these axes are defined, where only the above described effect is possible, i.e. absence of sliding between the friction surfaces during gear ratio control.

2 Theoretical Consideration

Figure 1 shows the centroid mechanism consisting of units (1) and (2), and two rotary and one contour pair. Lets assume that the centers of rotation O_1 and O_2 coincide whit the of the coordinate systems $O_1x_1y_1$ and $O_2x_2y_2$ in which the centroid contours k_1 and k_2 are defined.

In order for contours to be centroid, it is necessary that the arcs describing the contact point in the own coordinate systems be equal, or

$$(A - r_1)d\theta_2 = -r_1 d\theta_1 \,. \tag{1}$$

To define the centroid contours, it is necessary to know the equations of the lines $g_1 \equiv O_1x_1$, $g_\tau \equiv \tau - \tau$ and $g_2 \equiv O_2x_2$ in the inertial coordinate system $O_1x_0y_0$ as well as differential dependencies directly following from Fig. 1, namely

$$\mu_2 = \pi - \mu_1 \,; \qquad \frac{dr_1\,(\theta_1)}{d\theta_1} = \frac{r_1\,(\theta_1)}{\tan\mu_1} \,; \qquad \frac{d\theta_2\,(\theta_1)}{d\theta_1} = -\frac{r_1\,(\theta_1)}{A - r_1\,(\theta_1)}.$$

The equations of the lines $g_1\,(x_1)$, $g_\tau\,(x)$ and $g_2\,(x)$ are represented by

$$\begin{aligned} g_1\,(x)\,y &= x\,\tan\theta_1 \,, \\ g_\tau\,(x)\,y &= r_1 - x\,\cot\mu_2 \,, \\ g_2\,(x)\,y &= A - x\,\tan\theta_2 \,, \end{aligned} \tag{2}$$

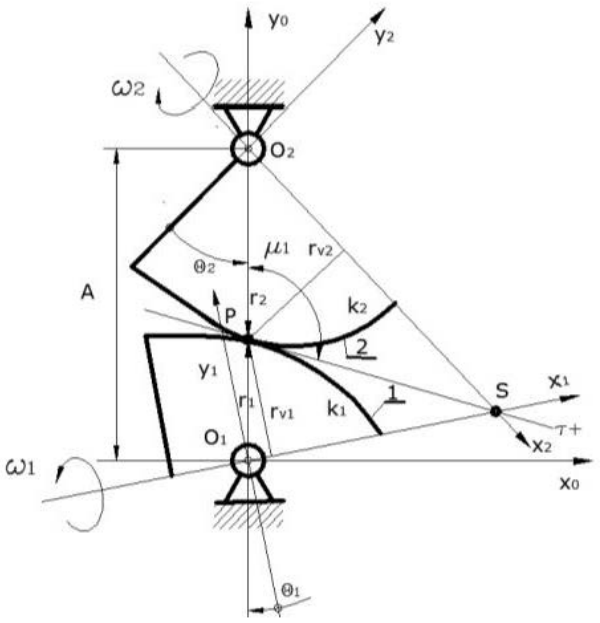


Fig. 1. Elemental centroid mechanism.

which lie in the plane $O_1x_0y_0$, or

$$\det \begin{pmatrix} -\tan\theta_1 & 1 & 0 \\ \cot\mu_2 & 1 & -r_1 \\ \tan\theta_2 & 1 & -A \end{pmatrix} = 0\,. \tag{3}$$

After successively solving (3) and matching with above-mentioned geometrical dependencies, we arrive at the following system of differential equations of the first order

$$\begin{aligned} &\{[A - r_1(\theta_1)]\tan\theta_1 - r_1(\theta_1)\tan\theta_2(\theta_1)\}\,d\theta_1 = A\frac{dr_1(\theta_1)}{r_1(\theta_1)}\,, \\ &[A - r_1(\theta_1)]\,d\theta_2(\theta_1) = -r_1(\theta_1)d\theta_1\,, \end{aligned} \tag{4}$$

whose solutions have the form

$$r_1\,(\theta_1) = \frac{A\,C_1}{C_1 - \left[\sqrt{1 - C_1^2\ln^2\dfrac{1+\sin\theta_1}{\cos\theta_1}}\right]\cos\theta_1} \tag{5}$$

and

$$\theta_2(\theta_1) = \arcsin\left(C_1 \ln \frac{1+\sin\theta_1}{\cos\theta_1} + C_1\right). \tag{6}$$

The equation (5) defines the centroid contour $k_1(\theta_1)$ in polar coordinates and the second (6) – the function of position of the centroid mechanism from Fig. 1.

The conjugate contour $k_2(\theta_1)$ of the centroid pair is obtained directly (Fig. 1) from

$$r_2(\theta_2) = A - r_1(\theta_1)\ . \tag{7}$$

A rotation of 2π along any straight line passing through the points O_1 and O_2 will determine two rotating surfaces Φ_1 and Φ_2, but this does not solve the problem of sliding in the contact zone during the change of the gear ratio. To eliminate it, the necessary and sufficient condition is the axes of symmetry (rotation) of the surfaces Φ_1 and Φ_2 must intersect always on the common tangent of the two contours k_1 and k_2.

3 Friction Surfaces

For the selected values of $A = 0.2\,[m]$ and $r_1(0) = 0.05\,[m]$ from (5), (6) and (7) for contours k_1 and k_2 of the centroid pair, contacting at the point P (momentum center of rotation) – Fig. 1, and function of position $\theta_2 = \theta_2(\theta_1)$ the following dependencies are obtained in polar coordinates:

$$r_1(\theta_1) = \frac{0.6}{\sqrt{9 - \ln^2 \frac{1+\sin\theta_1}{\cos\theta_1}}}, \tag{8}$$

$$r_2(\theta_2) = 0.2 - r_1(\theta_1)\ , \tag{9}$$

$$\theta_2(\theta_1) = \arcsin\left(-\frac{1}{3}\ln\frac{1+\sin\theta_1}{\cos\theta_1}\right)\ , \tag{10}$$

whose graphs are shown in Fig. 2, Fig. 3 and Fig. 4.

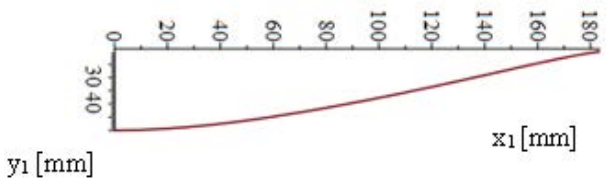


Fig. 2. Graph of the contour $k_1(\theta_1)$.

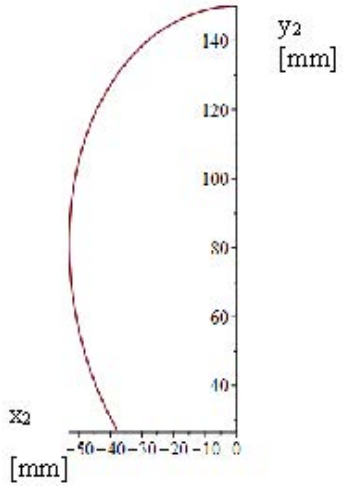


Fig. 3. Graph of the contour $k_1\,(\theta_1)$.

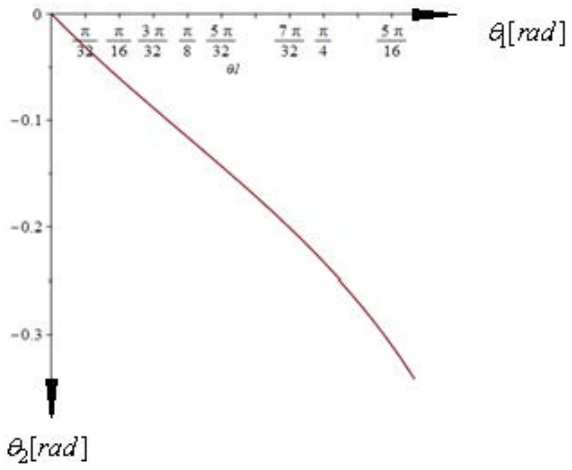


Fig. 4. Graph of the function of positioning $\theta_2 = \theta_2\,(\theta_1)$.

It necessary to note that obtaining the conjugate surfaces satisfying the necessary and sufficient condition for pure rolling – equation (3) is done by rotating the contours k_1 and k_2 are rotated around the axes O_1x_1 and O_2x_2 by 2π. The rotational surfaces Φ_1 and Φ_2 are formed future frictional surfaces satisfying the of zero geometric sliding during the change of the transmission ratio of the variator. Fig. 5 and Fig. 6 show the frictional surfaces an axonometric view.

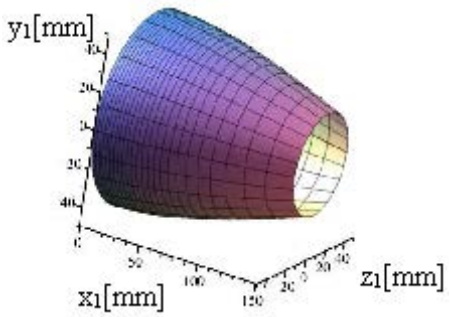


Fig. 5. Friction surface Φ_1.

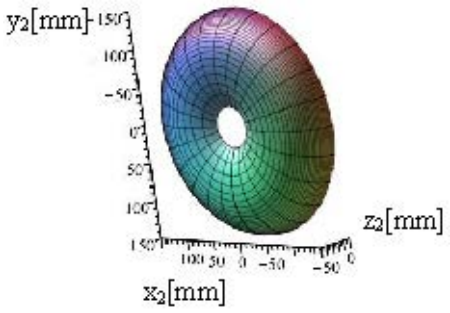


Fig. 6. Friction surface Φ_2.

4 Transmission Ratio

The transmission ratio of the so synthesized friction variator in accordance with Fig. 1 can be represented by

$$i_{V_{12}} = \frac{\omega_1}{\omega_2} = \frac{r_{V_2}}{r_{V_1}} = \frac{r_2 \cos\theta_2(\theta_1)}{r_1 \cos\theta_1}, \tag{11}$$

where ω_1 and ω_2 are the angular velocities of units 1 and 2 and r_{V_1} and r_{V_2} are the contact radiuses of the friction surfaces.

With $r_1(\theta_1)$, $r_2(\theta_2)$ and $\theta_2 = \theta_2(\theta_1)$ already known, it is not difficult to calculate the change in the transmission ratio of the variator, as well as its range.

Figure 7 shows the graph of the variation of the transmission ratio.

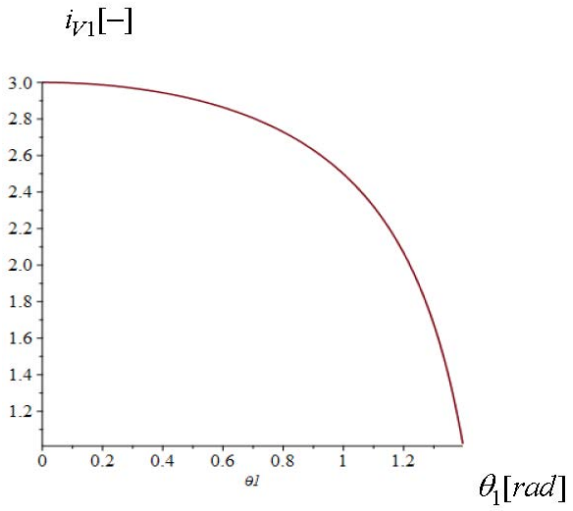


Fig. 7. Graph of the transmission ratio.

5 Structural Scheme and Control

Figure 8 shows the kinematic scheme of a friction variator with practical zero geometrical slip during gear ratio change, in accordance with aforementioned methodology .The movement from the input shaft 1 to the output shaft 12 is transmitted through the friction surfaces 2 and 6. The first one 2 performs the pure rotation around axis of the shaft 1, and the axis of the second 6 performs a general planar movement when 6 rolls on 2. The axes of the links 3 and 9 perform also a general planar movements, provided by ball guides 4 and 8 . The link 13 has a pure translation and unit 12 – rotation.

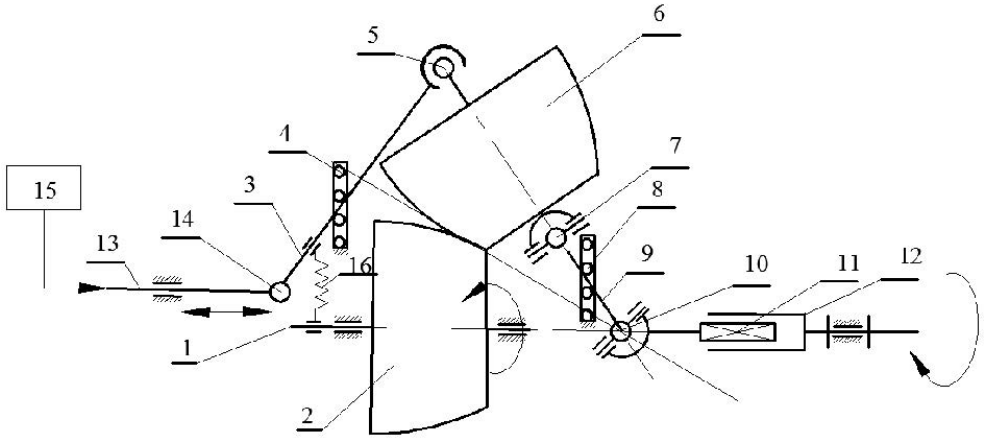


Fig. 8. Kinematic scheme of the variator with practical zero slip during regulation of the gear ratio.

The links 6 and 11 are connected by the shaft 9 and two cardan couplings 7 and 10. Between the links 11 and 12 there is a spline connection, providing a relative translation between them.

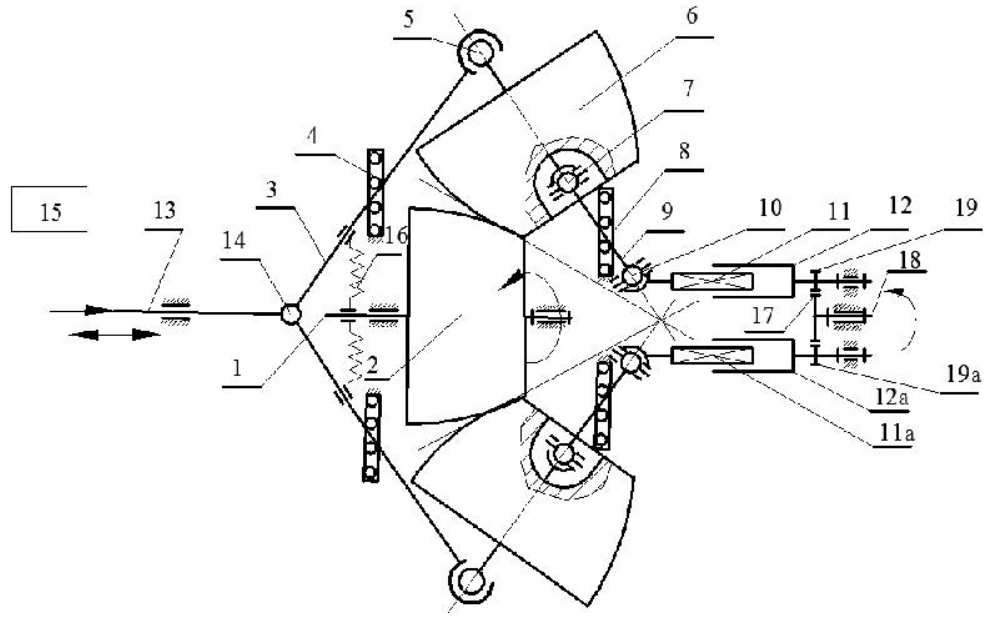


Fig. 9. Kinematical scheme of the variator with two power flows.

The spring 16 serves to ensure a normal pressure between the friction surfaces 2 and 6 during the transmission of torque between them. The change of the transmission ratio takes place according to a signal from control system 15 by the moving the link 13 and following centroid rolling (rubbing) of the friction surfaces 6 on 2.

The proposed scheme allows a significant increase in the load [18] carrying capacity of the variator, through two, three etc., the power flows, basically shown for two flows in Fig. 9 In this particular case , a summing mechanism is needed, conditionally represented by the gears 17, 19, 19a and the output shaft 18 of the variator [19].

6 Conclusion

The synthesis of the kinematic and structural schemes of a variator with zero geometric slip during the change of the transmission ratio presents compelling evidence for some important circumstances improving the quality indicators of the transmission, namely:

- the efficiency factor of the transmission system increases;
- the wear of the friction surfaces is reduced, and from there the reliability and durability are increased; this effect is of particular importance in cases where the operating mode of the machines requires frequent speed regulation;
- the structural scheme of the innovative variator is relatively simple to implement and does not require significant investments, which makes it attractive for industrial application;
- last but not leas possibility of increased load capacity when using a second, third, etc. secondary friction surfaces, i.e. realization of many flows of power in presence of a summing mechanism at the output.

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