

## Analysis of the stability of a contracting jet in a liquid-liquid system

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The stability of a liquid jet in a liquid medium has been investigated within framework of the known Rayleigh postulates for jet stability [3] in a previous paper [1]. The solution obtained generalized the results of Tomotika [11] and Scheele and Meister [6] by including in the Orr-Sommerfeld equations both the inertia and the viscosity terms.

Strictly speaking, the solutions stated above are applicable to cylindrical jets with uniform velocity profiles of undisturbed flows in the jet and in the surrounding liquid. In a real case, however, the loss of stability occurs under the conditions of a contracting jet. It was of interest therefore to extend the solution of [1] to the problem about the stability of a contracting jet. From a physical standpoint, using [1] would mean accounting for the influence of the inertial and viscous forces upon the jet stability.

The present paper deals with the stability of a contracting jet, taking as a starting point the approximate solution for the jet radius and the velocity profiles, proposed in [7]. An analogous idea has been used in [10], where the results of Weber [12] have been employed to the case of a liquid jet stability in a gaseous medium. In the work cited above [7], a similar analysis (for a case of a liquid-liquid system) has been made on the supposition that the growth rate of disturbance is independent of the average velocity of jet outflow.

### 1. Determination of jet length and drop size

Rayleigh's postulates [3], formulated for a cylindrical jet stability will now be applied locally for a contracting jet, considering the jet as composed of cylinders, having a sufficiently small height  $dz$  and a radius  $a(z)$  (Fig. 1). In this case the growth rate of disturbance  $\alpha$  and the wave number  $k$  are functions of  $z$ , in contrast to the case of a cylindrical jet, where they are constants. If  $\xi$  designates the amplitude of the proposed symmetrical disturbance at the jet surface, then by definition we have

$$(1) \quad \ln \left( \frac{\xi}{\xi_0} \right) = \alpha t,$$

wherein  $\xi_0$  is the initial amplitude of disturbance.

For a change of the logarithm form (1), corresponding to a length  $dz$  of the unit cylinders, one could write

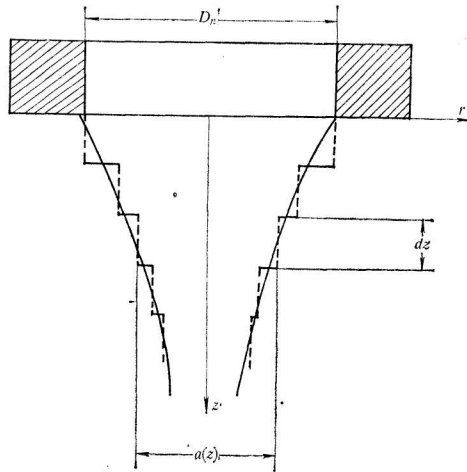


Fig. 1. Diagram of a contracting jet

$$(2) \quad d \ln \left( \frac{\xi}{\xi_0} \right) = \alpha dt = \alpha \frac{dz}{u_1}$$

wherein  $u_1$  is the interfacial velocity of the undisturbed main flow in the jet (which is also the velocity of disturbance propagation).

Written in an integral form, (2) is transformed into

$$(3) \quad \ln \left( \frac{\xi}{\xi_0} \right) = \int_0^z \frac{\alpha}{u_1} dz.$$

Let  $L$  signify the jet length, while  $a_N$  and  $a_L$  are respectively the nozzle radius and the jet radius at the break-up point. Applying Rayleigh's condition for the jet break-up [3]

$$a(L) = \xi(L)$$

one obtains for the unbroken jet length the following equation:

$$(4) \quad \frac{a_L}{a_N} = \exp \left[ \int_0^L \frac{\alpha}{u_1} dz - \ln \left( \frac{a_N}{\xi_0} \right) \right].$$

Using (4) as an equation for calculating jet length, we pre-suppose that  $\alpha$ ,  $a$ , and  $u_1$  are known functions of  $z$ . In the general case, (4) is a non-linear equation of  $L$ , with a complex structure, in which  $L$  participates as an upper integrational limit.

The growth rate of disturbance as a function of  $z$  has been determined in [1] for the case of a viscous cylindrical jet in a liquid-liquid system. It will be noted that the velocity of the surrounding liquid in the equation of Orr-Sommerfeld (designated by  $u_1$  in [1]) is here assumed to be equal to  $u_1$ . In contrast to [7], it is not necessary to consider  $\alpha$  from equation (4) as a constant independent of the average velocity of jet outflow.

As already mentioned, we shall use for the jet form the equation proposed in [7, 8]:

$$(5) \quad \bar{a}^4 \left( g A_0 D_N \bar{z} + \rho_2 u_N^2 + \frac{8T}{D_N} \right) - \bar{a}^3 \left( \frac{8T}{D_N} \right) = \rho_2 u_N^2.$$

When, as shown in [2, 4, 5], a viscous liquid jet flows out into a gas, due to the small resistance of the surrounding medium, the jet profile is quickly equilized and may with sufficient accuracy be assumed as flat. In that case, one may assume that the velocity of the jet surface is equal to the average jet velocity at the corresponding cross-section and thus easily calculate the jet length with the help of an algorithm — a combination between the stability equations of a cylindrical jet [1] and equations (4) and (5). The iterative determination of  $L$  from (4) is organized in such a way that at each step one calculates the jet radius, the corresponding average velocity, and the growth rate of disturbance and then, by numerical integration one determines the value of the disturbance amplitude. The iteration process is discontinued when in (4) the left-hand side (with a pre-determined accuracy) becomes equal to the right-hand side of the equation.

For the liquid-liquid systems, due to the comparable orders of magnitude of the viscosities of the jet and the surrounding medium no equalization of the jet velocity profile is observed. This makes inapplicable the replacement of  $u_1$  by the average velocity and imposes the use of a better approximation for  $u_1$  when calculating the length of the unbroken jet. From [7] we have for  $u_1$ :

$$(6) \quad \frac{u_1}{u_A} = [1 + \exp(-A\bar{z})][1 - \exp(-B\bar{z})],$$

wherein the unknown functions  $A(z)$  and  $B(z)$  are evaluated from the algebraic equation

$$(7) \quad [1 + \exp(-A\bar{z})] \left[ 1 - \frac{\exp(-B\bar{z})}{2} \right] = 1$$

and from the ordinary differential equation:

$$(8) \quad \frac{\mu_1 A_0 u_N a_N [1 - \exp(-B\bar{z})]^2 \left\{ \left[ 2B + 2z \frac{dB}{dz} \right] - \left[ A + z \frac{dA}{dz} \right] [1 - \exp(-B\bar{z})] \right\}}{\mu_2^2 [2 - \exp(-B\bar{z})] [\exp(-B\bar{z})]} = 1.$$

On substituting  $u_1$  from (6) through (8) in (4), for establishing  $L$  one again applies the iterative procedure described above.

It is evident from (4) that  $\ln \left( \frac{a_N}{\xi_0} \right)$  is the only term containing the amplitude  $\xi_0$  of initial jet disturbance, introduced in (1). In the general case, it is a function of a number of factors and especially of the given experimental set-up. Its concrete numerical value may be determined only on the basis of experimental data. Hence, we shall assume  $\ln \left( \frac{a_N}{\xi_0} \right)$  to be a known numerical parameter and for the sake of definiteness, in the numerical results (given further down) obtained for it in [7] we took the value of 6. The problem of determining the  $\ln \left( \frac{a_N}{\xi_0} \right)$  value, corresponding to a given apparatus, is of special interest but it is not the subject of the present paper.

Besides the length of the unbroken jet, it is of practical significance to establish the size of the drops, formed on breaking it up. It is generally accepted to determine the volume of the drops as the volume of the liquid in-

cluded between two subsequent minima of the disturbance wave with a length  $\lambda$ .

$$(9) \quad V = \pi a_L^2 \lambda.$$

For the diameter of an equivalent spherical drop with a volume  $d_e$ , we have

$$(10) \quad d_e = \sqrt[3]{6a_L^2 \lambda}.$$

In our numerical experiments for  $d_e$  we shall use (10), with  $L$  determined from (4) and  $\lambda$  from (1).

## 2. Numerical results

The results of the numerical experiments are shown in Figs. 2, 3 and 4. Fig. 2 juxtaposes the results obtained for the length of the cylindrical jet in three different ways: from the equation of Smith and Moss [9], from (4) without accounting for the velocity profiles, and again from (4) with the velocity profile from (6), (7) and (8). The three curves follow the known trend of the experimental results with a well shaped maximum, characteristic of liquid-liquid systems. However, the maximum with the cylindrical jet is in the region of relatively low outflow velocities, at which no jet will be formed.

Comparing curves 2 and 3 in Fig. 2, one could also conclude that neglecting the velocity profiles is not founded in the cases of liquid-liquid system. A confirmation of the above conclusion are also Figs. 3 and 4 which show the alteration of the growth rate of disturbance  $\alpha$  and the wave length  $\lambda$  as func-

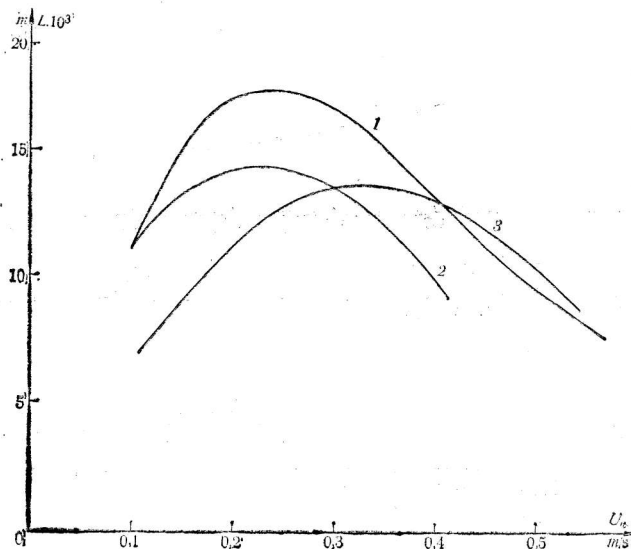


Fig. 2. Jet length versus velocity of outflow:  
1 — cylindrical jet; 2 — contracting jet with a flat profile; 3 — contracting jet with a real velocity profile

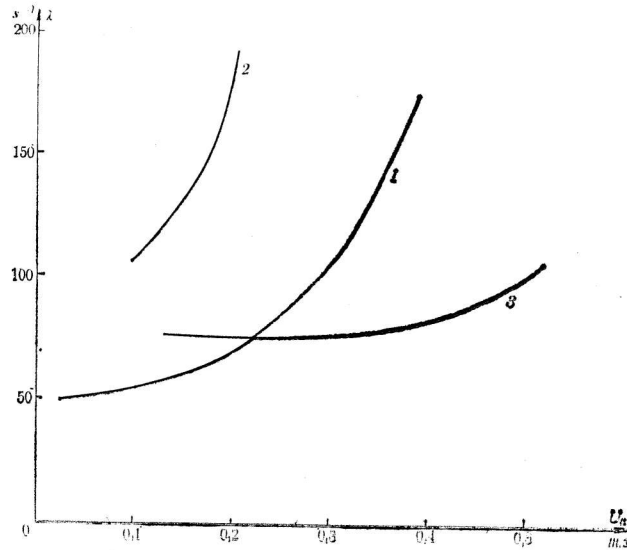


Fig. 3. Growth rate of disturbance versus velocity of outflow:

1 — cylindrical jet; 2 — contracting jet with a flat profile; 3 — contracting jet with a real velocity profile

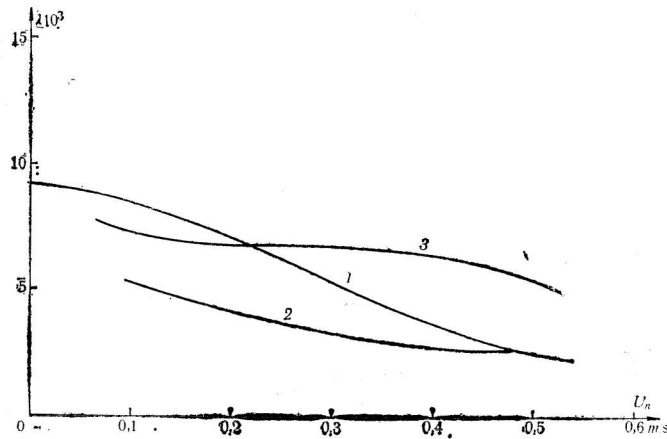


Fig. 4. Wave length of disturbance versus velocity of outflow:

1 — cylindrical jet; 2 — contracting jet with a flat profile; 3 — contracting jet with a real velocity profile

tions of the average velocity at the nozzle exit  $u_N$ . It has experimentally been established [7] that the growth rate of disturbance is only slightly increased with the increase of velocity  $u_N$ . From this point of view, the approximation proves the only applicable means for liquid-liquid systems, supposing a contracting jet and accounting for the velocity profiles (Fig. 3). The length of the disturbance wave with a cylindrical jet (Fig. 4) is appreciably greater in the beginning (small  $u_N$ ) and is sharply decreased with the increase of  $u_N$ . With a contracting jet (with or without accounting for the velocity profiles) this decrease is much less strongly expressed.

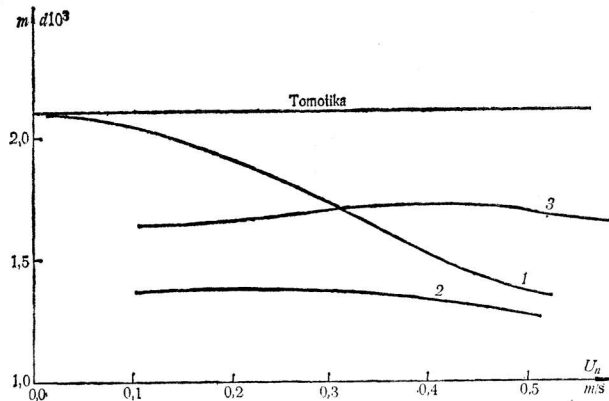


Fig. 5. Diameter of escaping drop versus outflow velocity: 1 — cylindrical jet; 2 — contracting jet with a flat profile; 3 — contracting jet with a real velocity profile

The diameter of the drops (Fig. 5), from the cylindrical jet depends only on the length of the disturbance wave  $\lambda$  and decreases markedly with the increase of outflow velocity  $u_N$ . With a contracting jet, the drop diameter is affected also by the jet radius at the break-up point, which is increased with the increase of  $u_N$ . As a result of superposition of these two opposite effects, the equivalent drop diameter increases (though slightly) with the increase of velocity  $u_N$ .

### Notation

- $A$  — coefficient in equation (6)
- $a$  — jet radius, m
- $a_L$  — jet radius at the break-up point, m
- $a_N$  — nozzle radius, m
- $B$  — coefficient in equation (6)
- $D_N$  — nozzle diameter, m
- $d_e$  — equivalent diameter of escaping drop, m
- $g$  — acceleration of gravity,  $m/s^2$
- $k = \frac{2\pi}{\lambda}$  — wave number,  $1/m$
- $L$  — jet length, m
- $P_T$  — surface tension,  $N/m^2$
- $r$  — radial coordinate, m
- $\bar{r} = \frac{r}{a_N}$  — dimensionless radial coordinate
- $t$  — time, s
- $T$  — coefficient of surface tension,  $N/m$
- $u_1$  — average velocity of the surrounding medium,  $m/s$
- $u_2$  — average jet velocity,  $m/s$
- $u_N$  — average velocity at the nozzle exit,  $m/s$
- $u_i$  — interfacial jet velocity,  $m/s$
- $V$  — volume of escaping drop,  $m^3$

$z$  — axial coordinate, m  
 $\bar{z} = \frac{z}{a_N}$  — dimensionless axial coordinate

### Greek letters

$\alpha$  — growth rate of disturbance, 1/s  
 $\lambda$  — wave length of disturbance, m  
 $\mu_1, \mu_2$  — viscosities of medium and jet, respectively, Ns/m<sup>2</sup>  
 $\xi$  — amplitude of disturbance at the jet surface, m  
 $\xi_0$  — initial amplitude of disturbance, m  
 $\rho_1, \rho_2$  — densities of medium and jet, respectively, kg/m<sup>3</sup>

### Indices

$j$  — takes the value 1 for the medium and 2 for the jet.

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### References

1. Асенов, А., И. Пенчев, С. Радев. ТПМ, № 2 (1977).
2. Маркова, М. П., В. Я. Шкадов. МЖГ, № 3, 30 (1972).
3. Рэлей, Д. В. Теория звука, т. II. ОГИЗ, 1944, Москва.
4. Duba, R. L., J. S. Vrentas. Chem. Eng. Sci., **22**, 855 (1967).
5. Grant, R. P., S. Middelman. AIChE J., **12**, 699 (1969).
6. Meister, B. J., G. F. Scheele. AIChE J., **13**, 682 (1967).
7. Meister, B. J., G. F. Scheele. AIChE J., **15**, 689 (1969).
8. Shiffler, D. A. Ph. D. thesis, Cornell Univer., Ithaca, N. Y. (1965).
9. Smith, S. W., H. Moss. Proc. Royal Soc., **A-93**, 373 (1917).
10. Takahashi, T., Y. Kitamura, Kagaku, Kagaku, **35**, 1229 (1971).
11. Tomotika, S. Proc. Royal Soc., **A-150**, 322 (1935).
12. Weber, C. ZAMM; **11**, 136 (1931).