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# A microcontinuum description of heat transfer in flowing suspensions

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## 1. Introduction

It has been experimentally verified that the suspended particles are responsible for the observed complex mechanical behaviour of a fluid suspension through such mechanism as the deformation of particles, the rotation and migration of particles, the radial redistribution of particles in tube flows etc. For instance in dilute suspensions a necklace-like particle aggregation occurs at about 0.6 radius from the tube centre (the Segré-Silberberg phenomenon). In concentrated suspensions, axial accumulation occurs resulting in reduction of particles or even establishment of a particle-free zone along the tube wall.

Using the microcontinuum theory of fluids which takes into account the local motions and deformations of the primitive elements of fluid, the various authors have shown that by means of the microcontinuum description the above mentioned phenomena may be predicted [1, 2].

However, the suspended particles are responsible not only for the complex mechanical behaviour of fluid suspensions but also, of course, for complex heat transfer. Since the microcontinuum theory of fluids renders it posible to include not only the mechanical effects but also the thermal effects of the primitive elements of fluids, it is of interest to investigate the ability of the microcontinuum theory [3], which takes into account the different temperatures of the primitive elements of fluids, to represent heat transfer.

The formulation of the specialized microcontinuum theory [3] have been accomplished in a formal way and no clear physical insight has been indicated. Then as a beginning, considering the complexity of the problem of a microcontinuum description of heat transfer in flowing suspensions, it seems appropriate to investigate only the ability of the microcontinuum theory to describe the effect of the radial redistribution of particles in the process of heat transfer from a tube wall to a flowing suspension. To this purpose we consider a Poiseuille flow between two parallel plates with simultaneous heat transfer arising as the result of the difference between the temperatures of plates.

## 2. Basis equations

Application of microcontinuum theory [3] to the problem of heat transfer in fluid suspensions flowing between two heated parallel plates due to the pressure gradient requires the determination of the following thermomechanical fields: velocity $v_k$, microrotation $\nu_k$, temperature $\theta$ and temperature variations $\vartheta_k$. To accomplish this requires the solution of the field equations governing the flow under consideration. Using a rectangular Cartesian system in which $x$ axis coincides with the centre line between the plates, the $y$ axis is normal to the flow and $z$ axis is perpendicular to the plane of the flow and assuming the dependence of thermomechanical fields only on the $y$ coordinate, the incompressibility, the microisotropy of a fluid and the negligible viscous dissipation, the equations governing the Poiseuille flow have the form

(1) $$(\mu_v+k_v)\frac{d^2v_x}{dy^2}+K_v\frac{d\nu_z}{dy}-\frac{dp}{dx}=0,$$

(2) $$\gamma_v\frac{d^2\nu_z}{dy^2}-2K_v\nu_z-K_v\frac{dv}{dy}=0,$$

(3) $$K\frac{d^2\theta}{dy^2}+\omega\frac{d\vartheta_y}{dy}=0,$$

(4) $$K_0\frac{d\theta}{dy}-\sigma\frac{d^2\vartheta}{dy^2}+\omega_0\vartheta_y=0.$$

The effect of the local concentration of particles on the thermomechanical fields may be included into the above model by the introduction of the diffusion equation which under the assumption of the independency of flow and thermodynamic fields has the form [1]

(5) $$\frac{dc}{dy}=\left(\beta_1-\alpha_1\frac{d\nu_z}{d_y}\right)\left(\nu_z-\frac{1}{2}\frac{dv_x}{dy}\right).$$

In the above the local concentration $c$ represents the mass of the particles contained in a suspension volume of unit mass and the constitutive coefficients $\mu_v$, $K_v$, ... etc. are considered to be functions of the local concentration only.

To complete the specification of the problem under consideration we must prescribe the boundary conditions. Assuming for the velocity $v_x$ the usual no slip conditions and for the temperature $\theta$ fixed uniform values on the plates the boundary conditions for these quantities are

(6) $$\begin{aligned} y=h: &\quad v_x=0, \quad \theta=\theta_1 \\ y=-h: &\quad v_x=0, \quad \theta=\theta_2. \end{aligned}$$

The specification of the proper boundary conditions for the microrotation $\nu_z$ and the temperature variation $\vartheta_y$ in the case of suspension flows represents a more complicated topic. Following here the analysis presented in [4] and [5], we suggest the boundary conditions for the quantities $\nu_z$ and $\vartheta_y$ in the following form:

(7) $$\begin{aligned} y=h: &\quad \nu_z=-s\frac{dv_x}{dy}, \quad \vartheta_y=-m\frac{d\theta}{dy}, \\ y=-h: &\quad \nu_z=-s\frac{dv_x}{dy}, \quad \vartheta_y=-m\frac{d\theta}{dy}, \end{aligned}$$

where $s$ and $m$ are parameters. Finally we prescribe the mean concentration $c_0$ and the condition at the wall representing the fact that particle can never be in real contact with the wall:

$$c_0=\frac{1}{2h}\int_{-h}^{h} c\,dy, \quad c_w=0. \tag{8}$$

Applying conditions (6)—(8) to equations (1)—(5), the thermomechanical fields may be calculated using either the numerical integration when the dependence of the constitutive coefficients on the local concentration is taken into account or the method of elimination leading to the analytical solution when the formal constancy of the coefficients is assumed. We adopt here the latter approach so that the solution of the above set of equations has the nondimensional form

$$\frac{v_x}{v_0}=1-\xi^2+\varepsilon\frac{K_v}{\mu_v+K_v}\frac{\cosh\lambda h}{\lambda h\sinh\lambda h}\left(\frac{\cosh\lambda h\xi}{\cosh\lambda h}-1\right), \tag{9}$$

$$\frac{v_z h}{v_0}=\xi-\varepsilon\frac{\sinh\lambda h\xi}{\sinh\lambda h}, \tag{10}$$

where

$$\xi=\frac{y}{h},\quad v_0=-\frac{\mathrm{d}p}{\mathrm{d}x}\frac{h^2}{(2\mu_v+K_v)},\quad \varepsilon=\frac{1-s}{1-\frac{s\omega}{2}},\quad \lambda=\left(\frac{2\mu_v+K_v}{\mu_v+K_v}\frac{K_v}{\gamma_v}\right)^{1/2},$$

$$\frac{\theta-\theta_m}{\theta_d}=\left(1-\frac{\omega}{K}\frac{K_0}{\omega_0}\alpha\beta\right)^{-1}\left(\xi-\frac{\omega}{K}\frac{K_0}{\omega_0}\alpha\beta\frac{\sinh\delta h\xi}{\sinh\delta h}\right), \tag{11}$$

$$\frac{\vartheta_y h}{\theta_d}=-\frac{K_0}{\omega_0}\left(1-\frac{\omega}{K}\frac{K_0}{\omega_0}\alpha\beta\right)^{-1}\left(1-\alpha\beta\frac{\delta h\cosh\delta h\xi}{\sinh\delta h}\right), \tag{12}$$

where

$$\theta_m=\frac{\theta_1+\theta_2}{2},\quad \theta_d=\frac{\theta_1-\theta_2}{2},\quad \delta=\left(\frac{K\omega_0-K_0\omega}{K\sigma}\right)^{1/2},\quad \alpha=\frac{1-m\frac{\omega_0}{K_0}}{1-m\frac{\omega}{k}},\quad \beta=\frac{1}{\delta h}\frac{\sinh\delta h}{\cosh\delta h},$$

and

$$\begin{aligned} c={}&c_0+(b-a)\left(\frac{1}{2}\frac{\mu_v}{\mu_v+K_v}-1\right)\frac{\varepsilon}{\lambda h\sinh\lambda h}\left(\cosh\lambda h\xi-\frac{\sinh\lambda h}{\lambda h}\right)\\ &+a\left(\frac{1}{2}\frac{\mu_v}{\mu_v+K_v}-1\right)\frac{\varepsilon^2}{2\sinh^2\lambda h}\left(\cosh^2\lambda h\xi-\frac{1}{2\lambda h}\sinh\lambda h\cosh\lambda h-\frac{1}{2}\right), \end{aligned} \tag{13}$$

where

$$a=a_1\frac{v_0^2}{h^2},\quad b=\beta_1 v_0.$$

## 3. Numerical results and discussion

Let us compute now the theoretical fields using the equations (9)—(13). To accomplish the computation it is first necessary to determine the values of the constitutive coefficients. In this respect, since the effect of the particle redistribution on the fields (9)—(13) may be thus at least approximately predicted, we employ the relations between the constitutive coefficients and the volume concentration of suspended particles as well as the viscous and thermal properties of a fluid obtained in [4—7]. These relations have the form for the viscous and gyroviscous coefficients [4, 6, 7]

$$(14)\qquad \mu_v+\frac{K_v}{2}=\mu_s,\quad \frac{K_v}{2\mu_v+K_v}=\frac{1.5c}{1+2.5c},\quad \gamma_v=\frac{\mu_a-\mu_s}{\mu_a}\frac{2}{3}\frac{\mu_s h^2}{\left[\varepsilon^2(c/c_m)^{-1}-\frac{1}{2}\right]},$$

$$\frac{\mu_a}{\mu_s}=\exp(0.026c) \text{ for blood},\quad \frac{\mu_a}{\mu_s}=1+2.5c+11.7c^2 \text{ for polystyrene spheres},$$

$$s=\frac{1}{2}\left(1-\frac{5}{16}c\right)(1-c),$$

for the heat-conduction coefficients [5]

$$(15)\qquad \begin{aligned} &K=K_s+\omega,\quad \omega=(K_m-K_s)\frac{c}{c_m},\quad \frac{K_0}{\omega_0}=1-\frac{c}{c_m},\quad \alpha\beta=\frac{c}{c_m},\\ &\delta h=\operatorname{cotg}\left(\frac{\pi}{2}\frac{c}{c_m}\right),\quad \bar{K}=\left(K-\frac{K_0}{\omega_0}\omega\right)\left(1-\frac{\omega}{k}\frac{k_0}{\omega_0}\alpha\beta\right)^{-1}, \end{aligned}$$

and for the coefficients $a$ and $b$ [8]

$$(16)\qquad a=a_1' c(1-c)^n,\quad b=b_1 c(1-c),$$

where $\mu_s$ represents the classical shear viscosity of the suspending fluid, $\mu_a$ — the apparent viscosity of a fluid suspension, $c$ — the local concentration, $K_s$ — the thermal conductivity of the suspending fluid, $K_m$ — the thermal conductivity of the fluid suspension at the maximum volume concentration of particles, $c_m$ — the maximum volume concentration of particles and $a_1$, $b_1$ and $n$ are parameters. Finally, $\bar{K}$ denotes the mean conductivity.

The computation of the theoretical fields (9), (10) and (13) together with (11) and (12) will be carried out here for two water suspensions, namely human blood and polystyrene spheres in water, to satisfy approximately the assumption concerning the thermal independency of viscous and gyroviscous constitutive coefficients. The values of physical constants necessary in calculations of the relations (14) and (15) are as follows: Human blood [5, 7], $\mu_s=1.2\ cP$, $p=5.10^{-5}\,m$ (particle radius), $K_s=0.137\ \mathrm{cal\,m^{-1}\,s^{-1}\,{}^\circ C^{-1}}$, $c_m=0.85$, $K_m=0.116$; polystyrene spheres in water [9], $\mu_s=1$, $p=2.5\,10^{-4}$, $K_s=0.143$, $c_m=0.6$ and $K_m=0.066$.

Unfortunately at present there is no physical basis on which it is possible to assign values to parameters $a_1$, $b_1$ and $n$ in relations (16). However, since $0\le c(\xi)\le 1$, following [8, 10] one can obtain restrictions on the values of the diffusion coefficients $a$ and $b$ for dilute suspensions

$$(17)\qquad a\gg b,\quad 0\le c_0+aA(\xi)\le 1,\ \text{i. e. } a\le\frac{c_0}{\max|A(\xi)|},$$

where from (13)

$$A(\xi)=\left(\frac{1}{2}\frac{\mu_v}{\mu_v+K_v}-1\right)\frac{\varepsilon}{\sinh\lambda h}\left[\frac{\varepsilon}{\sinh\lambda h}\left(\cosh^2\lambda h\xi-\frac{1}{2\lambda h}\sinh\lambda h\cosh\lambda h-\frac{1}{2}\right)\right.$$
$$\left.-\frac{1}{\lambda h}\left(\cosh\lambda h\xi-\frac{\sinh\lambda h}{\lambda h}\right)\right],$$

and for concentrated suspensions

$$(18)\qquad b\gg a,\quad 0\le c_0+bB(\xi)\le 1,\ \text{i. e. } b\le\frac{c_0}{\max|B(\xi)|},$$

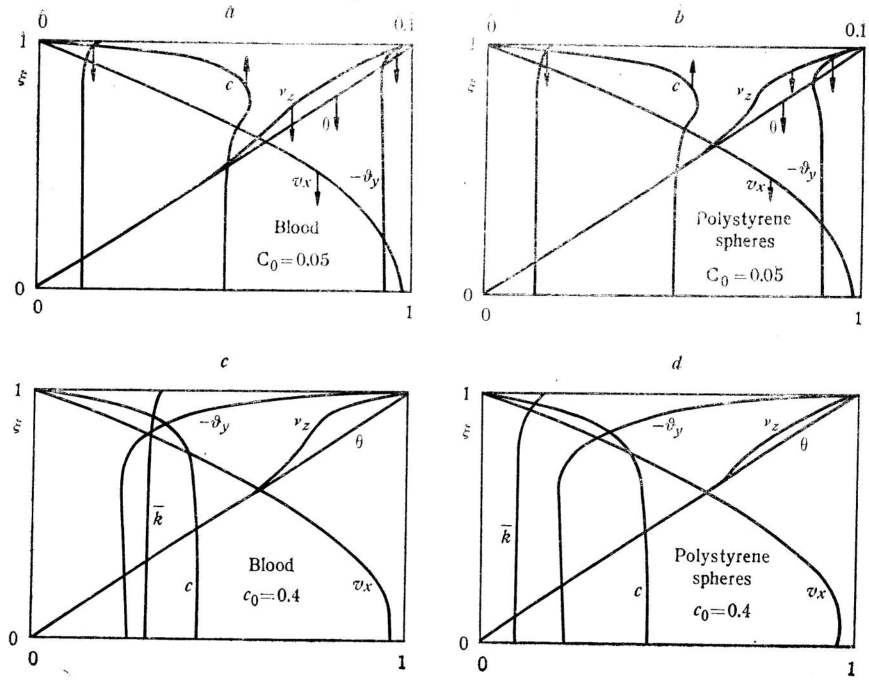


Fig. 1. The velocity $v_x$, microrotation $\nu_x$, temperature $\Theta$, temperature variation $\vartheta_y$, concentration $c$ and mean thermal conductivity $\bar{k}$ profiles between two plates 2 mm apart

where

$$B(\xi)=\left(\frac{1}{2}\frac{\mu_v}{\mu_v+K_v}-1\right)\frac{\varepsilon}{\lambda h \sinh \lambda h}\left(\cosh \lambda h\xi-\frac{\sinh \lambda h}{\lambda h}\right).$$

These restrictions are respected in all computations. For dilute suspensions the values of the parameters from (16) are considered to be $a_1=15$, $b_1=0$, $n=1$. For the concentrated blood $a_1=0$, $b_1=40$ and for the concentrated polystyrene $a_1=0$ and $b_1=60$.

The results of computations are shown in Fig. 1 for human blood and polystyrene spheres of 5% and of 40% concentration flowing between plates 2 mm apart. The essential feature of these results is that in dilute suspensions the deviation of the local mean thermal conductivity $\bar{k}$ from the uniform distribution in stationary suspensions is not very meaningful. Consequently the heat flux is not practically influenced. On the other hand, in concentrated suspensions owing to the radial accumulation of particles the maximum deviation of the local value of $\bar{k}$ from the uniform distribution in stationary suspensions is about 10%, in particular in the case of polystyrene spheres, i. e. in the case when the thermal conductivities of particles and a suspending fluid differ substantially. As a result the heat flux is reduced in polystyrene suspension.

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