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Some problems of interaction of magnetic field and diffusion in solids

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I. Introduction

We consider an isotropic linear elastic ideal conductor. A constant magnetic field H acts along the x-axis in the medium. In the plane $ax+by=0$ acts a source of diffusion with intensity varying in time harmonically. The start point of our considerations are the equations of thermodiffusion and equations of magnetoelasticity given by W. Nowacki [1, 2]. The aim of this report is to obtain the concentration and displacements on the basis of the established above conditions. We assume that all changes are sufficiently slow to neglect the inertial terms $\varrho\ddot{u}_i$.

II. Basic equations

The equations of electrodynamics yield [2, 3]

(2.1)
$$-\nabla^2 h_i=\beta\left[\left(\operatorname{rot}\left(\frac{\partial \mathbf{u}}{\partial t}\times H\right)\right)_i-\frac{\partial h_i}{\partial t}\right].$$

In the above equations the terms $\frac{\partial \mathbf{E}}{\partial t}$ and $\varepsilon_0\mu_0\left(\frac{\varepsilon}{\varepsilon_0}-1\right)$ are omitted. The equations of motion have the form [2]

(2.2)
$$\sigma_{ji,j}+\mu_0[\operatorname{rot}\mathbf{h}\times\mathbf{H}]_i+X_i=0.$$

Here the terms $(\operatorname{rot} h\times b)_i$ are neglected as small quantities. This assumption limits the materials to the dia- and paramagnetics. Taking into consideration the diffusion effects the form of constitutive equations is

(2.3)
$$\sigma_{ij}=2\mu\varepsilon_{ij}+(\lambda\varepsilon_{kk}-\gamma c)\,\delta_{ij}.$$

The set of equations (2.1)—(2.3) is completed by the coupled equation of diffusion [1]:

(2.4)
$$(\varkappa\nabla^2-\partial_t)\,c-\chi\nabla^2\operatorname{div}\boldsymbol{u}=\sigma_1.$$

The following notation is used:

$\mathbf{u}$ — displacement vector
σ_{ij} — stress tensor
$\mathbf{h}$ — magnetisation vector
$b=\mu_0 h$ — magnetic induction vector
c — concentration
σ_1 — intensity of concentration source
X_i — body forces
λ, μ — elastic constants
$\varkappa, \chi, \gamma$ — diffusion-elastic constants
μ_0 — magnetic permeability of vacuum
ε_0 — electric permittivity of vacuum
ε_0 — dielectric constant
$\beta = \lambda_0 \mu_0$.

The equations (2.2) and (2.3) can be reduced to the displacement equations. Finally the equations describing the problem take the form

$$\mu u_{i,kk}+(\lambda+\mu)\, u_{k,ki}+\mu_0\,(h_{i,j}-h_{j,i})H_j+X_i=\gamma\, c_i,$$

(2.5)
$$h_{i,jj}=\beta\,(\dot{h}_i-\dot{u}_{i,j}\, H_j+\dot{u}_{j,j}\, H_i),$$

$$c_{,jj}-\frac{1}{\varkappa}\,\dot{c}+d_\varepsilon\, u_{k,kjj}=\sigma,$$

where $d_\varepsilon=\frac{\chi}{\varkappa}$, $\sigma=\frac{\sigma_1}{\varkappa}$.

III. Solution

For the considered problem we have

$$\mathbf{H}=(H, 0, 0),\ \mathbf{X}=(0, 0, 0),\ \sigma=\sigma_0\, e^{i\omega t}\,\delta(ax+by),$$
$$a=\cos\alpha,\quad b=\sin\alpha,\quad \mathbf{u}=(u, v, w).$$

The problem is a plane one, so all functions depend on x and y only. The only cause which disturbs the medium is the source of diffusion. This change concentration c, displacements u and v and magnetic field h_x and h_y. w and h_z do not change, so we can assume $w=0$ and $h_z=0$. If we put

$$u=\Phi_{,x}+\Psi_{,y},\quad v=\Phi_{,y}-\Psi_{,x}$$

and restrict the considerations to an ideal conductor ($\beta=\infty$), then the set of equations take the form

$$(\lambda+2\mu)\nabla^2\Phi+\mu_0 H^2\Phi_{,yy}-\mu_0 H^2\Psi_{,xy}=\gamma c,$$
$$\mu\nabla^2\Psi-\mu_0 H^2\Phi_{,xy}+\mu_0 H^2\Psi_{,xx}=0,$$
$$\nabla^2 c-\frac{1}{\varkappa}\,\dot{c}+d_\varepsilon\nabla^2\nabla^2\Phi=\sigma_0\, e^{i\omega t}\,\delta\,(ax+by).$$

The solutions we assume in the form

$$\{\Phi(x, y, t),\ \Psi(x, y, t),\ c\,(x, y, t)\}=\{\Phi^*(x, y),\ \Psi^*(x, y),\ c^*(x, y)\}\, e^{i\omega t}.$$

Making use of the Fourier transform in respect to x and y we obtain

$$(\xi^2+\eta^2)\Phi^*_{FF}+A\eta^2\Phi^*_{FF}-A\xi\eta\Psi^*_{FF}+Cc^*_{FF}=0,$$

$$B\xi\eta\Phi^*_{FF}-(\xi^2+\eta^2)\Psi^*_{FF}-B\xi^2\Psi^*_{FF}=0,$$

$$d_\varepsilon(\xi^2+\eta^2)^2\Phi^*_{FF}-\left(\xi^2+\eta^2-\frac{i\omega}{\varkappa}\right)c^*_{FF}=\frac{2\pi\sigma_0}{a}\,\delta\left(\frac{b}{a}\xi-\eta\right),$$

where $A=\frac{\mu_0H^2}{\lambda+2\mu}$, $B=\frac{\mu_0H^2}{\mu}$, $C=\frac{\gamma}{\lambda+2\mu}$.

Using the inverse Fourier transform we obtain Φ^*, Ψ^*, c^*. For instance Φ^* has the form

$$\Phi^*=\frac{\sigma_0 a\,C(1+Ba^2)\varkappa}{B_1 i\omega}\left\{-\pi\,|ax+by|-\frac{i}{\sqrt{2}\sqrt{\frac{B_1\omega}{A_1\varkappa}}(1+i)}\left[\eta(ax+by)\exp\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i\right.\right.$$

$$\left.\left.-1)(ax+by)+\eta(-ax-by)\exp\left(-\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right]\right\},$$

where

$$A_1=1+Ab^2+Ba^2+(1+Ba^2)d_\varepsilon C,$$

$$B_1=1+Ab^2+Ba^2.$$

Finally the time independent factors of c and u are

$$c^*=\frac{\sigma_0 a(1+Ab^2+Ba^2)}{\sqrt{2}\sqrt{\frac{A_1B_1\omega}{\varkappa}}(1+i)}\left[\eta(ax+by)\exp\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right.$$

$$\left.+\eta(-ax-by)\exp\left(-\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right)\right],$$

$$u^*=\frac{\sigma_0 a^2(1+Ba^2)\varkappa}{Bi\omega}\left\{-\pi\,\mathrm{sgn}(ax+by)+\right.$$

$$-\frac{i}{\sqrt{\frac{2B_1\omega}{A_1\varkappa}}(1+i)}\left[\delta(ax+by)\exp\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right.$$

$$-\delta(ax+by)\exp\left(-\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right.$$

$$+\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)\left(\eta(ax+by)\exp\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right.$$

$$\left.\left.-\eta(ax+by)\exp\left(-\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right)\right)\right]$$

$$+\frac{\sigma_0 b^2 BC\varkappa\delta(ax+by)}{B_1 i\omega}\left[\exp\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right.$$

$$-\exp\left(-\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right.$$

$$+\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(\eta(ax+by)\exp\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)+$$

$$-\exp\left(-\sqrt{\frac{B_1\omega}{2A_1\varkappa}}(i-1)(ax+by)\right)\Big]\cdot$$

The similar form has v^*.

The influence of magnetic field is connected with the coefficients

$$A=\frac{\mu_0 H^2}{\lambda+2\mu},\quad B=\frac{\mu_0 H^2}{\mu}.$$

The obtained solutions:

i) indicate the type of singularities in the plane of source;

ii) allow in a simple way to test the results in dependence of reciprocal situation of magnetic field and source of diffusion.

References

1. Nowacki, W. MTIS, 13, 2/1975, p. 143.
2. Nowacki, W. Dynamiczne Zagadnienia Termosprężystości, PWN, 1966.
3. Parkus, H. Magneto-thermoelasticity, Udine, 1972.