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Modification of Coffin's low-cycle fracture criterion

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The known Coffin's criterion for low-cycle fatigue is generalized for the unstable alloys. The generalization is based on a proposed theory of plasticity for ageing solids taking into account the irreversible voluminal change. The attainment of certain critical value of such a voluminal change is used as a fracture criterion.

1. Basing on numerous experiments on cyclic extention-compression the following fracture criterion was formulated by Coffin [1] (see also [2]):

$$N_k(\Delta E_p)^2 = \text{const}, \tag{1}$$

where N_k is the number of cycles unitil the fracture occurs and ΔE_p is the plastic strain amplitude. This criterion is verified by many other authors who were usually obtaining satisfactory results. But there exists also systematic deviations from (1), firstly at large numbers of cycles [2]. For an explanation of these deviations it is necessary to take into account some considerations, concerning the thermodynamical stability and the unstability of alloys [3]. Basing on Coffin's experiments it could be affirmed that the stable alloys satisfy the criterion (1); essential deviations from (1) exist for the unstable alloys, for example, for some aluminium alloys and steels. The unstability of these alloys is a consequence of the fact that they are supersaturated solid solutions which have an inclination to a disintegration and formation of a stable solid solution. The experiments on the natural or artificial ageing show that the mechanical properties of the unstable materials are of immediate relevance to the kinetics of such a disintegration [4]. This connection could be well observed in the experiments on cyclic loading altered with ageing [5, 6]. That is why it is necessary to generalize the criterion (1) for the case of unstable alloys when the number of cycles until the fracture occurs depends on the strain history and on the alloy structure, too. Some considerations about the possible form of such a generalization are proposed in the lecture.

2. For the description of the decomposition of a supersaturated solid solution with the plasticity taken into account the following general equation has been proposed in [3]

$$d\lambda = f(\lambda, t)\ dt = d\,э^p, \tag{2}$$

where $d\,э^p = \int\sqrt{d\varepsilon^p_{\alpha\beta} d\varepsilon^p_{\alpha\beta}}$ is Odquist's parameter, $d\varepsilon^p_{ij}$ is the plastic strain increment tensor which as usually is assumed to be a deviator. The parameter λ

in (2) is interpreted as the "thermodynamical time" in the sense of Vakulenko [7] and it is connected with the energy dissipation at the plastic flow of an ageing material. For a stable body $f \equiv 0$ and the dissipation is caused only by the plastic flow so that $\lambda = \varepsilon^p$. When the plastic strain is constant $d\varepsilon^p = 0$ and the energy dissipation is caused then by the irreversible volumial change due to the ageing. In this case the parameter λ could be interpreted as the volume of the precipitated phase v through the ageing process $\lambda = v$ so (2) presents the well-known equation $dv = f(v, t)dt$ for the velocity of the ageing precipitations [4].

3. In the plasticity theories it is usually assumed that the irreversible volumial change (plastic dilatation) $\varepsilon^p_{kk} = 0$. However, there is enough reason to take into account the fact that the value of ε^d_{kk} can be considerable in the case of large ways of plastic strain which, for example, could be realized at cyclic loading process. Furthermore, the precipitations through the ageing process give also a noticable irreversible volumial change for an ageing specimen which flows plastically. That is why it is necessary to take into account the first invariant ε^p_{kk} in the plasticity of ageing solids. With that aim in view we introduce the system

$$(3) \qquad d\varepsilon^p_{ij} = \frac{d\lambda_0}{\sqrt{J^0_2}}(S_{ij} - r_{ij}), \quad d\lambda_0 = \sqrt{d\varepsilon^p_{\alpha\beta} d\varepsilon^p_{\alpha\beta}},$$

$$(4) \qquad J^{0\prime}_2 = (S_{\alpha\beta} - r'_{\alpha\beta})(S_{\alpha\beta} - r'_{\alpha\beta}) = C^2(\lambda),$$

$$(5) \qquad dr_{ij} = a_1 d\lambda \delta_{ij} + a_2 d\varepsilon^p_{ij},$$

where

$$d\varepsilon^p_{ij} = \frac{1}{3}\delta_{ij} d\varepsilon^p + d\,\varepsilon^p_{ij}, \quad d\varepsilon^p = d\varepsilon^p_{kk}$$

are the components of the plastic strain increment tensor with a deviatoric part $d\,э^p_{ij}$, S_{ij} is the stress deviator, $r'_{ij} = r_{ij} - \frac{1}{3} r\delta_{ij} (r = r_{kk})$ is the deviator of residual microstress tensor r_{ij}. Conventional Cartesian tensor notations are employed here; δ_{ij} represents the Kronecker delta; repeated indices indicate summation.

The thermodynamical time λ in the system (3)—(5) is connected here with the full energy dissipation due to the plastic flow, the irreversible volumial change and the ageing of the material

$$(6) \qquad d\lambda = f(\lambda, t) + d\lambda_0,$$

which coincides with (2) if the volumial change $d\varepsilon^p = 0$.

The system (3)—(6) is a natural generalization of the plasticity theories, considered early by Prager, Ishinskii, Kadashevich and Novozhilov, Arutyunyan and Vakulenko. For a stable material $f \equiv 0$ and this system is quite similar to the system, proposed in [8]. Note that the irreversible volumial change is connected here with the residual microstresses which is natural enough — see [8].

4. Consider the change of the dilatation ε^p through the plastic flow of an ageing body which satisfies the system (3) — (6). Contracting (3) with the Kronecker delta we get

$$(7) \qquad d\varepsilon^p = -\frac{d\lambda_0}{\sqrt{J^0_2}} r.$$

Taking into account (4) we obtain after simple calculations from (7) that

$$\frac{d\varepsilon^p}{d\text{э}^p}=\pm\frac{r}{C(\lambda)}\,. \tag{8}$$

It has to be noted that if the "natural" initial condition $\text{э}^p=0$ and $\varepsilon^p=0$ is satisfied then the residual microstresses $r_{ij}=0$. Thus, from (8) it follows that

$$\frac{d\varepsilon^p}{d\text{э}^p}=0 \quad \text{at} \quad \text{э}^p=0.$$

Then the irreversible volumial change is obviously a quadratic function on Odquist's parameter э^p for a small way of plastic deformation

$$\varepsilon^p\approx K(\text{э}^p)^2, \quad \text{э}^p\ll 1. \tag{9}$$

It has to be pointed out that this quadratic relation does not depend on the concrete conditions for the residual microstresses increment of the type (5).

So the noted in [8] quadratic dependence of ε^p on Odquist's parameter is also valid in the frameworks of the proposed system (3) — (6). That is quite natural, the ageing process in (3) and (4) yields only an isotropic hardening of the material (that was verified experimentally in [9]). The ageing effects change only the coefficient K in the relation (9).

Contracting (5) with the Kronecker delta we obtain

$$\frac{d\lambda}{d\text{э}^p}=\frac{f(\lambda,t)}{V}+\sqrt{1+\frac{1}{3}\left(\frac{r}{C(\lambda)}\right)^2}, \tag{10}$$

where $r=3a_1\lambda+a_2\varepsilon^p$ and $V=\frac{d\text{э}^p}{dt}$ is the velocity of loading. The equations (8) and (10) form the basic system determining the dependence of the irreversible volumial change ε^p on the way э^p of plastic strain. It has to be noted that the time t is excepted from the system (8), (10) for each given loading curve $\text{э}^p=\text{э}^p(t)$. For example, if $V=\text{const}$ then $t=\text{э}^p/V$ and we must insert this value of t into (8) and (10) and obtain the system which gives us the dependences $\varepsilon^p=\varepsilon^p(\text{э}^p)$, $\lambda=\lambda(\text{э}^p)$ for each known V. But here we consider the system (8), (10) only in a qualitative way. In our mind such considerations are of immediate relevance to the fracture criterions of Coffin's type.

5. Suppose for simplicity that $C(\lambda)\equiv C_0=\text{const}$. Then from (8) and (10) for the coefficient K in (9) we receive

$$K=\frac{3a_1}{2C_0}\left(1+\frac{f(\lambda,t)}{V}\right). \tag{11}$$

Let consider an experiment on alternative cyclic loading with given plastic strain amplitude ΔE_p and constant loading velocity V. It is obvious that $\Delta E_p=\text{э}^p$ in each half-cycle, where э^p is the plastic strain path. According to (9) and (11) the irreversible volumial change ε^p accumulated through N cycles is proportional to the quantity $(\Delta E_p)^2\sum_{n=1}^{2N}\left(1+\frac{f(\lambda_n,t_n)}{V}\right)$ on the assumption that the amplitude ΔE_p is small enough; here $t_n=n\Delta t$; $\Delta t=\Delta E_p/V$ is the time for one half-cycle, λ_n is the thermodynamical time (6) for the first n half-cycles.

Suppose that the accumulation of critical value of the irreversible volumial change is a fracture criterion (some considerations about such a criterion and its connection with (1) are given in [8]). Then for the number of cycles N_k until the fracture occurs we obtain

$$(12)\qquad (\Delta E_p)^2(N_k+F(N_k,\ \Delta E_p,\ V))=\text{const},$$

where

$$(13)\qquad F(N,\ \Delta E_p,\ V)=\frac{1}{V}\sum_{n=1}^{2N} f\left(\lambda_n,\ n\frac{\Delta E_p}{V}\right).$$

The relation (12) is the here proposed modification of Coffin's criterion (1) for ageing materials. When the material is stable the function F vanishes and (12) is identical with (1). For unstable materials $F>0$ and the comparison of (12) to (1) shows that the number of fracture cycles decreases when the ageing process is taken into account.

Let replace now the sum in (13) by an integral

$$F(N,\ \Delta E_p,\ V)\approx\frac{1}{\Delta E_p}\int_0^{T_N} f(\lambda,\ t)dt,$$

where $T_N=2N\Delta t$ is the time for N cycles. Then (12) could be rewritten in the following equivalent form:

$$(14)\qquad \Delta E_p\,.\,\lambda_{N_k}=\text{const},$$

where λ_N means the thermodynamical time (6) for N cycles.

When $\lambda=\text{э}^p$ is Odquist's parameter (14) is equivalent to Coffin's criterion (1) as it is noted in [10]. With the thermodynamical time λ determined from (2) the criterion (14) has been proposed in [3] as a generalization of (1) for ageing solids.

It should be noted that the proposed generalization (12) of Coffin's criterion (1) describes well the influence of the velocity of loading V on the number of fracture cycles. Viz. the function F in (13) vanishes if $V\to\infty$, so (12) reduces to (1). And that is natural enough — the ageing effects are as smaller as faster the loading process is. When the velocity V decreases the function F increases, so the number of cycles until the fracture occurs decreases.

The proposed considerations allow to expect that the criterion (12) would be valid for the low-cycle fracture for a large class of unstable materials. This criterion obviously needs a thorough experimental verification.

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