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Free Convection Effects on Oscillatory Flow of an Elastico-Viscous Fluid past an Infinite Vertical Plate with Variable Section

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Introduction

Oscillatory flows past horizontal bodies have been studied by Lighthill [1], Stuart [2], Messiha [3], for Newtonian fluid and by Kaloni [4], Soundalgekar and Puri [5] for elastico-viscous fluids. Stuart studied the case of constant suction and Messiha studied the variable suction case. The motion of the Newtonian fluid in upward direction past a vertical plate, in the presence of free-stream oscillating, has been studied by Soundalgekar [6] in case of variable suction. We now study the effects of variable suction on the flow of an elastico-viscous fluid past a vertical plate.

Mathematical Analysis

From ref. [5] and [6], we can show that the flow is governed by the following equations:

$$\varrho'\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=-\frac{\partial p'}{\partial x'}-\varrho' g_x+\eta_0\frac{\partial^2 u'}{\partial y'^2}-k_0\left(\frac{\partial^3 u'}{\partial y'^2\partial t'}+v_0\frac{\partial^3 u'}{\partial y'^3}\right) \tag{1}$$

$$\frac{\partial u'}{\partial y'}=0 \tag{2}$$

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{\lambda}{\varrho c_p}\frac{\partial^2 T'}{\partial y'^2} \tag{3}$$

Then following as in ref. [6], these equations reduce to the following non-dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial u}{\partial t}=\frac{1}{4}\frac{\partial u}{\partial t}+G\Theta+\frac{\partial^2 u}{\partial y^2}-k\left(\frac{1}{4}\frac{\partial^3 u}{\partial y^2\partial t}-(1+\varepsilon Ae^{i\omega t})\frac{\partial^3 u}{\partial y^3}\right) \tag{4}$$

$$\frac{P}{4}\frac{\partial\Theta}{\partial t}-P(1+\varepsilon Ae^{i\omega t})\frac{\partial\Theta}{\partial y}=\frac{\partial^2\Theta}{\partial y^2} \tag{5}$$

and the boundary conditions are

$$u=0,\ \Theta=1 \text{ at } y=0,\ u=U(t),\ \Theta=0 \text{ as } y\to\infty,$$

Hence the non-dimensional quantities are defined as follows:

(6)
$$y = y'v_0/\nu,\ t = t'v_0^2/4\nu,\ \omega = 4\nu\omega'/v_0^2,\ u = u'/U_0,$$
$$u = U'/U_0,\ \Theta = T' - T'_\infty/(T'_w - T'_\infty),\ k = k_0 v_0^2/\nu^2\varrho,$$
$$p = \eta_0 c_p/\lambda,\ G = g\beta\nu((T'_w - T'_\infty)/U_0 v_0^2.$$

To solve the equations (4)—(6) we assume

(7)
$$u = u_0 + \varepsilon\, e^{i\omega t} u_1 \text{ and } \Theta = \Theta_0 + \varepsilon\, e^{i\omega t}\Theta_1.$$

Substituting (7) in (4) and (6), equating the harmonic and the non-harmonic terms, we get

(8)
$$\Theta_0'' + p\Theta_0' = 0,\ \Theta_1'' + p\Theta_1' - \frac{i\omega p}{4}\Theta_1 = -PA\Theta_0'$$

and

(9)
$$ku_0''' + u_0'' + u_0' = -G\Theta_0,$$
$$ku_1''' + \left(1 - \frac{i\omega k}{4}\right)u_1'' + \left(1 - \frac{i\omega k}{4}\right)u_1 = -\frac{i\omega}{4} - G\Theta_1 - A(u_0' + ku_0''))$$

with the boundary conditions as

(10)
$$u_0(0) = 1,\quad u_1(0) = 0,\quad \Theta_0(0) = 1,\quad \Theta_1(0) = 0,$$
$$u_0(\infty) = 1,\quad u_1(\infty) = 1,\quad \Theta_0(\infty) = 0,\quad \Theta_1(\infty) = 0.$$

Equations (9) are the third-order differential equations for $k \neq 0$ and they reduce to the case of Newtonian fluid when $k = 0$. Hence, we conclude that due to the elastic property of the fluid the order of the governing equations is increased from two to three and hence, the given boundary conditions are not sufficient for an unique solution. To overcome this difficulty, following Beard and Walters (7), we expand u_0, u_1 in powers of k as $k \ll 1$. This is also possible physically for Walters (8) has derived these equations for liquids with short memories on the assumption that k is very small. Thus, we now expand

(11)
$$u_0 = u_{01} + ku_{02},\ u_1 = u_{11} + ku_{12}.$$

We substitute (11) in (9), equate the coefficients of different powers of k and we derive the solutions of the coupled linear differential equations. To save space, the solutions are not mentioned here. Substituting for u_0, u_1, Θ_0, Θ_1 in (7), we get the expressions for the velocity and temperature field. The expressions for transient velocity can be written as follows:

(12)
$$\left.\begin{array}{l} u = u_0 - \varepsilon M_i,\quad \omega t = \pi/2 \\ \text{where}\quad M + M_i = u_1 \end{array}\right\}.$$

The transient velocity u is shown on Fig. 1. Now, while carrying out the numerical calculations, all the real values of G are taken into account. This is because the value of $G = \nu g\beta(T_w - T_\infty)/u_0 v_0^2$ depends upon the temperature difference $T'_w - T'_\infty$. When $T'_w - T'_\infty > 0$, the free convection currents travel from the plate to the free-stream and hence the plate is cooled by the free-convection currents. Thus $G > 0$ corresponds to cooling of the plate whereas $G < 0$ then corresponds to heating of the plate. We observe from Fig. 1 that for $G > 0$, the velocity profiles are all positive. An increase in ω leads to a decrease in the transient velocity. Greater cooling of the plate causes a rise in the transient velocity of the fluid. The velocity profiles for $G < 0$ are interesting. Greater

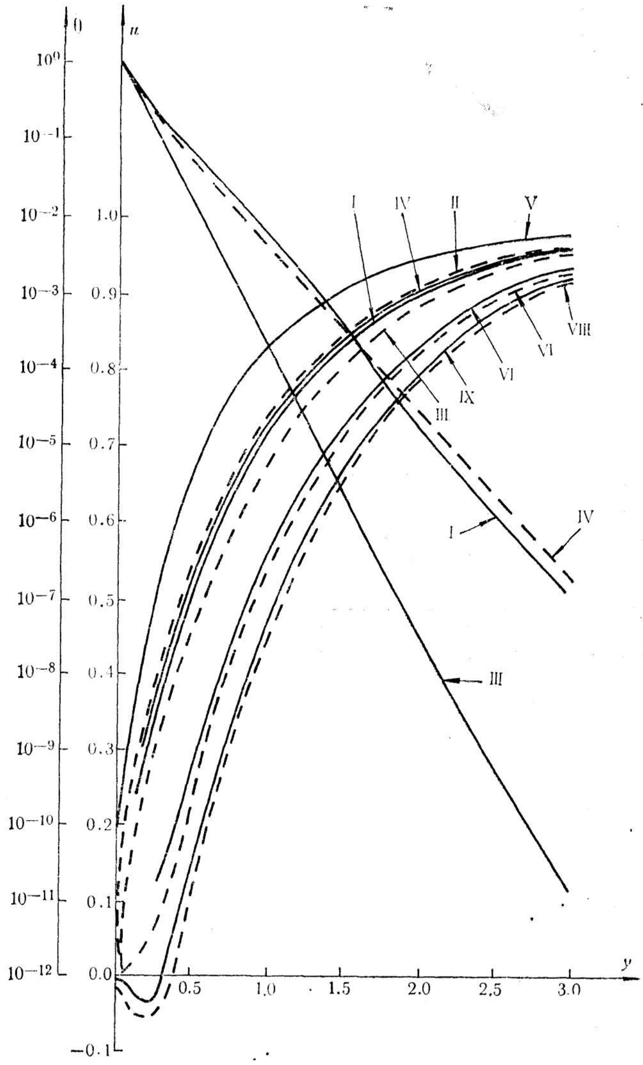

Fig. 1. Transient velocity and temperature profiles

heating of the plate by free-convection currents or an increase in A leads to a fall in the transient velocity such that it becomes negative near the plate. This indicates that there exists a reverse flow near the plate when $G<0$.

The expression for the transient temperature can be derived as

$$\Theta = \Theta_0 - \varepsilon T_i, \quad T_r + iT_i = \Theta_1, \quad \omega t = \pi/2, \tag{13}$$

where T_r and T_i are the fluctuating parts of the unsteady temperature. The transient temperature profiles are also shown on Fig. 1 for $G>0$. We observe from this figure that due to the presence of the elastic property of the fluid there is a fall in the transient temperature. However, an increase in ω leads to a rise in the transient temperature.

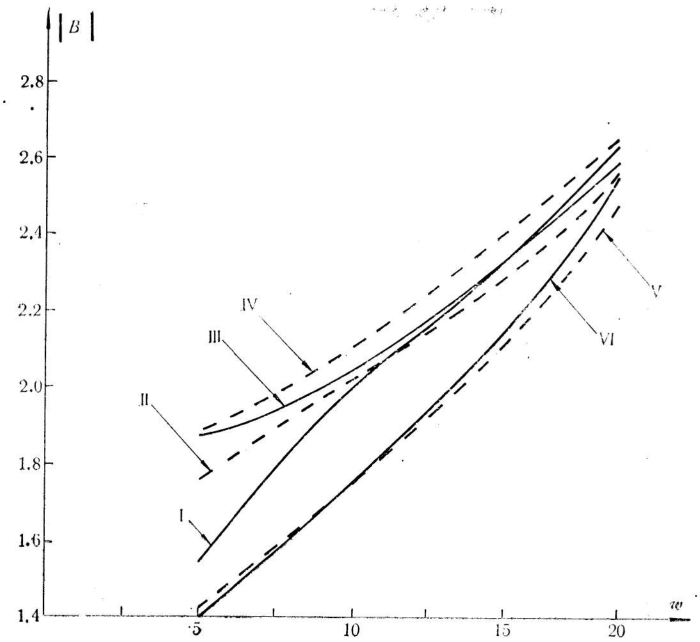

Fig. 2. Amplitude of skin friction

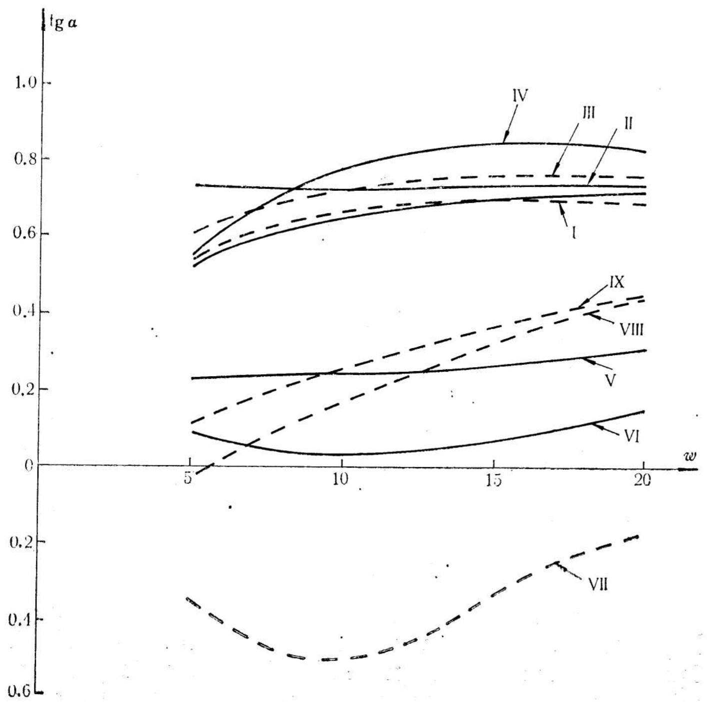

Fig. 3. Phase of skin friction

Knowing the velocity field we now calculate the shearing stress. For this fluid it is given in non-dimensional form as

$$P_{xy}=\left.\frac{du}{dy}\right|_{y=0}-k\left(\frac{1}{4}\frac{\partial^2 u}{\partial y\partial t}-\frac{\partial^2 u}{\partial y^2}\right)_{y=0}. \tag{14}$$

We can express it in terms of the amplitude and phase as

$$P_{xy}=(P_{xy})_{y=0\,(\text{mean})}+\varepsilon\,|B|\cos(\omega t+a), \tag{15}$$

where

$$B=B_1+iB_i=\left(u'_{11}+k\left(u'_{12}-\frac{i\omega}{4}u'_{11}+u''_{11}\right)\right)_{y=0} \tag{16}$$

and $\operatorname{tg} a=B_i/B_r$.
$|B|$ and $\operatorname{tg}\alpha$ are shown on Figures 2 and 3 respectively. $|B|$ is shown for $G<0$. We observe from this figure that both in case of Newtonain and the elastico-viscous fluid, the amplitude of the skin-friction increases with increasing ω. Due to the presence of elastic property in the fluid, the amplitude of the skin-friction decreases. Figure 3, we conclude that there is always a phase-lead except when A and $(-G)$ are large in which case there is a phase-lag

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