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Finite Elements for the Solution of Static Crack Problems in Two and Three Dimensions

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1. Finite elements for the determination of crack-parameters in the elastic case

In a lot of cases the engineer will try to approximate the real crack configuration by a two dimensional mesh, such for thin-walled objects loaded by crack extension modes I and II (normal tension, in-plane shear). The computer code CRACK2D has bean written for two dimensional crack problems (elastic case) with automatic mesh generation. For the near crack

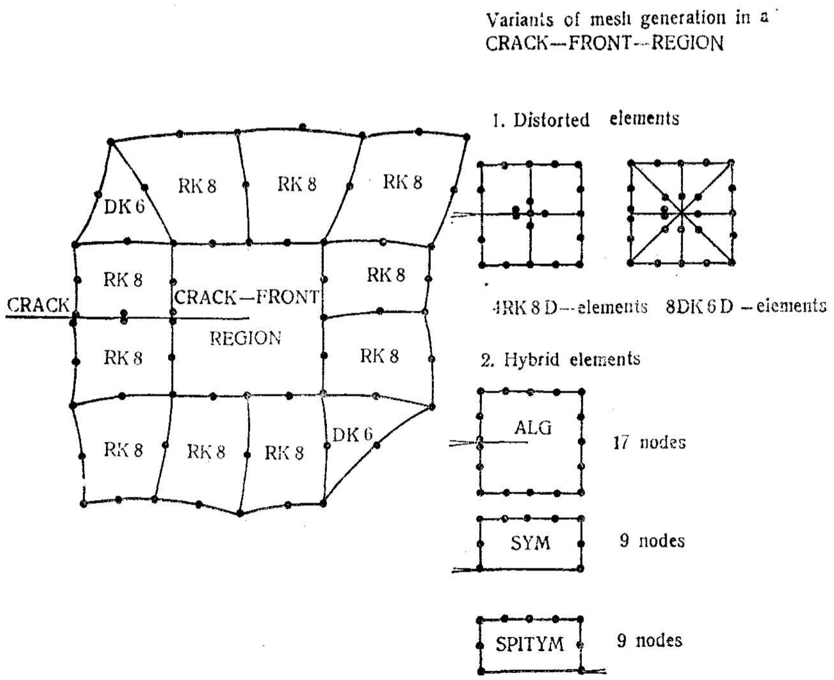

Fig. 1. Embedding of crack fronts by elements of the element catalogue CRACK 2D

front region special displacement (PK8D, DK6D) and hybrid type (ALG, SYM, SPISYM) crack elements are used which embed the crack front in the manner shown in Fig. 1. These elements are compatible.
The crack tip elements PK8D and DK6D were derived from the standard isoparametric ones (PK8 ane DK6) by use of the effect to be observed when distorting the midside nodes into a quarter point position (/1/). To increase accuracy and get the crack parameters K_I, K_{II} and J directly hybrid stress type elements have been added to the element catalogue CRACK2D (/2/). A quadratic polynomial variation for the boundary displacements ensures compatibility to adjacent isoparametric standard elements.
If there is no possibility to get a two dimensional approach for the real crack problem the engineer is to use a three dimensional finite element model in order to get accurate results for the stress field in the neighbourhood of the crack front (thickness direction).
In general there can be observed a superposition of the possible three crack opening modes. The computer code CRACK3D permits FEM-calculations for three dimensional crack problems in linear elastic-plastic materials. The internal element catalogue CRACK3D is based on hexahedron and triangular prism elements as showed in Fig. 2. All elements are of displacement type.
The special subparametric crack element SI45 guarantees a displacement variation proportional $r^{1/2}$ in the vicinity of the crack front. Its construction has

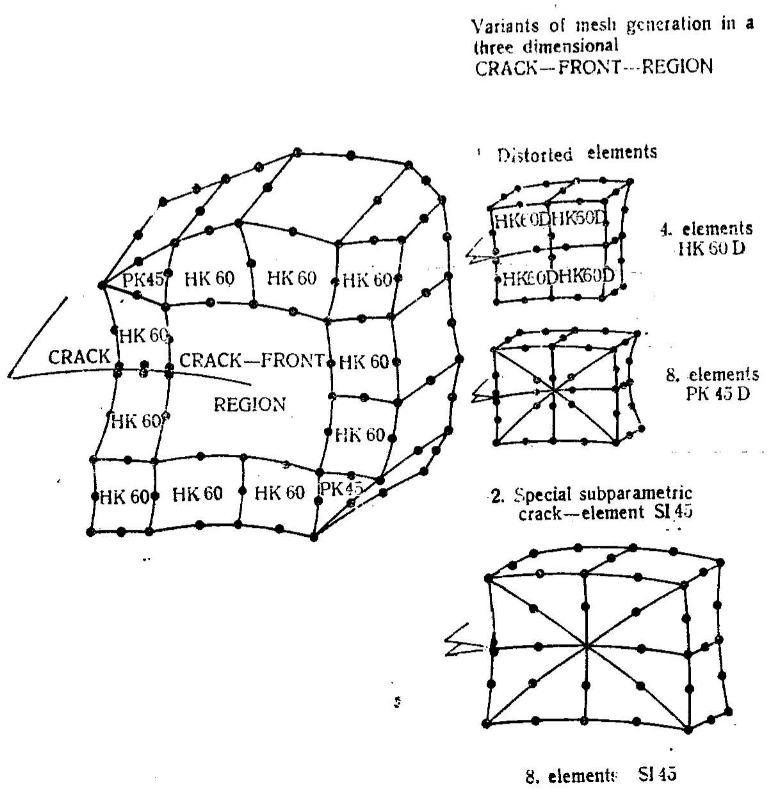

Fig. 2 Embedding of crack fronts by elements of the element catalogue CRACK 3D

been done with the help of the 7-point GAUSSian integration over the average triangles of the triangular prism (15 nodes per element). Its compatibility to the standard isoparametric elements like HK60 (20 nodes) and PK45 (15 nodes) is ensured by suitable shape-functions) of triangular coordinates. By distortion of the midside nodes of HK60 and PK45 towards the crack front we get a simulation of the $r^{-1/2}$ stress singularity again (see HK60D, PK45D, Fig. 2).
Numerical results showed the same acceptable degree of approximation to exact solutions as for PK45D and SI45. So the engineer is immediately able to applicate such FEM-programmes for crack analyses who possess only standard isoparametric elements with quadratic displacement formulations.

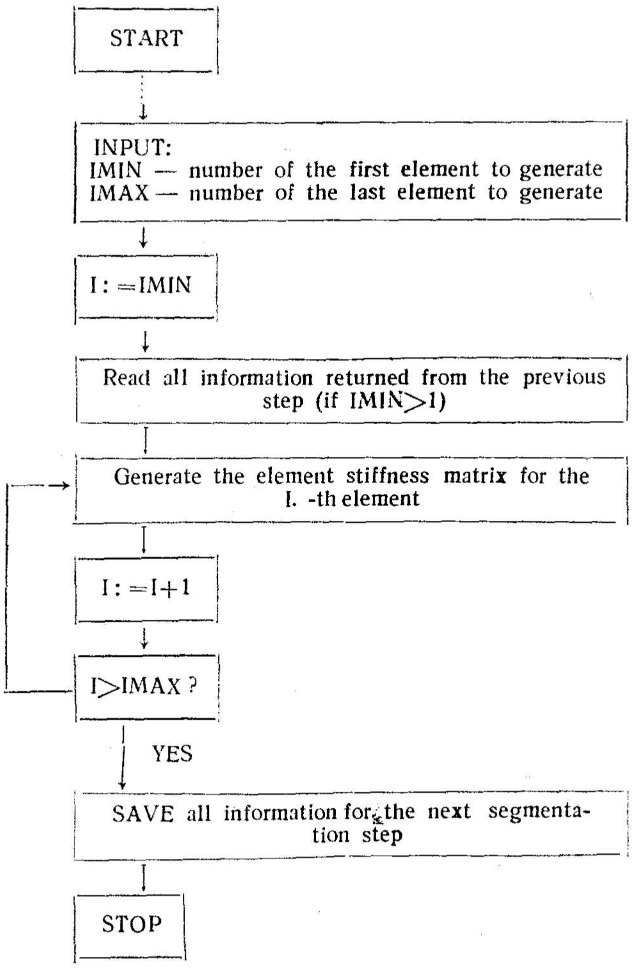

Fig. 3 Flow chart for the segmentation technigue illustrated for the generation of the systems stiffness matrix as an example

2. Segmentation procedure for three dimensional problems

In order to increase the computable number of degrees-of-freedom it is possible to divide the whole computional effort into single steps. This division can successfully be used for the generation of stiffness matrices and for the solution of large linear equation systems by the CHOLESKY method. Fig. 3 illustrates the flow chart of this technique. It can be shown that segmentation techniques are an acceptable alternative to a use of superelement concepts (/3/, /4/).

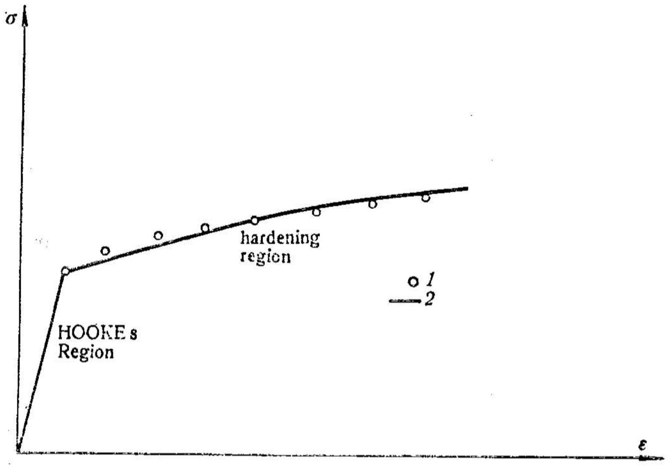

Fig. 4 Use of the GAUSSian least square method to compute a steady function $\sigma(\varepsilon)$ from discrete points $\sigma_i(\varepsilon_i)$

1) discrete function $\sigma_i(\varepsilon_i)$ resulting from the experiment

2) Polynomial opprach $\sigma(\varepsilon)=\sum_i P_i E^i$ (least square method;

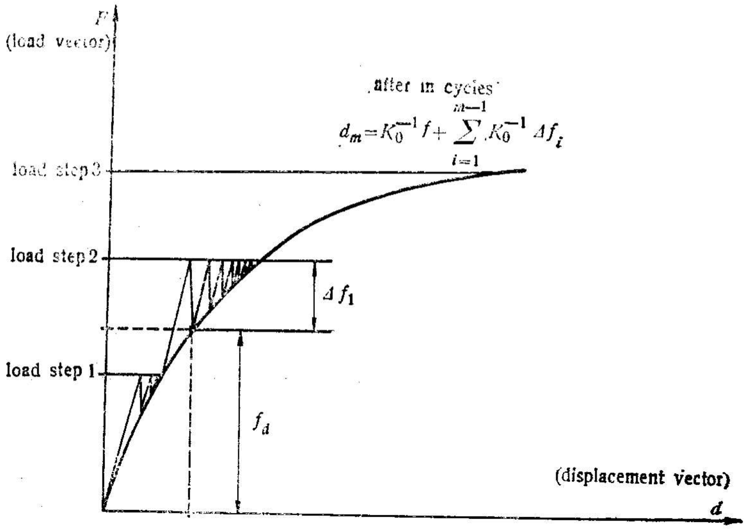

Fig. 5 Mixed procedure for elastic-plastic behaviour

3. A Tixed procedure for elastic-plastic materials in three dimensions

The method described here is based on the use of the iterative procedure in various load steps which must increase monotonous.
First a number of σ-ε points, resulting from the experiment are to put into a steady function $\sigma(\varepsilon)$ by means of the GAUSSian least method (Fig. 4).

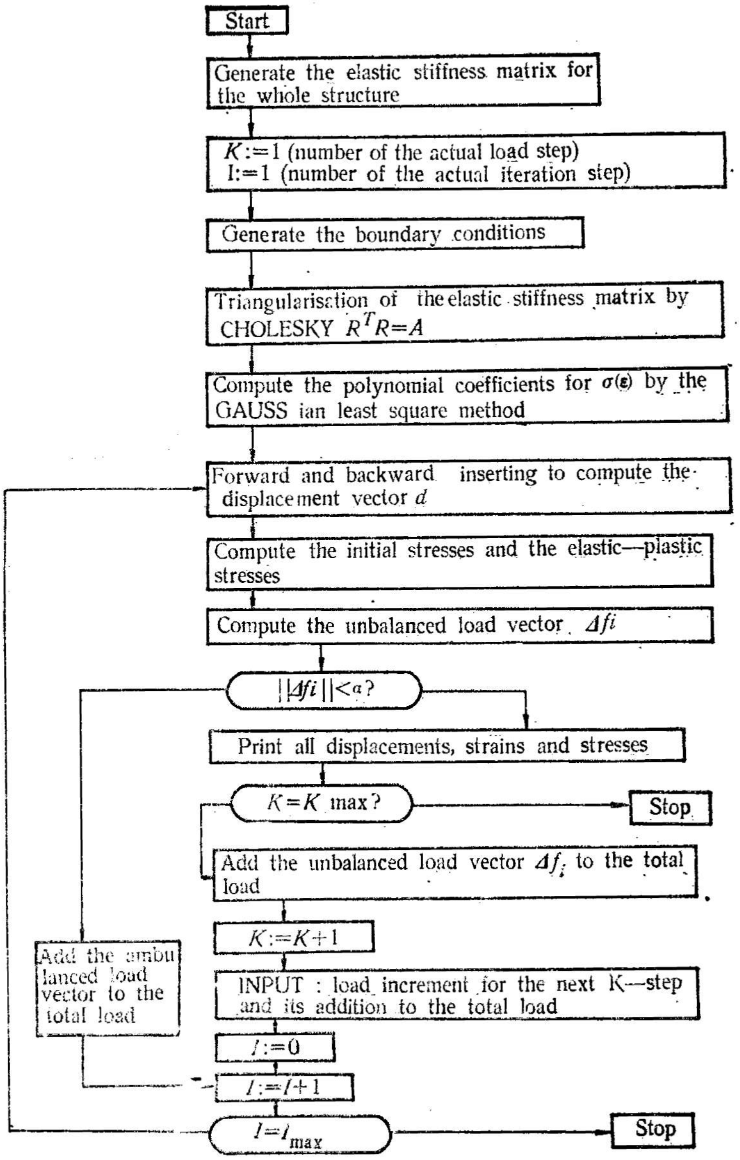

Fig. 6 Flow chart for the mixed procedure

About the principle of the mixed procedure see /5/. The use of constant stiffness matrices has been seen advantageous. From every iteration step there is resulting an unbalanced load vector Δfi for the finite element structure which can be added to the previous vector f to compute a new displacement vector (Fig. 5).

The advances of this method are:

— It is not necessary to generate a "tangent" stiffness matrix in the second and further iteration or load steps. The elastic stiffness matrix can be used in all iteration steps.

— At the end of the iteration the complete displacement vector referenced to the actual load and secant modulus permits the calculation of the stress/strain field in the structure.

In order to get the unbalanced load vector in each step the initial stress approach was taken (/6/).

Fig. 6 illustrates the flow chart of the mixed procedure established in the computer code CRACK3D.

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