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Механика на разрушението

On Crack Problems for Ductile Porous Media

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Introduction

Different porous materials have found in recent years large areas of industrial applications. The interest in problems concerning the influence of the specific structure of these materials on their mechanical properties and behaviour has correspondingly rapidly grown. The well known model of a porous medium has proved to be an effective basis for treating certain fundamental problems of the mechanics of real porous materials. The model of a plastically dilatant porous medium [1, 2] has received considerable development. Important problems concerning processes of fabrication as well as certain industrial applications of a range of ductile sintered steels have been successfully considered by means of this model in [3, 4].

In a previous author's work [5], an extended version of which is in print, the general plane strain problem has been studied within the frames of the same model. The results obtained in [5] have shown to imply [6] possibilities for modifying the known Dugdale model of a crack [7] to problems of opening mode cracks in ductile porous media under plane strain conditions.

It is shown in the present paper that on the basis of a standard dilational plasticity approach [1, 2] and on associated with this approach results [5, 6] as well as on known J — integral applications [8] and on experimental observations on cracks in real ductile popous materials such as sintered steels [9, 10, 11] one comes in a natural way up with a reasonable criterion of crack growth initiation in such materials. A theoretical estimation is given in the paper as well as of the experimentally observed [9, 10, 11] influence of the porosity on the plane strain fracture toughness of the sintered steels investigated.

1. Plasticity Aspects for Ductile Porous Media under Plane Strain Conditions

Consider briefly some aspects of the analysis developed in [5]. An isotropic porous medium with a uniformly distributed porosity is given. It consists of a solid matrix of an elastic-perfectly-plastic material surrounding the pores. The medium is assumed to obey an yield condition of the form [2]

$$(1) \qquad \frac{3}{2} s_{ij}s_{ij}+\alpha^2(\theta)\sigma^2-\beta^2(\theta)\bar{\sigma}_{YS}^2=0,$$

where s_{ij}, i, $j=1$, 2, 3 are the deviatoric stress components, $\bar{\sigma}_{YS}$ is the tensile yield stress of the matrix material, σ is the mean stress, α and β are functions of the porosity θ. Forms for these functions are proposed in [2, 3]. It is further assumed that the stress — plastic strain rate relations for the medium considered follow from the concept of the associated flow rule with the yield function (1) serving as a plastic potential.

As it is shown in [5] when considered under these assumptions the plane strain problem for the medium considered is much analogous to the classical plane stress problem of the incompressible plasticity theory. It is shown, in particular, that regions of elliptic, hyperbolic and parabolic stress states may occur within the porous body considered. Two families of real characteristics are shown to exist in the two latter cases. For both the stress and velocity fields the characteristic lines are defined by the equations

$$\frac{dx_2}{dx_1}=\operatorname{tg}(\Phi\mp\Psi), \tag{2}$$

where Ox_1x_2 is assumed to be the plane of deformation and the quantities Φ and Ψ are introduced through the relations

$$\operatorname{tg}^2\Phi=\frac{3\alpha^2}{9+\alpha^2}, \tag{3}$$

$$\Psi=\frac{1}{2}\left[\pi-\arccos\frac{\operatorname{ctg}\omega}{\operatorname{ctg}\Phi}\right], \tag{4}$$

$$\cos\omega=\frac{\sigma\alpha(9+\alpha^2)^{1/2}}{3\beta\bar{\sigma}_{YS}}. \tag{5}$$

The stresses in the points of the characteristic lines satisfy the relation

$$\sigma_t=\sigma_n\cos 2\Phi, \tag{6}$$

where t and n denote the tangential and normal to the characteristic lines directions, respectively.

It is further demonstrated in [5] that due to the plastic compressibility of the medium considered and in contrast to the classical plane strain problem of the incompressible plasticity theory discontinuities in the normal velocity component are permissible. The occurence of such discontinuities is possible only along the characteristic directions, Eq. (2), and is identical with the experimentally observed [1] formation of thin dilational bands. A process of intensive plastic dilation associated with intensive energy dissipation develops within these bands giving thus way to the final separation of the body along lines (surfaces) initially located within such bands. The actual location of the fracture surfaces is thus predetermined by the characteristic fields, Eq. (2).

2. Dugdale Crack Model Applied to a Porous Medium

It has been shown in [6] that there exists a possibility for introducing the known Dugdale crack model [7] to plane strain problems of opening mode cracks in ductile porous media. It is important to underline that the question considered in [6] is not of a formal but of a mechanically well justified modification of the model. The possibility for applying the model to cracks in porous media is shown to follow from experimental observations [1, 9, 10, 11] when viewing the latters in the light of the analysis developed in [5]. The above mentioned analogy between the problem considered and the plane stress problem of the incompressible plasticity is used in the considerations in [6] as well. It is shown with the aid of this analogy that the stress field at the crack tip is of the type constructed in [12] for the classical plane stress problem. In both cases the continuation of the crack line appears to be a line of a parabolic stress state. In the classical plane stress problem the latter fact explains and reflects at the same time the necking of the plate just ahead of the crack and thus forms the mechanical ground for the classical Dugdale crack model. In the plane strain problem here considered the occurence of such a line of parabolic stress state is identical with the formation of a dilational band along this line [5]. The band could be thus viewed as playing the role of the thin plastic zone of the classical Dugdale crack model.

Let the crack be viewed as a cut along the segment $|x_1|\leq L$, $2L$ being the crack length, and let a uniformly distributed tensile stress $\sigma_{22}(x_1, \pm\infty)=p$ be applied at infinity. Then, in accordance with the considerations described above and following the associated with the Dugdale's analysis [7] standard procedure one comes up with the following result. As it follows from equations (1) and (6) as well as from the symmetry in the problem the stress $\sigma_{22}(x_1, 0)$, $|x_1|\leq L+s$ acting within the plastic zone, that is within the dilational band, is defined as

$$\sigma_{22}(x_1, 0)=\frac{\beta\bar{\sigma}_{YS}}{\sqrt{3}\sin\Phi}, \quad |x_1|\leq L+s, \tag{7}$$

where $2(L+s)$ is the length of the associated with the very sense of the Dugdale model "imaginary" crack and s is thus the plastic zone length which is easily found to be

$$s=L\left(\sec\frac{\pi p\sqrt{3}\sin\Phi}{2\beta\bar{\sigma}_{YS}}-1\right). \tag{8}$$

The plastic zone length s depends on the porosity through the functions $\alpha(\theta)$ and $\beta(\theta)$, the first of which enters equation (8) through the quantity Φ in accordance with equation (3). As it is mentioned in [6] one may approximately consider the porosity θ in equation (8) to equal the initial one and thus use could be made of the forms of the $\alpha(\theta)$ and $\beta(\theta)$ functions proposed in [2] or [3]. The forms proposed in [3] read

$$\alpha(\theta)=2.5\theta^{1/2}, \quad \beta(\theta)=(1-\theta)^{5/2}. \tag{9}$$

Since the plastic zone in the model considered is identical with a dilational band then an intensive process of energy dissipation develops within i due to plastic dilation. As it is usual for cracks in ductile media one should expect that in the case of a porous medium the crack behaviour will strongly depend upon the parameters of this process as well. To obtain a reasonable estimation of these parameters becomes thus a problem of primary importance. As it will be shown in the following section such estimations are derivable from the considered simple modification of the Dugdale crack model.

3. J — Integral and a Crack Propagation Criterion

The path-independent J-integral is known to possess a simple energy interpretation. This results in the possibility of its reliable evaluation even by means of simple elastic-plastic crack models since such models, even if not exact in the details, imply nevertheless reliable estimations of certain integral parameters of the crack problems, for example of the energy variations [8].

Such a simple model is the one just considered. This model implies by means of a procedure similar to that used in [8] the following J-integral expression for the plane strain problem of an opening mode crack in a porous medium

(10) $$J=-\frac{8(1-\nu^2)}{\pi E}\,\frac{\beta^2\bar{\sigma}^2_{YS}L}{3\sin^2\Phi}\ln\left(\cos\frac{\pi p\sqrt{3}\sin\Phi}{2\beta\bar{\sigma}_{YS}}\right),$$

where E and ν are the Young's modulus and the Poisson's ratio of the porous medium considered.

At the same time the J-integral value J_e corresponding to the same crack problem when the latter is viewed as a linear -elastic one is

(11) $$J_e=\frac{1-\nu^2}{E}K_I^2,$$

where K_I is the stress intensity factor. Because of the path-independency of the J-integral equations (10) and (11) imply immediately the relation

(12) $$K_I^2=-\frac{8\beta^2\bar{\sigma}^2_{YS}L}{3\pi\sin^2\Phi}\ln\left(\cos\frac{\pi p\sqrt{3}\sin\Phi}{2\beta\bar{\sigma}_{YS}}\right)$$

and thus define the actual value of K_I which accounts now for the plasticity properties of the medium considered.

The importance of the latter relation becomes evident when viewing it in the light of the results reported in [9, 10, 11]. Basing upon experimental observations the authors of the works just cited come up with the conclusion that similarly to the case of compact materials the known in the crack mechanics K_{IC}-concept applies to problems of determination of the crack growth initiation conditions for ductile porous materials as well and that standard procedures are applicable for experimental determination of the plane strain fracture toughness values for such materials. Once this is the case then upon equating the value K_{IC} of the plane strain fracture toughness of the porous medium considered and the value K_I of the stress intensity factor as defined by equation (12) one obtains directly the critical tensile stress value at which crack growth initiation should take place.

In this regard another of the results reported in [9, 10, 11] deserves due attention. This is the observed in these works proportionality between the tensile yield stress σ_{YS} of the porous medium and its K_{IC} values. The stress σ_{YS} is given in accordance with equation (1) by the relation

(13) $$\sigma_{YS}=\beta\bar{\sigma}_{YS}\frac{3}{\sqrt{9+\alpha^2}}.$$

The condition of proportionality may be simply presented in the form

(14) $$K_{IC}=\sigma_{YS}L_{IC}^{1/2},$$

where the proportionality factor L_{IC} thus introduced has obviously a dimension of length. According to [9, 10, 11] equation (14) with L_{IC} being a constant is valid over a range of porosity (and temperature) values and applies independently of whether the simultaneous proportional increase (or decrease) in both K_{IC} and σ_{YS} values is caused by changes in the porosity or the temperature. Consequently, one may view L_{IC} as a specific for the porous medium length dimension parameter, that is a specific length. A number of questions of interest could be considered in principle regarding the quantity L_{IC}. But it will be sufficient for the purposes of the present analysis just to keep in mind the very existence of such a specific for the porous medium length.

Thus, in accordance with [9, 10, 11] and making use of equations (13) and (14) one obtains from equation (12) the critical stress p_{cr} to be

(15) $$p_{cr}=\frac{2\beta\bar{\sigma}_{YS}}{\pi\sqrt{3}\sin\Phi}\arccos\left[\exp\left(-\frac{L_{IC}}{L}\frac{27\pi\sin^2\Phi}{8(9+\alpha^2)}\right)\right].$$

As defined by the latter expression the critical stress depends directly on the dimensionless ratio of the specific length L_{IC} and the crack size L. It should be mentioned that the crack growth initiation criterion (15) could be derived in a similar way when using as a starting point the known J-integral applications to problems of establishing a J_{IC}-criterion for crack propagation [13].

4. Influence of Porosity on the Plane Strain Fracture Toughness.

It has been observed in the experiments described in [9, 10, 11] that the plane strain fracture toughness of ductile porous media such as the sintered steels investigated decreases with increasing porosity. When viewed from most general positions this fact is quite a natural one. A question of definite interest in this regard is to obtain an analytical estimation of this influence of the porosity on the fracture toughness values.

A possible approach to the problem is proposed in the following. Let $\bar{K}_{IC}$ be the plane strain fracture toughness of the matrix material. Then one may consider the following relation to hold true

(16) $$K_{IC}=F(\theta)\bar{K}_{IC},$$

where $F(\theta)$ is a dimensionless function of the porosity. The latter equation implicitly assumes that with exception of the porosity all the other parameters of the experiment such as matrix material, temperature, type of loading, etc., are kept the same. In accordance with equation (14) one may rewrite equation (16) in the form

(17) $$F(\theta)=\frac{L_{IC}^{1/2}\sigma_{YS}}{\bar{K}_{IC}}.$$

Further upon introducing a specific length dimension $\bar{L}_{IC}$ for the matrix material as

(18) $$\bar{L}_{IC}^{1/2}=\frac{\bar{K}_{IS}}{\bar{\sigma}_{YS}}$$

one comes immediately up with the relation

(19) $$F(\theta)=\frac{\sigma_{YS}}{\bar{\sigma}_{YS}}\left(\frac{L_{IC}}{\bar{L}_{IC}}\right)^{1/2}.$$

Numerous questions are considered in [14] concerning the existence, the nature, the possible practical applications, and the experimental determination of the quantity $\bar{L}_{IC}$ with regard to compact ductile materials such as the material of the matrix of the porous medium under consideration. Similar problems could be considered by analogy with [14] regarding L_{IC}. Important for the present analysis is that under the conditions considered both L_{IC} and $\bar{L}_{IC}$ and thus their ratio are constants. Then, by means of equation (13) one obtains from equation (19) the expression

(20) $$F(\theta)=\frac{3\beta}{\sqrt{9+\alpha^2}}\left(\frac{L_{IC}}{\bar{L}_{IC}}\right)^{1/2}.$$

This expression reveals in fact the desired analytical form of the K_{IC} versus θ dependence which appears thus finally to be

$$K_{IC}=\left(\frac{L_{IC}}{\bar{L}_{IC}}\right)^{1/2}\frac{3\beta}{\sqrt{9+\alpha^2}}\bar{K}_{IC}, \tag{21}$$

where $\bar{K}_{IC}$ and $L_{IC}/\bar{L}_{IC}$ are now constants.

It is clear from the very nature of the α(θ) and β(θ) functions that (cf. yield condition (1)) α(θ) is an increasing and β(θ) is a decreasing function of the porosity. The latter fact is proved, in particular, by the forms of these functions in equation (9) since they are obtained by means of experimental data processing and reflect thus the behaviour of real typical ductile porous materials. Because of these properties of the α(θ) and β(θ) functions one easily observes that the θ-depending term in presentation (21) is a decreasing function of the porosity. Equation (21) proves thus at least qualitatively the experimentally established K_{IC} versus θ dependence and appear as a possible analytical form of the latter.

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