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Vibrations of Rigid Bodies in the Visco-Elastic Model

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This report concerns with the non-linear vibrations of rigid bodies in the visco-elastic model and the effect of dynamic absorber the mechanical model for which is shown in Fig. 1. In the first paragraph the self-excited vibration of a rigid body caused by "negative" friction is considered. The second paragraph is devoted to the forced vibration of a rigid body when the mass M is acted on by the harmonic excitation. In the last two paragraphs the effect of dynamic absorber for damping the mentioned vibrations are studied.

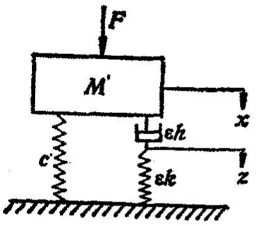

Fig. 1

§1. Self-excited Vibration

The motion equations of the mechanical system shown in Fig. 1 are of the form

$$M\ddot{x}+cx+\varepsilon\beta x^3+\varepsilon h(\dot{x}-\dot{z})=\varepsilon F,$$
$$\varepsilon h(\dot{z}-\dot{x})+\varepsilon kz=0, \tag{1.1}$$

where ε is small parameter, F is the force acting on the mass M, the term $\varepsilon\beta x^3$ characterizes the nonlinearity of the main spring. The first equation describes the motion of the mass M, while the second one — the motion of the second spring (εk).

Eliminating z between equations (1.1) we get

$$\dddot{x}+\xi\ddot{x}+\Omega^2\dot{x}+\xi\Omega^2 x=\varepsilon R, \tag{1.2}$$

here

$$R=\frac{1}{M}[\dot{F}+\xi F-3\beta x^2\dot{x}-k\dot{x}-\xi\beta x^3],\ \xi=\frac{k}{h},\quad \Omega^2=\frac{c}{M}. \tag{1.3}$$

Thus, in contrast with the vibration of a rigid body in the elastic Hook's model here the motion of mass M is described by the differential equation of third order. This kind of equation and its solution have been investigated in several papers of the author [1—3].

Now, let us consider the case when $\beta=0$ and F is "negative" friction of the type

$$F=p\left(1-\frac{r}{3}\dot{x}^2\right)\dot{x}, \tag{1.4}$$

where p, r are positive constants. Using the asymptotic method [1—4] we receive in the first approximation the solution of equation (1.2) in the form

$$x=a\cos(\Omega t+\psi), \tag{1.5}$$

here a and ψ are unknown and determined from the system of averaged equations

$$\begin{aligned}\frac{da}{dt}&=-\frac{\varepsilon}{\Omega(\xi^2+\Omega^2)}[\xi\langle R_0\sin\varphi\rangle+\Omega\langle R_0\cos\varphi\rangle],\\ \frac{d\psi}{dt}&=\frac{\varepsilon}{\Omega a(\xi^2+\Omega^2)}[\Omega\langle R_0\sin\varphi\rangle-\xi\langle R_0\cos\varphi\rangle],\end{aligned} \tag{1.6}$$

$\langle\ \rangle$ is averaged operator, $\varphi=\Omega t+\psi$,

$$R_0=R(x=a\cos\varphi,\quad \dot{x}=-a\Omega\sin\varphi,\quad \ddot{x}=-a\Omega^2\cos\varphi).$$

Substituting into (1.6) the expression (1.3) we obtain

$$\begin{aligned}\frac{da}{dt}&=\frac{\varepsilon a}{2M(\xi^2+\Omega^2)}\left[p(\xi^2+\Omega^2)-\xi k-\frac{1}{4}pr\Omega^2(\xi^2+\Omega^2)a^2\right],\\ \frac{d\psi}{dt}&=\varepsilon k\Omega/2M(\xi^2+\Omega^2).\end{aligned} \tag{1.7}$$

From here it follows that if $\xi k>p(\xi^2+\Omega^2)$ the equilibrium $a=0$ is stable and there is no self-excited vibration of mass M. If $\xi k<p(\xi^2+\Omega^2)$ the mass M vibrates with the amplitude

$$a_0=2\sqrt{\frac{p(\xi^2+\Omega^2)-\xi k}{pr\Omega^2(\xi^2+\Omega^2)}}. \tag{1.8}$$

It is worth mentioning that for the corresponding Hook's model

$$M\ddot{x}+cx=\varepsilon p\left(1-\frac{r}{3}\dot{x}^2\right)\dot{x} \tag{1.9}$$

the mass M accomplishes self-excited vibration with larger amplitude

$$A_0=\frac{2}{\Omega\sqrt{r}}. \tag{1.10}$$

§2. Forced Vibration

For the case $F=P\sin\gamma t$ the motion equation (1.2) takes the form

$$\dddot{x}+\xi\ddot{x}+\Omega^2\dot{x}+\xi\Omega^2x=\frac{\varepsilon}{M}p(\gamma\cos\gamma t+\xi\sin\gamma t)-\frac{\varepsilon}{M}[(k+3\beta x^2)\dot{x}-\xi\beta x^3]. \tag{2.1}$$

Let us consider the resonant case, when

$$\Omega^2=\gamma^2+\varepsilon\sigma. \tag{2.2}$$

Two parameters family of periodic solution of equation (2.1) is found in the form [1, 2]

(2.3) $$x = a\cos(\gamma t+\psi)+\varepsilon u_1(a,\ \psi,\ \gamma t)+\varepsilon^2 u_2(a,\ \psi,\ \gamma t)+\ \ldots,$$

(2.4) $$\frac{da}{dt}=\varepsilon A_1(a,\ \psi)+\varepsilon^2 A_2(a,\ \psi)+\ \ldots,$$
$$\frac{d\psi}{dt}=\varepsilon B_1(a,\ \psi)+\varepsilon^2 B_2(a,\ \psi)+\ \ldots$$

Following the method presented in [1, 2] we receive in the first approximation

(2.5) $$x=a\cos(\gamma t+\psi),$$

and its refinement

(2.6) $$x=a\cos(\gamma t+\psi)+\frac{\varepsilon}{32M\gamma^2}\,\beta a^3\cos 3(\gamma t+\psi),$$

here a and ψ satisfy the system of equations

(2.7) $$\frac{da}{dt}=-\varepsilon\left[\frac{\gamma r_{11}+\xi r_{21}}{2\gamma(\gamma^2+\xi^2)}+\frac{P}{2\gamma M}\cos\psi\right],\quad r_{11}=-\frac{1}{M}\left(M\xi\sigma a+\frac{3}{4}\xi\beta a^3\right),$$
$$\frac{d\psi}{dt}=\frac{\varepsilon}{a}\left[\frac{\gamma r_{21}-\xi r_{11}}{2\gamma(\gamma^2+\xi^2)}+\frac{P}{2\gamma M}\sin\psi\right],\quad r_{21}=\frac{1}{M}\left(M\gamma\sigma a+\frac{3}{4}\gamma\beta a^3+k\gamma a\right).$$

In Fig. 2 the resonant curve for the case $\frac{\varepsilon}{M}=0.01$, $\Omega=\xi=k=1$, $P^2=\frac{0.5}{3}$, $\beta=10$ is presented.

§3. Dynamic Absorber for Forced Vibration

To damp the forced vibration of the main mass M a dynamic absorber with mass m is put on (fig. 3). The motion equations of the system considered will be

(3.1) $$\begin{aligned} &m\ddot{y}+d(y-x)=0,\\ &M\ddot{x}+cx+d(x-y)+\varepsilon h(\dot{x}-\dot{z})=P\sin\gamma t,\\ &\varepsilon h(\dot{z}-\dot{x})+\varepsilon kz=0. \end{aligned}$$

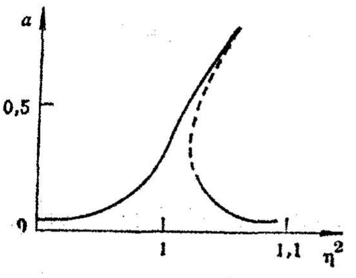

Fig. 2

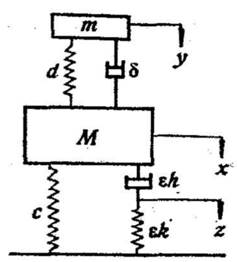

Fig. 3

Eliminating z in the last two equations and denoting

(3.2) $$\omega^2=\frac{d}{m},\quad \frac{m}{M}=\mu$$

we have

$$(3.3)\qquad \dddot{x} +\xi\ddot{x}+\Omega^2\dot{x}+\xi\Omega^2 x+\mu\omega^2(\dot{x}-\dot{y})+\mu\omega^2\xi(x-y)=\frac{1}{M}(\xi P\sin\gamma t$$

$$+\gamma P\cos\gamma t)-\frac{\varepsilon}{M}k\dot{x},\quad \ddot{y}+\omega^2(y-x)=0.$$

The solution of these equations will be found in the form

$$(3.4)\qquad x=x_0+a\sin\gamma t+b\cos\gamma t,\quad Y=A\sin\gamma t+B\cos\gamma t,$$

here x_0, a, b, A, B are constants. Substituting (3.4) into (3.3) and comparing the coefficients of $\sin\gamma t$, $\cos\gamma t$ we receive the following equations for unknowns

$$a\gamma(\Omega^2-\gamma^2+\mu\omega^2)+b\xi(\Omega^2-\gamma^2+\mu\omega^2)-\mu\omega^2(\gamma A+\xi B)=\frac{\gamma}{M}P-\frac{\varepsilon}{M}k\gamma a,$$

$$(3.5)\qquad a\xi(\Omega^2-\gamma^2+\mu\omega^2)-b\gamma(\Omega^2-\gamma^2+\mu\omega^2)-\mu\omega^2(\xi A-\gamma B)=\frac{\varepsilon}{M}P+\frac{\varepsilon}{M}k\gamma b,$$

$$a\omega^2=(\omega^2-\gamma^2)A,\quad b\omega^2=(\omega^2-\gamma^2)B,\quad x_0=0.$$

From here it follows that if the free frequency ω of the dynamic absorber is chosen equal to the frequency γ of external force $\omega=\gamma$ then

$$a=b=0,\quad B=0,\quad A=-P/m\omega^2$$

and therefore

$$x=0,\quad y=-\frac{P}{m\omega^2}\sin\gamma t.$$

The forced vibration of mass M is damped.

§4. Dynamic Absorber for Self-Excited Vibration

Let us consider now a dynamic absorber with small mass (εm) and weak spring (εd). The motion equations take the form

$$(4.1)\qquad \begin{aligned} &\dddot{x}+\xi\ddot{x}+\Omega^2\dot{x}+\xi\Omega^2 x=\varepsilon R,\\ &\ddot{y}+\omega^2 y+\delta_1\dot{y}=\omega^2 x+\delta_1\dot{x},\end{aligned}$$

where $\delta_1=\delta/m$

$$(4.2)\qquad R=\frac{1}{M}[\dot{F}+\xi F-k\dot{x}-d(\dot{x}-\dot{y})-\xi d(x-y)-\xi\delta(\dot{x}-\dot{y})-\delta(\ddot{x}-\ddot{y})].$$

A solution of the system (4.1) is found as

$$(4.3)\qquad x=a\cos\varphi,\quad y=a(L\cos\varphi+N\sin\varphi),$$

here

$$(4.4)\qquad L=\frac{(\omega^2-\Omega^2)\omega^2+\Omega^2\delta_1^2}{(\omega^2-\Omega^2)^2+\Omega^2\delta_1^2},\quad N=\frac{\Omega^3\delta_1}{(\omega^2-\Omega^2)^2+\Omega^2\delta_1^2}$$

In the first approximation we have the following equations for a and ψ [1, 2]

$$\frac{da}{dt}=\frac{-\varepsilon}{\Omega(\xi^2+\Omega^2)}[\xi\langle R_0\sin\varphi\rangle+\Omega\langle R_0\cos\varphi\rangle].$$

(4.5)
$$\frac{d\psi}{dt}=\frac{\varepsilon}{\Omega a\,(\xi^2+\Omega^2)}\,[\Omega\langle R_0\sin\varphi\rangle-\xi\langle R_0\cos\varphi\rangle].$$
$$R_0=R\,[x=a\cos\varphi,\quad y=a(L\cos\varphi+N\sin\varphi)].$$

Substituting here R from (4.2) we get

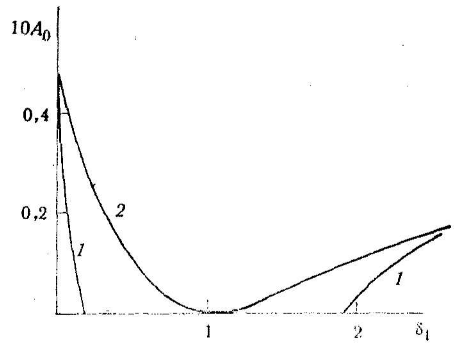

Fig. 4

(4.6)
$$\frac{da}{dt}=\frac{-\varepsilon a}{2M}\left[p\left(\frac{r}{4}\,\Omega^2a^2-1\right)+\frac{\xi k}{\xi^2+\Omega^2}+\frac{m\Omega^4\delta_1}{(\omega^2-\Omega^2)^2+\Omega^2\delta_1^2}\right].$$

From here it follows that

1) The regime $a=0$ and therefore the equilibrium $x=0$ of mass M is stable if $p<\frac{\xi k}{\xi^2+\Omega^2}+m\Omega^4\frac{\delta_1}{(\omega^2-\Omega^2)^2+\Omega^2\delta_1^2}$

2) There exists a self-excited vibration of mass M with the amplitude a_0 determined by

(4.7)
$$A_0=\frac{1}{4}\,pr\Omega^2a_0^2=p-\left[\frac{\xi k}{\xi^2+\Omega^2}+\frac{m\Omega^4\delta_1}{(\omega^2-\Omega^2)^2+\Omega^2\delta_1^2}\right]>0.$$

The effect of dynamic absorber is characterized by the term containing δ_1 in (4.7). If $\omega\neq\Omega$ this term will be very small when δ_1 is either very small or high enough and therefore in the interval $(0, \infty)$ the coefficient δ_1 takes the values corresponding to minimal values of a_0. If $\omega=\Omega$ the effect of the dynamic absorber is high when δ is small.

Figure 4 shows the dependence of the amplitude a_0 on the coefficient of friction δ for the case $p=0.1$, $k=0.1$, $m=0{,}1$ $\xi=\Omega=1$, $\delta_1=\delta/m$. Curve 1 corresponds to the value $\omega^2=1.5\Omega^2$ and curve 2 to $\omega^2=2\Omega^2$. From (4.6), (4.7) it is easy to verify that the points of axis δ_1 which correspond to $A_0>0$ ($A_0<0$) are unstable (stable).

Thus, it is posible to use the dynamic absorber to damp the forced vibration of mass M in the Hook — Maxwell's visco-elastic system. For this purpose it is nesessary to choose the free frequency of absorber equal to the frequency of external force. To damp the self-excited vibration of mass M we have to choose the suitable coefficient of friction (δ) of the absorber.

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