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Boundary Layer Growth on a Rotating Circular Cylinder in a Planar Stagnation Flow

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1. Introduction

In a previous paper (1) we study the problem of boundary layer growth on a static circular cylinder in a semi-infinite fluid. Interaction between the boundary layers and the outer inviscid flow as well as effect of the plane wall on the hydrodynamic force which the cylinder exerts on the fluid have been accounted for.

The rotatory motion of a body in a fluid confined to various surfaces has been repeatedly investigated. Jeffery (2) develops a general method for defining two-dimensional steady Stokes flows confined to the angular region between two non-concentric cylinders. He uses his method to calculate the torgue on a cylinder rotating next to a plane wall. In a recent paper Jeffery and Onishi (3) solve explicitly three problems, namely a cylinder translating parallel and/or perpendicular to a plane wall, and rotating next to it. O'Neill and Ranger (4) consider the slow flow problem of a rotating sphere in the presence of an interface. Bispherical coordinates are made use of and the torgue is calculated in the form of an infinite series.

In two consecutive papers we consider the problem of boundary layer growth on a rotating circular cylinder in an infinite viscous fluid (5) and in a semi-infinite incompressible viscous fluid. The flow field far from the cylinder is assumed to be

(i) a uniform flow;

(ii) a planar stagnation flow.

The full Navier-Stokes equations are solved approximately by using the method of inner and outer expansions.

2. Statement of the problem

A viscous incompressible fluid with kinematic viscosity ν and density ρ is started from rest in a direction perpendicular to an infinite solid plane with a velocity

$$V'(t)=U_\infty\left(\frac{t'}{T_0}\right)^\alpha \quad ; \quad V(t)=\frac{V'(t)}{U_\infty}=t^\alpha .$$

An infinite circular cylinder, positioned parallel to the plane, rotates with an angular velocity

$$\Omega'(t)=\beta\frac{U_\infty}{L}\left(\frac{t'}{T_0}\right)^\alpha \quad ; \quad \Omega(t)=\frac{\Omega'(t)L}{U_\infty}=\beta t^\alpha .$$

Here t' is the time since motion has begun, T_0 and U_∞ are a characteristic time and a characteristic velocity, respectively, $\alpha \geqq 0$ and $\beta \geqq 0$ are nondimensional parameters, and the radius of the cylinder is taken for a characteristic length L.

The assumptions to be made are:

(i) the fluid is slightly viscous, that is the nondimensional parameters of the flow as well as both the Reynolds $R = U_\infty L/\nu$ and the Strouhal $S = L/U_\infty T_0$ numbers are large;

(ii) the problem is solved for times close to the time of initial fluid detachment from the cylinder.

Thus we can infer a small parameter $\varepsilon = 1/S = 1/\nu' R$ where ν' is a constant of order one, and accordingly $U_\infty = \frac{1}{\varepsilon \nu'} \frac{\nu}{L}$ and $T_0 = \frac{1}{\nu'} \frac{\nu}{U_\infty^2}$. The pressure is normalized by $\rho U_\infty^2 S$.

We shall solve a two-dimensional problem since the flow pattern appears one and the same in all planes perpendicular to the cylinder axis. In any of these we shall apply bi-polar cylindrical coordinate system: bi-polar coordinates (ξ, η) are defined through the relation

$$x' + iy' = ic \operatorname{cotg}[1/2(\xi + i\eta)], \quad C = \text{const} > 0$$

by Lame's coefficients

$$h'_\xi = h'_\eta = h' = \frac{c}{\operatorname{ch}\eta - \cos\xi}; \quad h = \frac{h'}{L}.$$

The surfaces of the cylinder and the plane are described as $\eta = \eta_0 < 0$ and $\eta = 0$, respectively and thus solution ranges within $\eta_0 \leqq \eta \leqq 0$, $0 \leqq \xi < 2\Pi$ and at infinity $\xi = 0, \eta = 0$. The gap parameter is $d' = \frac{c(\operatorname{ch}\eta_0 - 1)}{\operatorname{sh}|\eta_0|}$, $d = \frac{d'}{L}$.

The nondimensional Navier-Stokes equations and the continuity equation then become

$$\text{(1)} \quad \frac{\partial u}{\partial t} + \varepsilon\left(\frac{u}{h}\frac{\partial u}{\partial \xi} + \frac{v}{h}\frac{\partial u}{\partial \eta} + \frac{uv}{h^2}\frac{\partial h}{\partial \eta} - \frac{v^2}{h^2}\frac{\partial h}{\partial \xi}\right) = -\frac{1}{h}\frac{\partial p}{\partial \xi} + \frac{\varepsilon^2 \nu'}{h}\frac{\partial}{\partial \eta}\left\{\frac{1}{h^2}\left[\frac{\partial}{\partial \xi}(hv) - \frac{\partial}{\partial \eta}(hu)\right]\right\},$$

$$\text{(2)} \quad \frac{\partial v}{\partial t} + \varepsilon\left(\frac{u}{h}\frac{\partial v}{\partial \xi} + \frac{v}{h}\frac{\partial v}{\partial \eta} + \frac{uv}{h^2}\frac{\partial h}{\partial \xi} - \frac{u^2}{h^2}\frac{\partial h}{\partial \eta}\right) = -\frac{1}{h}\frac{\partial p}{\partial \eta} - \frac{\varepsilon^2 \nu'}{h}\frac{\partial}{\partial \xi}\left\{\frac{1}{h^2}\left[\frac{\partial}{\partial \eta}(hu) - \frac{\partial}{\partial \xi}(hv)\right]\right\}$$

$$\text{(3)} \quad \frac{\partial}{\partial \xi}(hu) + \frac{\partial}{\partial \eta}(hv) = 0,$$

where u, v are the velocity components in the ξ and η direction, respectively.

We can introduce a stream function ψ through the velocity components

$$\text{(4)} \quad u = \frac{1}{h}\frac{\partial \psi}{\partial \eta}, \quad v = -\frac{1}{h}\frac{\partial \psi}{\partial \xi}.$$

The initial and the boundary conditions are

$$\text{(5)} \quad \Psi = 0 \quad \text{for} \quad t \leqq 0$$

$$\text{(6)} \quad \Psi = t^\alpha \operatorname{sh}^2|\eta_0|\left(-\frac{\operatorname{sh}\eta \sin\xi}{(\operatorname{ch}\eta - \cos\xi)^2} - \sum_{k=1}^{\infty} 2ke^{k\eta_0}\frac{\operatorname{sh}k\eta}{\operatorname{sh}k\eta_0}\sin k\xi\right) + B_0(t)\frac{\eta}{\eta_0} \quad \text{as} \quad t \to 0^+$$

$$\text{(7)} \quad \Psi \sim -t^\alpha \operatorname{sh}^2|\eta_0|\frac{\operatorname{sh}\eta \sin\xi}{(\operatorname{ch}\eta - \cos\xi)^2} \quad \text{as} \quad \xi \to 0, \eta \to 0, t \geqq 0$$

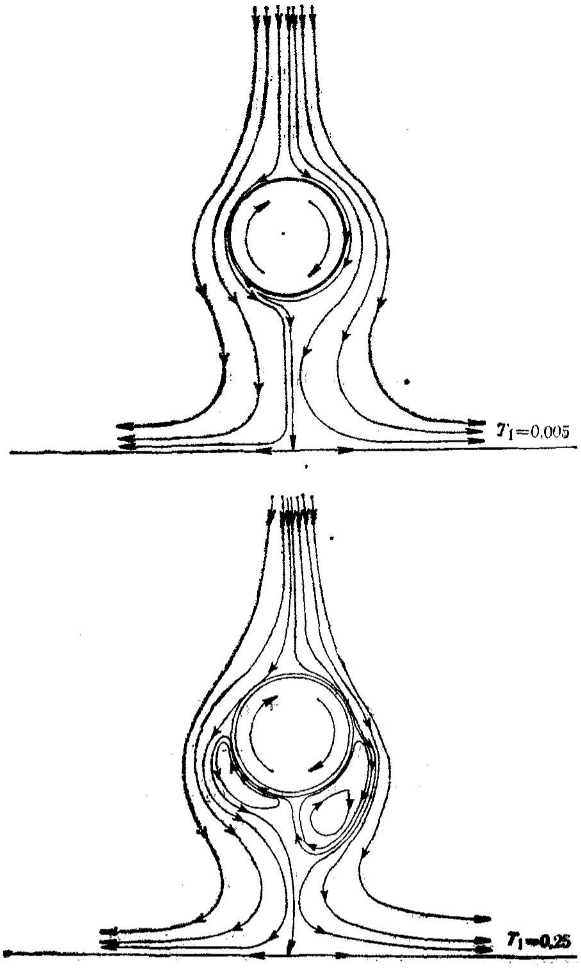

Fig. 1. Streamline pattern for an impulsive flow past a rotating circular cylinder $\beta=1$ next to a solid plane $d\approx 3$ at $R=100$ before and after detachment

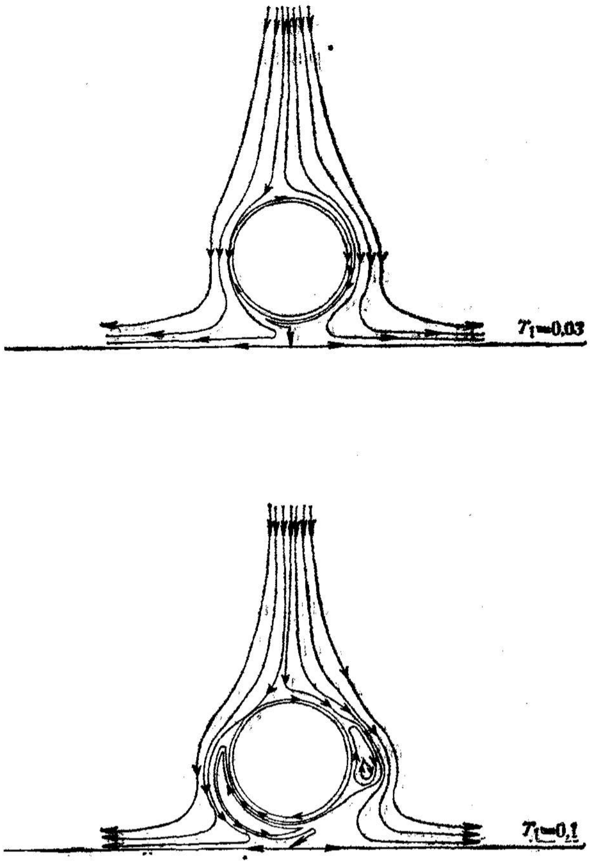

Fig. 2. Streamline pattern for a uniformly accelerated flow past a rotating circular cylinder $\beta=1$ next to a solid plane $d\approx 0.5$ at $R=100$ before and after detachment

(8)
$$\psi=B(t) \text{ and } \frac{\partial\psi}{\partial\eta}=\beta t^{\alpha}\frac{\operatorname{sh}|\eta_0|}{\operatorname{ch}\eta_0-\cos\xi} \quad \text{at} \quad \eta=\eta_0,$$

(9)
$$\bar{\psi}=\frac{\partial\bar{\psi}}{\partial\eta}=0 \quad \text{at} \quad \eta=0.$$

The initial condition (6) is a solution to the posed problem for an ideal fluid. Functions $B_0(t)$, $B(t)$ are temporarily unknown and will be specified further below. The pressure p, introduced by (1) and (2), is an arbitrary function for multi-connected regions. The condition for a single-valued pressure field

(10)
$$\int_0^{2\pi}(\partial p/\partial\xi)d\xi=0$$

causes restrictions which, in turn, prompt the exact definition of the flow field.

3. Inner and outer expansions

The problem is solved by using the inner and outer expansions method with reference to the boundary layers around the cylinder and the plane and beyond them. Overlapping of boundary layers is excluded from the solution.

The outer expansion is

$$\Psi=\Psi_0+\varepsilon\Psi_1+\cdots$$
$$U=U_0+\varepsilon U_1+\cdots \tag{11}$$
$$V=V_0+\varepsilon V_1+\cdots$$
$$P=P_0+\varepsilon P_1+\cdots$$

Here the fluid behaves as an ideal one and therefore the stream function satisfies the Laplace equation.

In the boundary layer around the cylinder

$$\psi=\varepsilon\psi_0+\varepsilon^2\psi_1+\cdots$$
$$u=u_0+\varepsilon u_1+\cdots$$
$$v=\varepsilon v_0+\varepsilon^2 v_1+\cdots \tag{12}$$
$$p=p_0+\varepsilon p_1+\cdots$$
$$h(\xi)=h_0(\xi)+\varepsilon h_1(\xi)\zeta+\cdots$$

In the boundary layer around the plane expansion of the hydrodynamic variables s similar. We shall introduce stretched variables $\zeta=\frac{\eta-\eta_0}{\varepsilon}$ and $\bar{\xi}=-\frac{\eta}{\varepsilon}$ values vary much more normally to the surfaces than along surface direction. $h_0(\xi)$, $h_1(\xi)$ and $\bar{h}_0(\xi)$, $\bar{h}_1(\xi)$ are the coefficients of a Taylor's expansion of Lame's coefficients for $\eta=\eta_0$ and $\eta=0$, respectively.

Substituting these expansions in (1) and (2) and comparing like powers of ε, we obtain the equations for the zeroth, the first, etc. approximations in all the three regions. Boundary conditions (6) and (7) refer to the outer inviscid flow, (8) and (9) to the boundary layers. The principle of matched asymptotic expansions is used to obtain the wanting boundary conditions (6).

The expansion coefficients of the function

$$B(t)=B_0(t)+\varepsilon B_1(t)+\cdots \tag{13}$$

are derived from the condition for a single-valued pressure of the corresponding approximations and are subsequently checked in the boundary layer around the cylinder.

4. Solution to the problem

At the moment of starting the flow is fully potential. The initial boundary condition (6) is assumed to be the zeroth approximation of the outer flow stream function.

Certain equations of the regions around the cylinder and the plane show that the pressure does not vary normally to the boundary layers in zeroth and first approximations. A single-valued pressure is ascertained by $B_0(t)=0$ and further on

$$u_0=t^a \operatorname{sh}|\eta_0|\left[(\operatorname{ch}\eta_0-\cos\xi)\sum_{k=1}^{\infty}2k^2e^{k\eta_0}\left(1-\frac{\operatorname{ch}k\eta_0}{\operatorname{sh}k\eta_0}\right)\sin k\xi\right]. \tag{14}$$

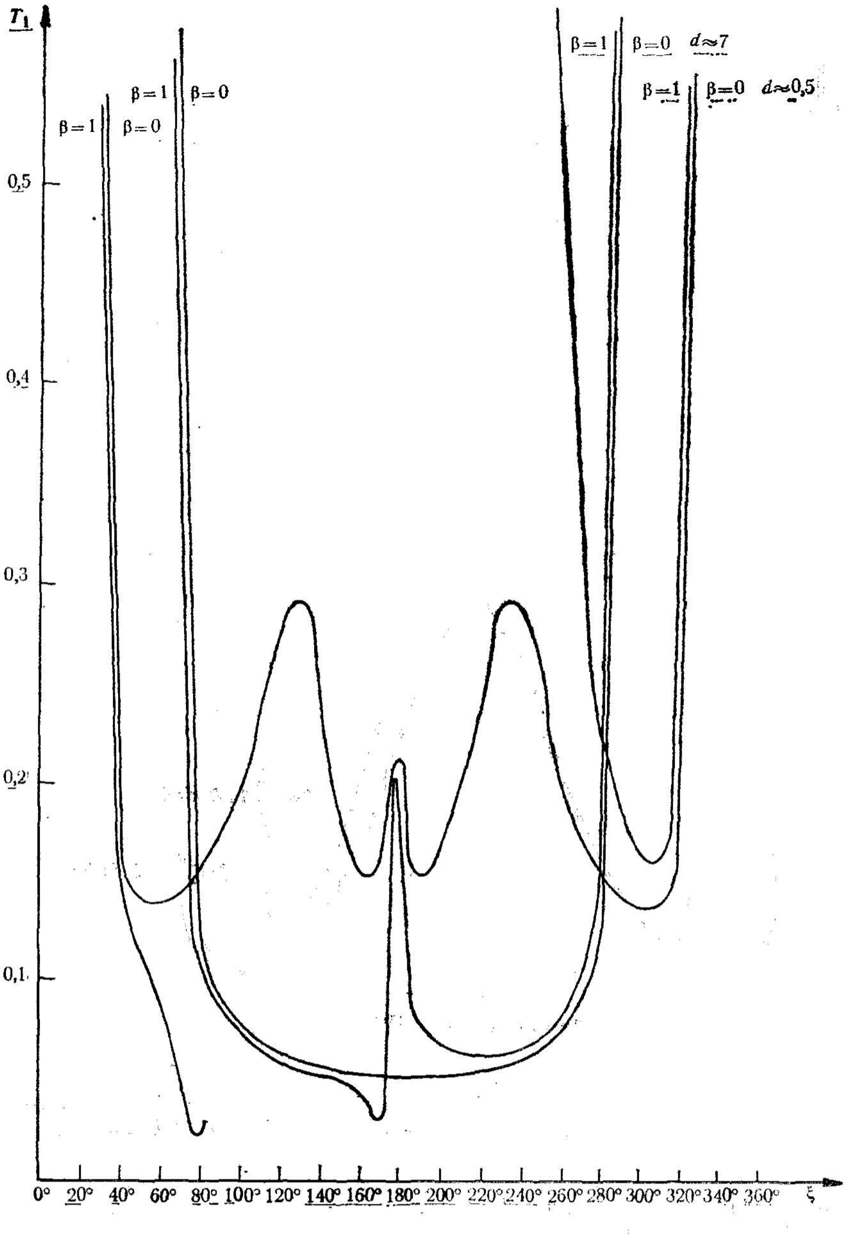

Fig. 3. Detachment points movement for an impulsive flow past a static (β=0) and a rotating (β=1) circular cylinder next to a solid plane at R=100 and various d

$$[1-2^{2\alpha}\Gamma(\alpha+1)G_\alpha(\tilde{\eta})]+\beta t^\alpha 2^{2\alpha}\Gamma(\alpha+1)G_\alpha(\tilde{\eta}),$$

where $\tilde{\eta}=\frac{\xi}{2\sqrt{\nu' t}}\,\frac{\mathrm{sh}\,|\eta_0|}{\mathrm{ch}\,\eta_0-\cos\xi}$, $G_\alpha(\tilde{\eta})=\frac{2}{\sqrt{\pi}\,\Gamma(2\alpha+1)}\int\limits_{\tilde{\eta}}^{\infty}(\gamma-\tilde{\eta})^{2\alpha}e^{-\gamma^2}d\gamma$ are generally accepted.

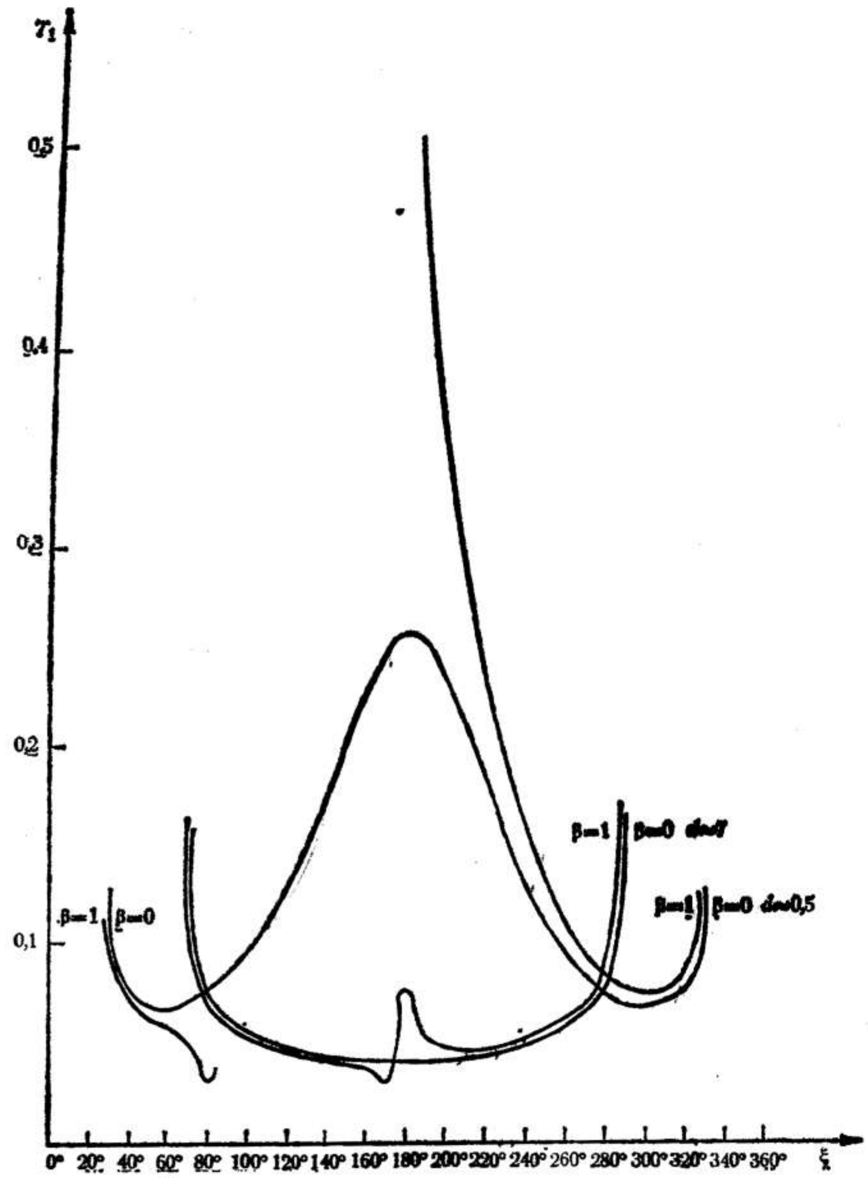

Fig. 4. Detachment points movement for a uniformly accelerated flow past a static ($\beta=0$) and a rotating ($\beta=1$) circular cylinder next to a solid plane at $R=100$ and various d

$$
(15)\qquad v_0=t^{\alpha+1/2}\sqrt{\nu'}\left\{-\left[(\operatorname{ch}\eta_0-\cos\xi)^2\sum_{k=1}^{\infty}2k^3e^{k\eta_0}\left(1-\frac{\operatorname{ch}k\eta_0}{\operatorname{sh}k\eta_0}\right)\cos k\xi\right]2\tilde{\eta}\right.
$$

$$
-\left[\sin\xi(\operatorname{ch}\eta_0-\cos\xi)\sum_{k=1}^{\infty}2k^2e^{k\eta_0}\left(1-\frac{\operatorname{ch}k\eta_0}{\operatorname{sh}k\eta_0}\right)\sin k\xi\right.
$$

$$
\left.+(\operatorname{ch}\eta_0-\cos\xi)^2\sum_{k=1}^{\infty}2k^3e^{k\eta_0}\left(1-\frac{\operatorname{ch}k\eta_0}{\operatorname{sh}k\eta_0}\right)\cos k\xi\right]\left[-\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)}+2^{2\alpha+1}\Gamma(\alpha+1)G_{\alpha+1/2}(\tilde{\eta}\right],
$$

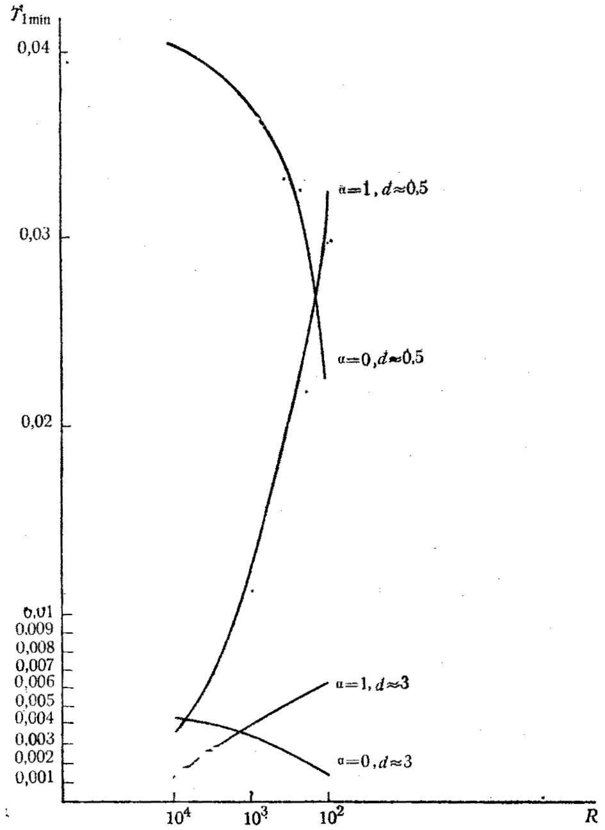

Fig. 5. Initial time of boundary layer detachment from a rotating cylinder ($\beta=1$) as a function of R at various α and d

$$-\left[\sin\xi(\operatorname{ch}\eta_0-\cos\xi)\sum_{k=1}^{\infty}2k^2 e^{k\eta_0}\left(1-\frac{\operatorname{ch}k\eta_0}{\operatorname{sh}k\eta_0}\right)\sin k\xi\right]2^{2\alpha+1}\Gamma(\alpha+1)\tilde{\eta}\tilde{G}_\alpha(\tilde{\eta})\Bigg\}$$

$$+\beta t^{\alpha+1/2}\sqrt{\nu'}\frac{\sin\xi}{\operatorname{sh}|\eta_0|}2^{2\alpha+1}\Gamma(\alpha+1)\tilde{\eta}\tilde{G}_\alpha(\tilde{\eta}).$$

The solution pertaining to the boundary layer around the plane is similar eliminating the β terms.

The stream function Ψ_1 satisfies the Laplace equation, the requirement for U_1 and V_1 to be zero at infinity as well as the conditions obtained by matching $\Psi_0+\varepsilon\Psi_1$ with $\varepsilon\Psi_0$ and $\Psi_0+\varepsilon\Psi_1$ with $\varepsilon\overline{\Psi}_0$ accurate to $O(\varepsilon)$ with the help of intermediate variables $\tilde{\tilde{\eta}}=\sqrt{\varepsilon}\tilde{\eta}$ and $\overline{\overline{\eta}}=\sqrt{\varepsilon}\,\overline{\tilde{\eta}}$ for fixed $\tilde{\tilde{\eta}}$ and $\overline{\overline{\eta}}$ and $\varepsilon\to 0$.

$$\Psi_1=t^{\alpha+1/2}\sqrt{\nu'}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)}\operatorname{sh}|\eta_0|\sum_{n=1}^{\infty}\left[\frac{\operatorname{sh}n(\eta-\eta_0)}{\operatorname{sh}n\eta_0}\bar{\gamma}_n-\frac{\operatorname{sh}n\eta}{\operatorname{sh}n\eta_0}\gamma_n\right]\sin n\xi \tag{16}$$

$$+\left[\beta t^{\alpha+1/2}\sqrt{\nu'}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)}+B_1(t)\right]\frac{\eta}{\eta_0},$$

where $\gamma_n=\operatorname{ch}\eta_0 2n^2e^{n\eta_0}\left[1-\frac{\operatorname{ch}n\eta_0}{\operatorname{sh}n\eta_0}\right]-(n-1)^2e^{(n-1)\eta_0}\left[1-\frac{\operatorname{ch}(n-1)\eta_0}{\operatorname{sh}(n-1)\eta_0}\right]$

$$-(n+1)^2 e^{(n+1)\eta_0}\left[1-\frac{\operatorname{ch}(n+1)\eta_0}{\operatorname{sh}(n+1)\eta_0}\right]$$

$$\bar{\gamma}_n=2+2n^2\frac{e^{n\eta_0}}{\operatorname{sh} n\eta_0}-(n-1)^2\frac{e^{(n-1)\eta_0}}{\operatorname{sh}(n-1)\eta_0}-(n+1)^2\frac{e^{(n+1)\eta_0}}{\operatorname{sh}(n+1)\eta_0}.$$

Ensuing from the condition for a single-valued pressure is

$$\frac{\partial B_1(t)}{\partial t}=-\beta t^{\alpha-1/2}\sqrt{\nu'}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1/2)}.$$

In the first approximation

$$(17)\qquad u_1=t^{\alpha+1/2}\sqrt{\nu'}\Big\{A(\xi)\left[-\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)}+2^{2\alpha+1}\Gamma(\alpha+1)G_{\alpha+1/2}(\tilde{\eta})\right]+B(\xi)\,[-2\tilde{\eta}$$

$$+2^{2\alpha-2}\Gamma(\alpha+1)G_{\alpha-3/2}(\tilde{\eta})-2^{2\alpha+1}(\alpha-1/2)\Gamma(\alpha+1)G_{\alpha-1/2}(\tilde{\eta})+2^{2\alpha+2}(\alpha^2-1/4)\Gamma(\alpha+1)G_{\alpha+1/2}(\tilde{\eta})]\Big\}$$

$$+t^{2\alpha+1}B(\xi)C(\xi)\Big\{2^{2\alpha-1}\frac{1}{2+\alpha}\Gamma(\alpha+1)G_{\alpha-1}(\tilde{\eta})-2^{2\alpha-1}\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+5/2)}G_{\alpha-1/2}(\tilde{\eta})$$

$$+2^{2\alpha+1}\frac{1-\alpha}{1+\alpha}\Gamma(\alpha+1)G_\alpha(\tilde{\eta})+2^{4\alpha+1}\Gamma^2(\alpha+1)G^2_{\alpha+1/2}(\tilde{\eta})-2^{4\alpha+1}\Gamma^2(\alpha+1)G_\alpha(\tilde{\eta})G_{\alpha+1}(\tilde{\eta})$$

$$-\left[\frac{2\alpha}{2+\alpha}-\frac{1}{2(\alpha+1)}-\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2)\Gamma(\alpha+5/2)}+\frac{2(1-\alpha)}{1+\alpha}+\frac{\Gamma^2(\alpha+1)}{2\Gamma^2(\alpha+3/2)}\right]2^{4\alpha+2}\Gamma(2\alpha+2)G_{2\alpha+1}(\tilde{\eta})\Big\}$$

$$+\beta t^{\alpha+1/2}\sqrt{\nu'}\,[-2^{2\alpha-2}\Gamma(\alpha+1)G_{\alpha-3/2}(\tilde{\eta})+2^{2\alpha+1}(\alpha-1/2)\Gamma(\alpha+1)G_{\alpha-1/2}(\tilde{\eta})$$

$$-2^{2\alpha+2}(\alpha^2-1/4)\Gamma(\alpha+1)G_{\alpha+1/2}(\tilde{\eta})]+\beta t^{2\alpha+1}C(\xi)\Big\{-2^{2\alpha-1}\frac{1}{2+\alpha}\Gamma(\alpha+1)G_{\alpha-1}(\tilde{\eta})$$

$$+2^{2\alpha-1}\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+5/2)}G_{\alpha-1/2}(\tilde{\eta})-2^{2\alpha+1}\frac{1/2-\alpha}{1+\alpha}\Gamma(\alpha+1)G_2(\tilde{\eta})-2^{4\alpha+1}\Gamma^2(\alpha+1)G^2_{\alpha+1/2}(\tilde{\eta})$$

$$+2^{4\alpha+1}\Gamma^2(\alpha+1)G_\alpha(\tilde{\eta})G_{\alpha+1}(\tilde{\eta})+\Big[\frac{2\alpha}{2+\alpha}-\frac{1}{2(1+\alpha)}$$

$$-\frac{\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2)\Gamma(\alpha+5/2)}+\frac{2(1/2-\alpha)}{1+\alpha}+\frac{\Gamma^2(\alpha+1)}{2\Gamma^2(\alpha+3/2)}\Big]2^{4\alpha+2}\Gamma(2\alpha+2)G_{2\alpha+1}(\tilde{\eta})\Big\},$$

where $A(\xi)=(\operatorname{ch}\eta_0-\cos\xi)\sum_{n=1}^{\infty} n\left(\frac{\operatorname{ch} n\eta_0}{\operatorname{sh} n\eta_0}\gamma_n-\frac{1}{\operatorname{sh} n\eta_0}\bar{\gamma}_n\right)\sin n\xi$,

$$B(\xi)=\operatorname{sh}|\eta_0|(\operatorname{ch}\eta_0-\cos\xi)\sum_{k=1}^{\infty}2k^2e^{k\eta_0}\left(1-\frac{\operatorname{ch} k\eta_0}{\operatorname{sh} k\eta_0}\right)\sin k\xi,$$

$$C(\xi)=\sin\xi\,(\operatorname{ch}\eta_0-\cos\xi)\sum_{k=1}^{\infty}2k^2e^{k\eta_0}\left(1-\frac{\operatorname{ch} k\eta_0}{\operatorname{sh} k\eta_0}\right)\sin k\xi$$

$$+(\operatorname{ch}\eta_0-\cos\xi)^2\sum_{k=1}^{\infty}2k^3e^{k\eta_0}\left(1-\frac{\operatorname{ch} k\eta_0}{\operatorname{sh} k\eta_0}\right)\cos k\xi.$$

The solution pertaining to the boundary layer around the plane is similar eliminating the β terms.

The uniformly valid solution accurate to $0(\varepsilon)$ becomes

$$\Psi = t^{\alpha} \operatorname{sh}^2 |\eta_0| \left[-\frac{\operatorname{sh}\eta \sin\xi}{(\operatorname{ch}\eta-\cos\xi)^2} - \sum_{k=1}^{\infty} 2ke^{k\eta_0} \frac{\operatorname{sh}k\eta}{\operatorname{sh}k\eta_0} \sin k\xi \right] \tag{18}$$

$$+\varepsilon t^{\alpha+1/2}\sqrt{\nu'} \left\{ \operatorname{sh}|\eta_0| \frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} \sum_{n=1}^{\infty} \left[\frac{\operatorname{sh} n(\eta-\eta_0)}{\operatorname{sh} n\eta_0} \bar{\gamma}_n - \frac{\operatorname{sh} n\eta}{\operatorname{sh} n\eta_0} \gamma_n \right] \sin n\xi \right.$$

$$\left. + 2^{2\alpha+1}\Gamma(\alpha+1) \left[(B(\xi)-\beta) G_{\alpha+1/2}(\tilde{\eta}) - \bar{B}(\xi) G_{\alpha+1/2}(\tilde{\tilde{\eta}}) \right] \right\}.$$

To that order the rotatory motion influences only the solution pertaining to the boundary layer around the cylinder.

5. Flow properties

Flow separation and reversal occurs at zero skin friction points. Since the problem is two-dimensional the hydrodynamic force interaction of the body and the flow is defined by the surface force vector F and the surface force torgue M.

The points marking boundary layer detachment from the cylinder accurate to zeroth and first approximations can be given by

$$\left.\frac{\partial u_0}{\partial \tilde{\eta}}\right|_{\tilde{\eta}=0} + \varepsilon \left.\frac{\partial u_1}{\partial \tilde{\eta}}\right|_{\tilde{\eta}=0} - \varepsilon t^{1/2} 2\sqrt{\nu'}\, u_0 |_{\tilde{\eta}=0} = 0. \tag{19}$$

The force and the torgue are figured from

$$F_x = \int_0^{2\pi} (p_0+\varepsilon p_1) \frac{\operatorname{sh}|\eta_0|(1-\operatorname{ch}\eta_0\cos\xi)}{(\operatorname{ch}\eta_0-\cos\xi)^2} d\xi + \varepsilon t^{-1/2} \frac{\sqrt{\nu'}}{2} \int_0^{2\pi} \frac{\operatorname{sh}|\eta_0| \operatorname{sh}\eta_0 \sin\xi}{(\operatorname{ch}\eta_0-\cos\xi)^2} \left.\frac{\partial u_0}{\partial\tilde{\eta}}\right|_{\tilde{\eta}=0} d\xi, \tag{20}$$

$$F_y = -\int_0^{2\pi} (p_0+\varepsilon p_1) \frac{\operatorname{sh}|\eta_0| \operatorname{sh}\eta_0 \sin\xi}{(\operatorname{ch}\eta_0-\cos\xi)^2} d\xi + \varepsilon t^{-1/2} \frac{\sqrt{\nu'}}{2} \int_0^{2\pi} \frac{\operatorname{sh}|\eta_0|(1-\operatorname{ch}\eta_0\cos\xi)}{(\operatorname{ch}\eta_0-\cos\xi)^2} \left.\frac{\partial u_0}{\partial\tilde{\eta}}\right|_{\tilde{\eta}=0} d\xi, \tag{21}$$

$$M = t^{-1/2} \frac{\sqrt{\nu'}}{2} \int_0^{2\pi} \left.\frac{\partial u_0}{\partial\tilde{\eta}}\right|_{\tilde{\eta}=0} d\xi. \tag{22}$$

We shall set $T_1 = \dfrac{\text{actual time}}{L\,|\,U_\infty} = \varepsilon t$ and from (19) we obtain the following equation for T_1

$$(\nu' R)^{\alpha} \left\{ B(\xi)C(\xi) \left[-\frac{4}{2+\alpha}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha-1/2)} + \frac{(2\alpha+1)(2\alpha+9/4)}{1+\alpha}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+5/2)} - \frac{4(1-\alpha)}{1+\alpha}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1/2)} \right.\right. \tag{23}$$

$$\left. -\frac{2\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} + \left(\frac{4\alpha}{2+\alpha} - \frac{1}{1+\alpha} - \frac{2\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2)\Gamma(\alpha+5/2)} + \frac{4(1-\alpha)}{1+\alpha} + \frac{\Gamma^2(\alpha+1)}{\Gamma^2(\alpha+3/2)} \right) \frac{\Gamma(2\alpha+2)}{\Gamma(2\alpha+3/2)} \right]$$

$$+\beta C(\xi) \left[\frac{4}{2+\alpha}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha-1/2)} - \frac{(2\alpha+1)(2\alpha+9/4)}{1+\alpha}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+5/2)} + \frac{4(1/2-\alpha)}{1+\alpha}\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+1/2)} + 2\frac{\Gamma(\alpha+1)}{\Gamma(\alpha+3/2)} \right.$$

$$\left.\left. - \left(\frac{4\alpha}{2+\alpha} - \frac{1}{1+\alpha} - \frac{2\Gamma^2(\alpha+1)}{\Gamma(\alpha+1/2)\Gamma(\alpha+5/2)} + \frac{4(1/2-\alpha)}{1+\alpha} + \frac{\Gamma^2(\alpha+1)}{\Gamma^2(\alpha+3/2)} \right) \frac{\Gamma(2\alpha+2)}{\Gamma(2\alpha+3/2)} \right] \right\} T_1^{\alpha+1}$$

$$-\frac{1}{\sqrt{R}} [2A(\xi)+B(\xi)+3\beta] T_1^{1/2} + [B(\xi)-\beta] \frac{2\Gamma(\alpha+1)}{\Gamma(\alpha+1/2)} = 0.$$

Certain computations have been made to find out the dependence of the solution on: (i) acceleration parameter α; (ii) parameter β, which is zero for a static cylinder and one for a rotating cylinder; (iii) the Reynolds number R, that is fluid viscosity; and (iv) gap parameter d.

Figures 1 and 2 show the streamline patterns for an impulsive and a uniformly accelerated flows at $R=100$ and various d. Flow and vortices are asymmetrical and highly dependent on the position of the cylinder with respect to the solid plane. There are two vortices at the back and not a single one to the front. Note that the separation streamline $\Psi=0$ forks on the plane at the stagnation point,

Figures 3 and 4 show the detachment time curves T_1 for an impulsive ($\alpha=0$) and a uniformly accelerated ($\alpha=1$) flows past a rotating ($\beta=1$) and a static ($\beta=0$) cylinder in the presence of a solid plane for $d\approx 0.5, 7$ and $R=100$. Having once begun detachment moves up and downstream. Body rotation lessens the minimal detachment time $T_{1\min}$.

Dependence of the initial time of boundary layer detachment from a rotating cylinder on the Reynolds number at $\alpha=0{,}1$ and $d\approx 0.5, 3$ has been studied and findings are plotted in Fig. 5. Function behaviour differs for the cases of an impulsive and a uniformly accelerated flows. Increase of fluid viscosity goes hand in hand with decrease of $T_{1\min}$ at $\alpha=0$ and increase of $T_{1\min}$ at $\alpha>0$ which only confirms our presumptions.

The F_y component normal to the flow is zero accurate to $0(\varepsilon)$. The established dependence of the torgue on parameters α, β and R is qualitatively similar to that in (5).

6. Conclusion

Flow behaviour is highly dependent on flow acceleration, body rotation, fluid viscosity and the position of the cylinder in relation to the plane.

Cylinder rotation affects the boundary layer around the plane wall accurate to $0(\varepsilon^2)$. This explains the fact that to a lower order $\Psi=0$ crosses the plane at the stagnation point and the force which the body exerts on the fluid is in a direction reverse to that of the flow.

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