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Synthesis of the primary pitch cones of hyperbolic gearings, one of them being of cylindric form

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The design of space gearings transforming a rotation between skew axes by means of high-order kinematic joints is related to the choice of the meshing point location, i. e. to the choice of the pitch point place between the tooth surfaces. This point is also a point of the primary pitch cones of the designed tooth mechanism. Therefore, the choice of the pitch point place determines their form, measures and mutual displacement which has a definite effect on the quality at the meshing point location reflecting on the efficiency of the gearing, on the loading capacity, on breaking strength, on the reliabilily and the durability, on the smoothness and noiseless of the gearing operation, and so on.

Statement of the problem

The necessary and sufficient conditions for the existence of the primary pitch cones H_i $(i=1, 2)$ are (fig. 1) [1, 2]

a) their axes of rotation should coincide with the axes of the elements of the gearing and should be firmly connected with them;

b) H_i have a common tangent plane T passing through a common contact point P (the pitch point), i. e. the following condition should be satisfied:

$$\bar{m}_i . \bar{V}_{12}=0, \quad (1)$$

where $\bar{m}_i$ is the unit normal vector of the primary pitch cone H_i; $\bar{V}_{12}$ is the sliding velocity vector between the contact tooth surfaces at the same point.

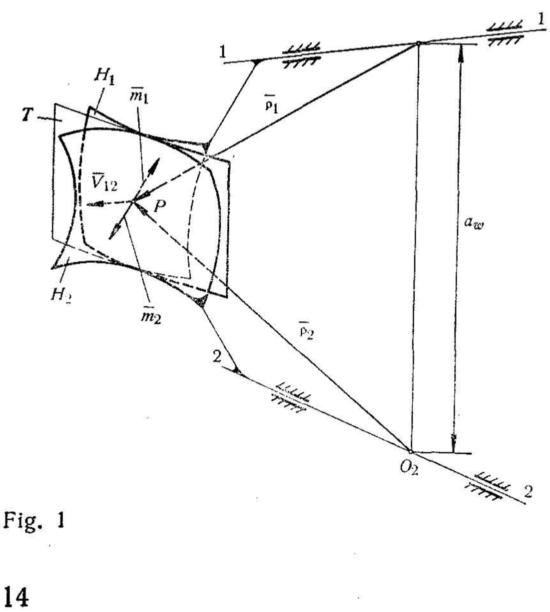

Fig. 1

Three approaches exist. They concern the choice of the primary pitch cones of the hyperbolic gearings [3]:

First approach: According to it, the primary pitch cones are parts of axoid hyperboloids of space mechanisms. It is necessary to orient their tooth surfaces so that when the rotation is transferred they should contact with one another along the generatrices of these hyperboloids. All this is related to the effort to synthesize hyperbolic gearings having a minimal sliding velocity between the tooth surfaces. However, in practice, due to certain technological arguments, the parts of the axoid hyperboloids are approximated to simple surfaces — cylinders or cones. Besides, it is practically impossible to relate the pitch of the teeth to the orientation of the straight-line generatrices of the hyperboloids. Hence, the gearing, actually accomplished, has primary pitch cones which differ considerably from the corresponding parts of the axoid hyperboloids, and hence, the sliding velocity is not minimal.

Second approach: The primary pitch cone of the pinion is a cylinder or cone and the primary pitch cone of the gear is an envelope of the first one when this primary pitch cone rotates round the axis of the second one. In this case the second primary pitch cone is nonlinear undevelopable. The determined undevelopable surface is approximated to the developable one in order to improve the technology of the designed space tooth mechanism. Obviously, the developable surface obtained by means of approximation is not an envelope of the first primary pitch cone.

Third approach: The primary pitch cones of the gearing have the most technological form — cones, cylinders or discs. The dependence between the parameters characterizing their measures and mutual displacement in space is determined by means of condition (1) of their contact in the pitch point.

It is better to utilize the third approach since the choice of the optimal position of the pitch point, and consequently, the choice of the optimal geometry of the primary pitch cones, is based on the criteria controlling a definite quality at the meshing point location of the designed gearing.

Condition (1) treating the existence of the primary pitch cones H_i (fig. 1) is equivalent to the vector equations [2]:

$$\bar{\rho}_1=\bar{\rho}_2, \quad \bar{m}_1=\bar{m}_2, \tag{2}$$

where $\bar{\rho}_i$ $(i=1, 2)$ is the radius-vector of the pitch point P_i as a point of H_i.

Investigation of the geometry of the primary pitch cones of hyperbolic gearings when one of them has a cylindric form

Vector equations (2) appears to be

$$\left|\begin{array}{l} -r_1\cos\theta_1=k_{a_2}a_2\cos\gamma\pm r_2\cos\theta_2\sin\gamma, \\ \pm r_1\sin\theta_1=-a_w+r_2\sin\theta_2, \\ -a_1=\pm k_{a_2}a_2\sin\gamma-r_2\cos\theta_2\cos\gamma, \\ \sin\theta_1=\cos\delta_2\sin\theta_2, \\ \sin\delta_2\sin\gamma-\cos\delta_2\cos\theta_2\cos\gamma=0, \end{array}\right. \tag{3}$$

when using the constructions for external and internal meshing shown on fig. 2 and fig. 3 in the general case (H_2 is a cone). Here r_i, θ_i, a_i $(i=1, 2)$ are the cylindric coordinates of the point P_i as a point of H_i; γ is the nonorthogonal angle of the skew axes 1—1 and 2—2; a_w is the shortest distance between the axes; δ_2 is one half of the pitch cone angle of H_2.

In system (3) and in the expressions that follow the upper sign is related to the case of externally meshed primary pitch cones, while the lower sign is associated with internally meshed ones; $k_{a_2}=-1$ for external meshing and in the case of internal meshing when the shortest distance a_w between the axes 1—1 and 2—2 is outside H_2, and $k_{a_2}=1$ for internal meshing when is inside H_2.

The unknowns in system (3) are 9: r_1, θ_1, a_1, r_2, θ_2, a_2, δ_2, γ and a_w. For the existence of a unique solution four of them are specified—a_w, γ, r_1 and a_1. Gearings

performing rotations between skew axes are under consideration. Therefore, $\theta_2 \in \left(0, \frac{\pi}{2}\right]$.
System (3) is transformed into:

$$
(4)\qquad
\begin{cases}
r_2 = (a_w \mp r_1 \sin\theta_1)/\sin\theta_2, \\
a_2 = -k_{a_2}(\pm a_1 \sin\gamma + r_1 \cos\theta_1 \cos\gamma), \\
a_1 \cos\gamma \mp r_1 \cos\theta_1 \sin\gamma = (a_w \mp r_1 \cos\theta_1) \operatorname{ctg}\theta_2, \\
\cos\delta_2 = \sin\theta_1/\sin\theta_2, \\
\sqrt{1 - \sin^2\theta_2 \cos^2\gamma}\,.\,\sin\theta_1 = \sin\theta_2 \sin\gamma.
\end{cases}
$$

It is natural to study the following cases:

I. $\gamma \neq 0$ and $\theta_2 \neq \frac{\pi}{2}$

Then the geometric parameters of the primary pitch cones are determined by the expressions:

$$
(5)\qquad
\begin{aligned}
&\theta_2 = \operatorname{arcctg}(a_1 \cos\gamma/a_w), \\
&\theta_1 = \arcsin(\sin\theta_2 \sin\gamma/\sqrt{1 - \sin^2\theta_2 \cos^2\gamma}), \\
&\delta_2 = \arccos(\sin\theta_1/\sin\theta_2), \\
&r_2 = (a_w \mp r_1 \sin\theta_1)/\sin\theta_2, \\
&a_2 = -k_{a_2}(\pm a_1 \sin\gamma + r_1 \cos\theta_1 \cos\gamma).
\end{aligned}
$$

Obviously, a hyperbolic gearing (with external or internal meshing) exists if and only if $a_2 > 0$. Moreover, in the case of a gearing with external meshing which is admitted as a patent for an invention (see fig. 2) [4] the condition $\theta_i < \arcsin(a_w/r_1)$ should be satisfied if $a_w < r_1$. Two gearings with external meshing are produced. Their speed ratios are 41 and 56.

2. $\gamma = 0$ and $\theta_2 \neq \frac{\pi}{2}$

Then the geometric parameters characterizing the primary pitch cones are determined by the expressions

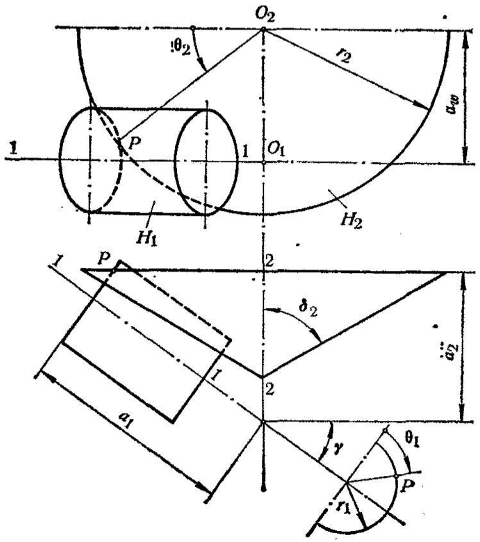

Fig. 2

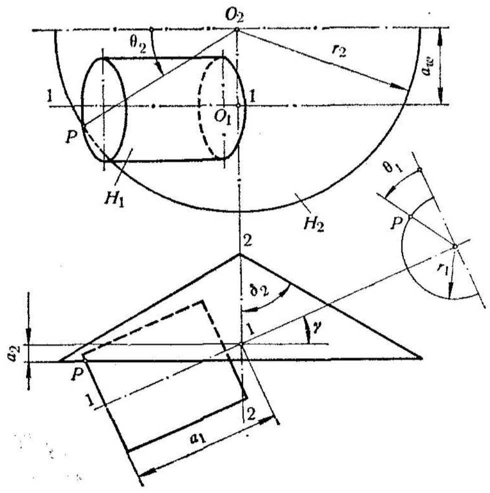

Fig. 3

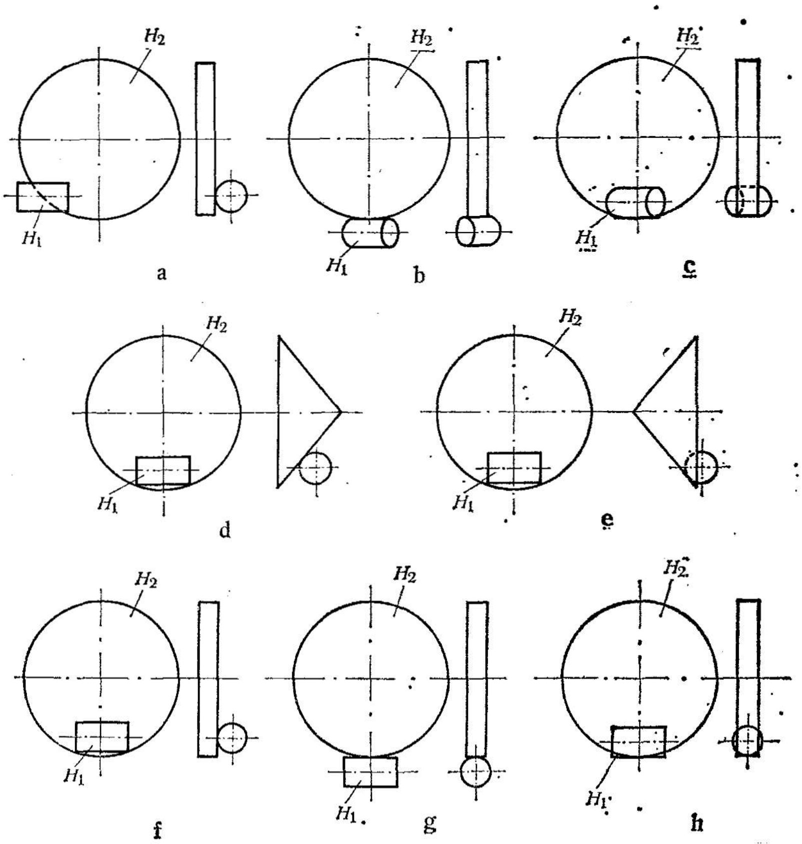

Fig. 4 a, b, c, d, e, f, g, h

$$\theta_2=\text{arcctg}\,(a_1/a_w),\quad \theta_1=0,\quad \delta_2=\frac{\pi}{2},$$
(6)
$$r_2=a_w/\sin\theta_2,\quad a_2=-k_{a_2}r_1.$$

The solution (6) of system (3) shows that it is impossible to have a gearing with internal meshing when a_w is inside the primary pitch cone H_2. Algorithm (6) fixes the measures and mutual displacement of the primary pitch cones of a gearing "helicon" when the cone H_2 is degenerated disc (Fig. 4, a).

3. $\gamma\neq0$ and $\theta_2=\frac{\pi}{2}$.

The the solution of system (3) is

(7) $$\theta_1=\frac{\pi}{2},\ \delta_2=0,\ a_1=a_2=0,\ r_2=a_w\mp r_1.$$

These parameters correspond to a non-orthogonal worm or screw gearing with external (Fig. 4, b) and internal (Fig. 4, c) meshing, when the cone H_2 is transformed into a cylinder. Obviously, the non-orthogonal worm gearing with external meshing exists only if $a_w>r_1$.

4. $\gamma=0$ and $\theta_2=\frac{\pi}{2}$

In this case, system (4) is transformed as follows:

$$(8)\qquad \left| \begin{array}{l} r_2 = a_w \mp r_1 \sin\theta_1, \\ a_2 = -k_{a_2} r_1 \cos\theta_1, \\ a_1 = 0, \\ \cos\delta_2 = \sin\theta_1, \\ 0 \,.\, \sin\theta_1 = 0. \end{array} \right.$$

This system has unlimited number of solutions which can be described by

$$(9)\qquad \delta_2 + \theta_1 = \frac{\pi}{2},\quad r_2 = a_w \mp r_1 \sin\theta_1,\quad a_2 = -k_{a_2} r_2 \cos\theta_1.$$

According to (9), a hyperbolic gearing with external meshing (shown on Fig. 4, d) exists only if $\sin\theta_1 < a_w/r_1$, and a $a_2 > 0$ hyperbolic gearing with internal meshing (Fig. 4, e) exists if a_w is outside H_2.

Describe the following two cases:

a) $\theta_1 = 0$. Then $\delta_2 = \frac{\pi}{2}$, $a_2 = r_1$, $a_2 = a_w$. This corresponds to the geometric parameters of a toroid gearing (Fig. 4, f);

b) $\theta_1 = \frac{\pi}{2}$. Whence, $\delta_2 = 0$, $a_2 = 0$, $r_2 = a_w \mp r_1$.

This solution fixes an orthogonal worm or screw gearing with external or internal meshing accordingly (Fig. 4, g and 4, h). It should be mentioned that an orthogonal worm gearing with external meshing exists if $a_w > r_1$ while it is impossible to produce an orthogonal worm gearing with internal meshing because the bearing of the pinion cannot be accomplished.

Conclusion

The present paper motivates the method we have accepted for the determination of the parameters characterizing the measures and mutual displacement of the primary pitch cones of this class of hyperbolic gearings on the basis of a survey we have made on the known approaches for synthesis of primary pitch cones. The investigation allowed to find the algorithms for calculating the geometry of the primary pitch cones of all hyperbolic gearings, when one of the primary pitch cones has a cylindric form.

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