БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ
ТЕОРЕТИЧНА И ПРИЛОЖНА МЕХАНИКА
София, 1987, год. XVIII, № 2

BULGARIAN ACADEMY OF SCIENCES
THEORETICAL AND APPLIED MECHANICS
Sofia, 1987, Year XVIII, No 2

Some Results of the Investigation of the Conditions for Non-existence of Ordinary Nodes on the Contact Lines of Spiroid Gearings

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1. Introduction

The spiroid gearings belong to the class of hyperbolic spatial mechanisms and occupy the field between the worm gearings and hypoid ones. This field is defined by the two following characteristics:

Geometric-kinematic ones. The meshing zone of the spiroid gearings is displaced from the offset line O_2O along the axis 1—1 of the first gear as well as along the axis 2—2 of the other one (Fig. 1). The small gear 1, called spiroid worm, is a conic worm with non-symmetric helical teeth and the large one 2, called spiroid gear, is a conic gear with non-symmetric spiral teeth [1, 2]. This defines the basic designation of the gearing under consideration that is a transmission of high reduction rotation between skew axes.

Technological ones. The tooth surfaces of the spiral teeth of the spiroid gear are generated by second Oliver's principle [3]. Due to this, the spiroid gear is cut by a hob whose form, measures, relative position and relative motion are like the ones of the spiroid worm. In result, a conjugate linear contact between the tooth surfaces is realized relatively easy.

2. Characteristics of the contact in the ordinary nodes

Spiroid gearings with orthogonally skew axes are considered in this article (Fig. 1).

It is known [3] that one of the basic problems of the geometric synthesis of the spiroid gearings is the synthesis of their tooth flanks $\Sigma_i^{(j)}$. The indexation $i=1, 2$ refers to the tooth surfaces of the spiroid worm and of the spiroid gear, respectively, while the indexation $j=1, 2$ relates to the surfaces the helical and spiral teeth operate with in low-side and high-side driving, respectively. On Fig. 1, the directions of the angular velocity vectors $\bar{\omega}_1$ and $\bar{\omega}_2$ correspond to low-side driving. The indexation $j=1, 2$ will be introduced only when necessary. Besides, the upper signs in the analytical relations that follow apply for $j=1$ and the lower signs apply for $j=2$.

The technology of manifacture of the spiroid gearings is based on the second Oliver's principle. The basic surface Σ_1 is a linear conic helicoid [1, 2]. Led by mo-

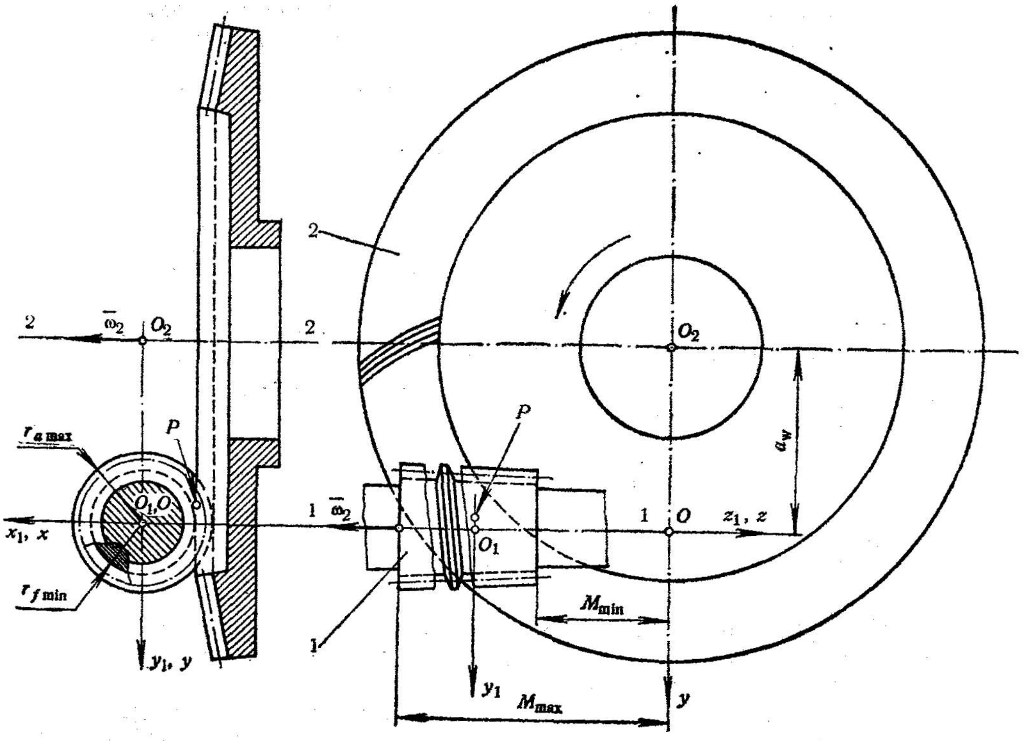
2
$\overline{\omega}_2$
O_2
2
O_2
$r_{a\,max}$
P
O_1, O
x_1, x
$r_{f\,min}$
y_1, y
1
$\overline{\omega}_2$
P
O_1
1
O
z_1, z
a_w
M_{min}
y_1
M_{max}
y
2
1

Fig. 1

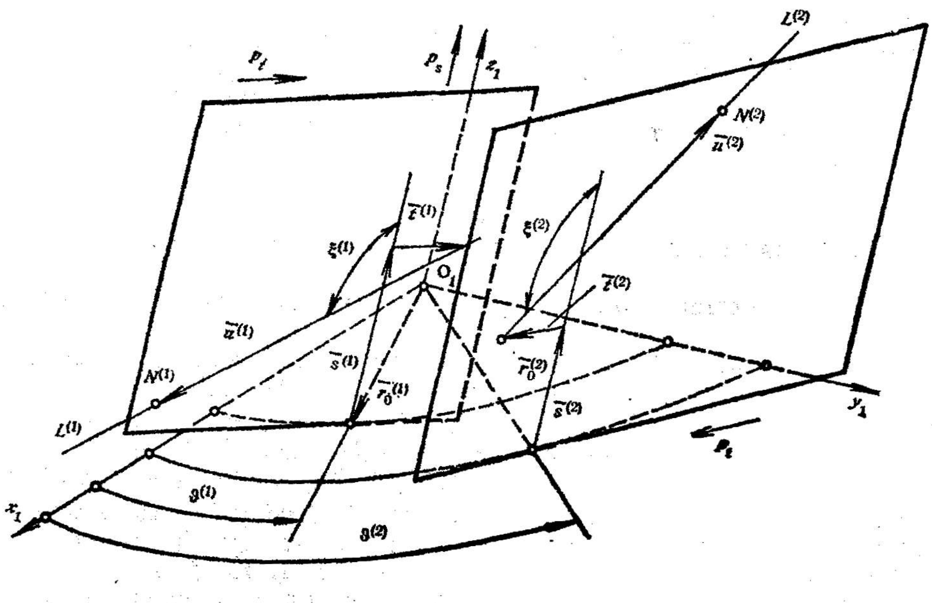

Fig. 2

tives aiming generality and according to I 1, 5 I the investigation will be performed when Σ_1 is a convolute helicoid. The two helicoids $\Sigma_1^{(1)}$ and $\Sigma_1^{(2)}$ used as tooth flanks of a spiroid worm with right-hand helical teeth are described by the straight-line generatrices $L^{(1)}$ and $L^{(2)}$ shown on Fig. 2. The straight line $L^{(j)}$ performs a right-hand helical motion consisting of I 3, 4, 5 I: a) helical motion along the axis O_1z_1 of the worm with a parameter p_s and b) helical motion with a parameter p_t in the plane which is a tangent to the cylinder having a radius $r_0^{(j)}$ equal to a constant and this motion is performed perpendicularly to the axis O_1z_1. The angle between $L^{(j)}$ and the positive direction of the axis O_1z_1 is $\xi^{(j)}$, $\frac{\pi}{2} < \xi^{(j)} < \pi$ $(j=1, 2)$.

Then the equations of the convolute helicoids $\Sigma_1^{(j)}$ $(j=1, 2)$ in the co-ordinate system S_1 firmly connected with the worm appear to be

$$x_1 = r_0 \cos\theta + R_0 \sin\theta$$
$$y_1 = r_0 \sin\theta - R_0 \cos\theta$$
$$z_1 = p_s\theta \pm u\cos\xi$$
$$R_0 = \pm(u\sin\xi - p_t\theta),$$

where u and θ are co-ordinates of Σ_1.

On the surface Σ_1 with a vector equation

(2) $$\bar{\rho}_1 = \bar{\rho}_1(u, \theta)$$

a family Q of contact lines appears when adding to (2) the basic equation of meshing 161

(3) $$\bar{n}_1 \cdot \bar{v}_{12} = f(u, \theta, \varphi) = 0,$$

where $\bar{n}_1$ is the normal vector to Σ_1, $\bar{v}_{12}$ is the sliding velocity vector between Σ_1 and Σ_2, φ is a parameter of meshing.

Using (1) it is easy to determine the projections of $\bar{n}_1$ in the system S_1

(4) $$n_{1,x_1} = \mp(h\sin\xi\cos\theta + R_0\cos\xi\sin\theta)$$
$$n_{1,y_1} = \mp(h\sin\xi\sin\theta - R_0\cos\xi\cos\theta)$$
$$n_{1,z_1} = \pm R_0\sin\xi.$$

Here $h = p_s \pm p_t \operatorname{ctg}\xi + r_0 \operatorname{ctg}\xi$ is a parameter of distribution 13, 41.

The sliding velocity vector in the pitch point P considered as a point belonging to Q is

(5) $$\bar{v}_{12} = \bar{v}_1 - \bar{v}_2 = \bar{\omega}_{12} \times \bar{\rho}_1 + \bar{\omega}_2 \times \bar{a}_W,$$

where ρ_i and v_i are the radius vector and the circumferential velocity vector of the contact point P_i belonging to the driving element i, respectively, $\bar{\omega}_{12} = \bar{\omega}_1 - \bar{\omega}_2$ is the relative angular velocity vector, $\bar{a}_W$ is the offset vector. In (5) it is assumed that $\omega_1 = 1\ s^{-1}$, whence $\omega_2 = i_{21} = 1/i_{12}\ s^{-1}$ (i_{12} is the angular velocity ratio of the gearing).

Equation (5) turns to be

(6) $$v_{12,x} = y, \quad v_{12,y} = i_{21}z - x, \quad v_{12,z} = -i_{21}(a_W + y)$$

in the stationary coordinate system S.

Let the surface Σ_1 is regular. The points belonging to the curve Q such that

(7) $$\bar{v}_{r,1} = \frac{\partial\bar{\rho}_1}{\partial u} \cdot \frac{du}{dt} + \frac{\partial\bar{\rho}_1}{\partial\theta} \cdot \frac{d\theta}{dt} = \bar{0},$$

when

(8) $$\bar{n}_1 = \frac{\partial \bar{\rho}_1}{\partial u} \times \frac{\partial \bar{\rho}_1}{\partial \theta} \neq \bar{0}$$

are called ordinary nodes. There should not exist ordinary nodes in the meshing zone since large wear and tear of the surfaces as well as loss of pressure in the oil film occur because of the large relative sliding in these points 161.

The differential of the function $f(u, \theta, \varphi)$ is

(9) $$\frac{df}{dt} = \frac{\partial f}{\partial u} \cdot \frac{du}{dt} + \frac{\partial f}{\partial \theta} \cdot \frac{d\theta}{dt} + \frac{\partial f}{\partial \varphi} \cdot \frac{d\varphi}{dt} = 0.$$

Obviously, if there exist ordinary nodes in the meshing zone of the spiroid gearings, then

(10) $$\frac{\partial f}{\partial \varphi} = 0$$

(see (7), (8) and (9)).

It is easy to prove that the condition

(11) $$\frac{\partial f}{\partial \varphi} \neq 0,$$

when (8) is satisfied, is a condition for non-existence of ordinary nodes on the contact lines of the gearing.

3. Condition for an avoidance of the ordinary nodes from the meshing zone of the spiroid gearings

When using (1), (4) and (6) as well as the limitations

(12) $$\sqrt{r_{f,\min}^2 - r_0^2} \leq |u \sin \xi - p_t \theta| \leq \sqrt{r_{a,\max}^2 - r_0^2}$$
$$M_{\min} \leq |z| \leq M_{\max}, \quad M_{\min} \neq 0$$

the condition (11) for the convolute helicoid is equivalent to the union of the following systems of conditions

(13) $$M_{\max} < r_0 |h|^{-1} \sqrt{r_{f,\min}^2 - r_0^2}$$
$$\frac{a_w - r_0}{p_1} + \frac{M_{\max} |h|}{p_1 \sqrt{r_{f,\min}^2 - r_0^2}} < i_{12} < \frac{a_w + r_0}{p_1} - \frac{M_{\max} |h|}{p_1 \sqrt{r_{f,\min}^2 - r_0^2}}$$

and

(14) $$M_{\min} > r_0 |h|^{-1} \sqrt{r_{a,\max}^2 - r_0^2}$$
$$\frac{a_w + r_0}{p_1} - \frac{M_{\min} |h|}{p_1 \sqrt{r_{a,\max}^2 - r_0^2}} < i_{12} < \frac{a_w - r_0}{p_1} + \frac{M_{\min} |h|}{p_1 \sqrt{r_{a,\max}^2 - r_0^2}}.$$

Here $r_{f,\min}$ and $r_{a,\max}$ are the minimal internal and maximal external radii of the spiroid worm, respectively, $M_{\min}$ and $M_{\max}$ are the minimal and maximal mounting displacement starting from the offset line O_2O of the gearing (Fig. 1), $p_1 = p_s \pm p_t \operatorname{ctg} \xi$.

This union fixes the domain of existence of the convolute spiroid gearings such that their contact lines have no ordinary nodes.

Having in mind the geometric characteristics of the involute and the archimedian spiroid gearings I 3, 4, 5 I the following conditions define the domain of existence of these gearings without ordinary nodes in their meshing zones, respectively.

(15)
$$\frac{a_W - r_0}{p_s \pm p_t \operatorname{ctg} \xi} < i_{12} < \frac{a_W + r_0}{p_s \pm p_t \operatorname{ctg} \xi}$$

(16)
$$\frac{a_W}{p_s \pm p_t \operatorname{ctg} \xi} - \frac{M_{\min}}{r_{a,\max}} < i_{12} < \frac{a_W}{p_s \pm p_t \operatorname{ctg} \xi} + \frac{M_{\min}}{r_{a,\max}}$$

(13), (14), (15) and (16) transform into conditions for an avoidance of the ordinary nodes on the contact lines of helicon gearings when it is assumed $p_t = 0$ in them.

4. Conclusion

When designing convolute, involute and archimedian spiroid gearings the conditions (13), (14), (15) and (16) make possible to look for such constructive solutions that there exist no ordinary nodes in their meshing zones. In result, the negative consequences as pitting, scoring and wear and tear of the tooth surfaces as well as the efficiency values and hydrodynamic loading capacity of the spatial gearings under consideration decrease. The created technical solutions of this class of gearings are objects of three patents which are admitted as inventions I 7, 8, 9 I.

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Received 12. 03. 1986.