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Dendritic Pattern Formation

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Extended Abstract

We present a numerical analysis of dendritic crystal growth. The growth of dendritic crystals occurs when the relevant transport of heat or matter takes place in the medium ahead of the advancing interface, such that the crystallization front moves into a supercooled or supersaturated region.

A dendrite then is an approximately parabolic needle-crystal with tree-like sidebranches. Under constant experimental conditions it advances at a well-defined velocity producing sidebranches at an approximately constant rate.

The system can be parameterized by three independent length scales: *a*) the capillary length d_0 being a measure of surface tension and stiffness, *b*) the critical radius R_c being a measure of the supercooling, *c*) the diffusion length l as a measure of diffusion speed.

Detailed definitions can be found in ref. [1], a series of precision experiments are reported in ref. [2]. The first approximate solution to this problem [3] ignoring surface-tension leads to a parabolic form of the growing crystal, the Ivantsov-parabola. It gives a relation $V \sim R_0^{-1}$ between the growth rate V and the radius of curvative R_0 at the tip of the parabola. Including surface tension in an approximate form, it was later argued [1], that the stability length $\lambda_s = 2\pi\sqrt{ld_0}$ plays a crucial role for the velocity selection and sidebranch spacing. More recently it was found [4—6], that crystalline anisotropy is needed to allow for a discrete set of stationary needle crystals to be solutions of the problem.

The fastest of these needle crystals was conjectured to describe the operating mode of the dendrite, as it is weakly stable against the formation of sidebranches [5, 7, 8].

The presently available analytical results, however, are still not conclusive about the sidebranching mode of operation, the usual experimental finding. We have, therefore, performed a numerical analysis using a Green's function technique [9]. The results are summarized as follows. The supercooling should be expressed by the Peclet number $p = R_0/l$, which seems to make the result independent of dimensionality. The growth rate then can be scaled for different supercoolings and anisotropies as

$$\sigma = \frac{d_v}{2Dp^2} v, \qquad (1)$$

with D being the diffusion coefficient. Then $\sigma = \sigma(\varepsilon)$ is a function of the relative anisotropy ε of the surface tension only (kinetic anisotropy was not considered), whose functional relation is given in [4, 10]. The wavelength λ of the sidebranch-spacing, defined as the distance travelled by the dendrite on average between generation of two neighboring branches, scales with the stability length [1, 6—8]

$$\lambda \gtrsim \lambda_s. \tag{2}$$

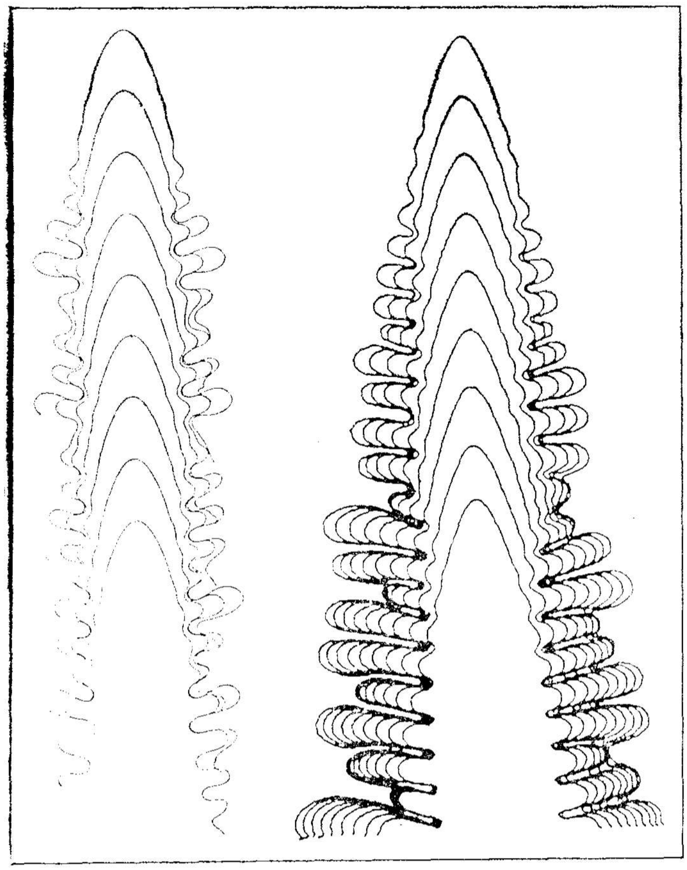

Fig. 1. Computed dendrite (left) [9] compared with experimental dendrite (right) [11]

The overall shape of the primary dendrite averaged over the sidebranches is almost parabolic, the deviation of tip radius from R_0 follows the prediction of ref. [4]. A comparison of our simulations [9] with recent image-processing results of experimental dendrites [11] shows striking similarity (Fig. 1). There are differences as the experiment shows a projection of a three-dimensional dendrite, where the sidebranches grow narrower than in our two-dimensional calculation. Our calculation was furthermore restricted to the tip region (with a smoothing approximation for the tail), while experimentally there is very little known about anisotropy and stiffness of the surface tension, not to speak of kinetic coefficients. Since our previous theory [1], ignoring the importance of anisotropy, gave the same scaling results for the dependence of V and R on the parameters, the numerical calculations presented here give the first confirmation for the validity of the more recent results concerning anisotropy.

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