

A Kinetic Model of Gas-Solids Suspension with Vapourizing Particles

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Let $f_i(r, \gamma_g, t)$, $f_{ia}(r, \gamma_a, t)$ — be smoothed in physically-infinitely small volume ΔV and time Δt functions of molecule distribution of i -kind gas both between particles and inside their pours, $h_a(r, \gamma_p, \Omega_p, t)$ — unary functions of distributions of a -kind solid particles; $\gamma = \{u, m\}$, u, m — velocity and angular momentum, t, r is tense and radius-vector. Instead of angular momentum angular velocity ω can be used. The magnitudes dealing with the gas molecules between particles and those in the pours and in the solid particles as well are denoted with the letters g, a and p subsequently. Variable Ω_p includes the inner energy of the particle A_p (or its temperature T_p), particle mass M_p and $N-1$ value, μ_{ia} i -kind gas mass absorbed by a -kind particle, where N is the number of gas molecules. If particles vaporize, Ω_p includes additionally value σ_p the particle surface with the constant particle temperature. Variable Ω_p can also include the surface and volumetrical particle porousness (β_p, ε_p). In case of considering the vaporizing particles as effective spheres, their volume can be excluded from the number of chance variables. The volume-time smoothing in the volume ΔV of the local functions distribution f_{ij} , satisfying Boltzmann equation and that of Enskog (1, 2) is obtained by way of calculation of the number of molecules of i -kind between the solid particles or inside their pours with values γ_g, γ_a from the given interval. The change of mathematical expectation of the considered gas molecules inside ΔV is due to their mutual collision between the articles and that of the element of the solid surface of the particle and the solid particle element belonging to a pour, and also to the "over-flow" of the smoothed gas molecule distribution through the particle surface. The latter is connected with the fact that functions f_i are continuous on the particle surface. The change of the mathematical expectation of i -kind gas molecules in the pours in a small interval of time is calculated in the same way, but the role of σ_p is played by the magnitude Σ_p which is the inner pour surface. The change of the mathematical expectation of the considered solid particles in a small interval of time is due to the collision of the particles. Hence, the system of kinetic equations of the vaporizing gas-solid suspensions will acquire the following appearance:

$$(1) \quad D_g f_i = \varepsilon J_{gg}(f, f) + J_{pi} + J_{pi} + c[\nabla \cdot (V_g f_i) + (\partial/\partial \gamma_g) \cdot (\dot{\gamma}_g f_i)]$$

$$(2) \quad D_a f_{ia} = c \varepsilon_a J_{aa}(f_a, f_a) + J_{\Sigma} - I_{ai}$$

$$(3) \quad C_p h_a = J_{pp}$$

where $\varepsilon = 1 - c$ is the volume porousness ΔV , c — is particle share of volume in it, ε_a is the porousness of all the particles in volume ΔV , ∇ — is the gradient operator, $V_g = u_g - v_g$ — is own velocity, v_g — is average mass gas velocity between the par-

ticles in the pores and solid particles depending on the index. The disjunctive point-stands for scalar product of two vectors or vector and tensor, and the two disjunctive points stand for a double-tensor product. The point above stands for differentiation in time along the trajectory. The differential operator

$$(4) \quad C_p h_\alpha = D_p h_\alpha + (\partial/\partial\Omega_p) \cdot (\dot{\Omega}_p h_\alpha),$$

and the differential operator D has the structure

$$(5) \quad D\varphi = \frac{\partial\varphi}{\partial t} + \nabla \cdot (u\varphi) + \frac{\partial}{\partial\gamma} \cdot (\dot{\gamma}\varphi).$$

The operators J_{gg} and J_{aa} , describing the collision of the gas molecules, have the similar appearance. In particular (1):

$$(6) \quad J_{gg}(f, f) = \sum_{j,k,l} \int \left\{ \frac{s_i s_j}{s_k s_l} \left(\frac{m_i m_j}{m_k m_l} \right)^3 Z_{ij}^g \left(r + \frac{1}{2} ed_{kl}^g \right) f_k(r, \gamma'_g, t) f_l(r + ed_{kl}^g, \gamma', t) - Z_{ij}^g \left(r - \frac{1}{2} ed_{ij}^g \right) f_i(r, \gamma_g, t) f_j(r - ed_{ij}^g, \gamma, t) \right\} |u_g - u(\sigma_{ij}^{kl}) d\omega_{kl} d\gamma, \quad (u_g - u) \cdot e > 0,$$

where σ_{ij}^{kl} is the differential collision cross-section, d_{ij}^g, e — is half-sum of molecule diameters and the direction of their centre lines, m_i — is molecule mass, s_i — is the statistical weight, Z_{ij}^g is the correlative gas molecule function. For both monoatomic and multi-atomic molecules, considered as rough spheres, the statistic weight is equal to unity. Provided the gas-molecule distribution function is averaged in rotations, that is $\gamma_g = u_g$, the statistic weight, generally speaking, is different from the unity. The operator of the double solid particle collision is obtained from (6) by the choice $s_i = 1$ and interaction cross-section looking as

$$|u_p - u| \sigma_{ij}^{kl} d\omega_{kl} = (d_{ij}^p)^2 \delta_{ik} \delta_{jl} (u_p - u) \cdot ede.$$

Here δ_{ij} is Kroneker symbol, d_{ij}^p is half-sum of particle diameters. The collision operator will contain the additional intergration by the variable Ω . Both the distribution and correlational functions are replaced by the similar magnitudes for the solid particle functions. The primed are the magnitude meanings resulting from collisions.

The interaction operator of the gas molecules with the solid particles describing the balance of the gas molecules which disperse on the particles, has the following appearance

$$(7) \quad J_{pi} = \sum_a \int d\gamma_p d\Omega_p \int_{\sigma_p} h_a \Gamma_{ia} d\sigma,$$

$$\Gamma_{ia} = (1 - \beta_p) \{ U \Psi_{ia} + J_g^\pm - (1 - U)/(u_g - w_p) \cdot n | f_i(r \in \sigma_p) \},$$

where $U = 1$, if $(u_g - w_p) \cdot n > 0$, and is equal to 0 otherwise; n — is the external normal to the particle surface: the term J_g^+ and J_g^- describe the interaction of two gas molecules with the element of particle surface (they are negligibly small in the rarified gas), the boundary function

$$(8) \quad \Psi_{ia} = \sum_k \int_{(u-w_p) \cdot n < 0} |(u-w_p) \cdot n| T_{ka}^i f_k(r \in \sigma_p) d\gamma + R_{ia}.$$

The term R_{ia} describes the surface particle emission, the effective boundary transformant is defined from the expression

$$T_{ka}^i = (\varepsilon \tilde{T}_{ka}^i f_{kl}(r+r', \dots) / f_k(r \in \sigma_p)),$$

\tilde{T}_{ka}^i is the density of probability of gas molecule dispersion on solid surface, and the procedure of volume-tense averaging is determined by the expression

$$\langle \varphi \rangle = \lim_{\Delta V, \Delta t \rightarrow \infty} \left\{ \frac{1}{\Delta V \Delta t} \int_0^{\Delta t} dt' \int_{\Delta V} \varphi(r+r', t+t', \dots) dr' \right\},$$

w_p is the velocity of the particle surface point. In the vicinity of the body (particle) the averaging in volume ΔV is replaced by averaging on surface element $\Delta S \rightarrow \infty$. In order to obtain J_g^\pm operator, the correlative gas molecule functions ought to be taken in the particle surface point in expression (6), by having the subintegral expression previously multiplied by $d_{ij}^g(e, n) > 0$. Interaction of gas molecules with pour is described by analogy with the help of distribution functions both falling (f_{ia}) on pour and reflected, that is gas molecules flying off it [1]. Provided the pour diametres are large in comparison with the average length of the freely moving of gas molecules,

$$(9) \quad J_{ai} = \sum_a \int \beta_p \tau_p h_a d\gamma_p d\Omega_p \nabla \cdot (V_a f_{ia} + J_g)$$

under the satisfactory extent of accuracy [2]. τ_p is particle volume. Vector J_g is derived from J_g^\pm by replacement of (e, n) by e .

The smoothed distribution functions f_i and f_{ia} are defined by the expressions $f_i = \langle \varepsilon f_{ii} \rangle$, $f_{ia} = \langle \varepsilon \varepsilon_a f_{ia}^a \rangle$. The operator J_Σ is calculated by analogy (7), but integration is performed along the pour surface inside particles, not on the particle surface, and the normal is considered to be directed into particles.

Since chance forces, force moments, etc. are taken in diffuse approximation, then

$$(10) \quad \begin{aligned} \dot{u}_g &= \dot{u}_g^{reg} - f_i^{-1} \nabla \cdot (B_1^g f_i), \quad \dot{m}_g = 0, \\ \dot{P} &= \dot{P}^{reg} - h_a^{-1} \left\{ \nabla \cdot (B_1^p h_a) + \frac{\partial}{\partial \gamma_p} \cdot (B_2^p h_a) + \frac{\partial}{\partial \Omega_p} \cdot (B_3^p h_a) \right\}, \quad P = M_p u_p, \dots, \end{aligned}$$

where \dot{u}_g^{reg} , $\dot{\gamma}_g^{reg}$ etc., are regular parts of the above-mentioned magnitudes. We've also got the analogous expressions for $\dot{\gamma}_a$. In particular

$$(11) \quad \dot{u}_g^{reg} = \dot{u}_a^{reg} + F_i, \quad \dot{P}^{reg} = M_p F_a - \sum_i \int_{\sigma_p} d\sigma \int m_i u_g \Gamma_{ia} d\gamma_g - \tau_p [\nabla \cdot \Pi_g - \beta_p \nabla \cdot (\Pi_a - \Pi_g)]$$

$$(12) \quad \begin{aligned} \dot{m}_p^{reg} &= - \sum_i \int_{\sigma_p} d\sigma \int (r_\sigma \times m_i u_g + m_g) \Gamma_{ia} d\gamma_g - \tau_p [\nabla \cdot \pi_g + \beta_p \nabla \cdot (\pi_a - \pi_g) \\ &+ i \times (\beta_p \Pi_a - (1 - \beta_p) \Pi_g) \cdot i + j \times (\beta_p \Pi_a + (1 - \beta_p) \Pi_g) \cdot j + k \times (\beta_p \Pi_a - (1 - \beta_p) \Pi_g) \cdot k], \end{aligned}$$

where F_i and F_a are forces of mass unity applied to both gas molecule and particle. It's quite easy to obtain general expressions for other values $\dot{\gamma}^{reg}$ and $\dot{\Omega}_p^{reg}$ (1) as well.

For diffusion tensors B_1^g , B_1^p , \tilde{B}_1^p and B_1 in (3) the following expressions are obtained:

$$(13) \quad \begin{aligned} B_1^g &= \tilde{B}_1^g + \frac{1}{2} I \times \dot{m}_g, \quad B_1^p = \tilde{B}_1^p + \frac{1}{2} I \times \dot{m}_p, \\ \tilde{B}_1^g &= -\frac{1}{6} I \sum_j \int n_j^g(r, t) Z_{ji}^g r_{jj} \frac{dU_j^g}{dr_{ji}} dr_{ji}, \end{aligned}$$

$$n_j^g = \int f_j d\gamma_g, \dots$$

Here I is a unit tensor. Integration on r_{ji} is carried out in the region $r_{ji} \geq \sigma$. In the point $r_{ji} = \sigma$ the interaction potential of gas molecules or particles vanishes. We have restricted ourselves to central potentials. Tensors B_1^g and B_1^p take into account attraction of gas molecules or particles as well as asymmetry of pressure in gas suspensions resulting from inter-phase interaction. These effects are tangible in concentrated gas suspensions particularly if the carrying phase is a dense gas. If particles happen to have no pores, $\beta_p = \varepsilon_p = \varepsilon_a = 0$, $f_{ia} = 0$. In this case the equation (2) is satisfied automatically.

For deriving of macroscopic equations describing the flow of each phase of gas suspension (that is air-mechanics phase equations) the methods of gas kinetic theory can be used [1]. The closed description of the media herewith is possible at the level of balance equations corresponding to additive invariant of the speedy process stages [1, 4]. As it is shown in [1], the stream members (pressure tensor, heat stream vector etc.) contain not only kinetic and potential parts but also corrections conditioned by, average values of diffusion tensors, standing in the kinetic system under three-dimensional gradients.

In particular pressure tensor in solid phase has the appearance

$$(14) \quad \Pi_p = \sum_{\beta} \int (M_p V_p V_p + B_1^p) h_{\beta} d\gamma_p d\Omega_p + \Pi_p^0.$$

If particles are not porous, and they are slowly vaporizing, the results considered in this monography [1] can be fully transferred to the particular case considered here. Weakly non-equilibrium solving of the system (1)---(3) is possible provided the particle concentration and gas rarefaction are not too small. In this case the usual modification of Chapman-Enskog method can be used [1, 4]. Provided the energy of the chaotic particle movement is small, the modification of the method under the random relation between the colliding and Focker-Plank terms of the kinetic equation [1] may be used. In this case one has to deal with integral-differential equation (unlike the linear-integral one in the case of the usual method of Chapman-Enskog). Fortunately, it is solvable, too.

The asymptotic analysis of the solving of the kinetic system of gas suspension equations under different rates of physical and chemical transformations can be found in the monograph [1]. Diffusion tensors and their regular parts γ and Ω_p are interconnected with their regular parts. Like in [1], these connections can be found by using equilibrium solving of the kinetic system. This will result into some differential system serving for determination of appropriate diffusion tensors. In case the mentioned tensors only depend on coordinates and time, we'll obtain expressions for their estimation considered in [1].

Statement of the boundary problem for gas suspension demands boundary conditions on streamlined surfaces. Some multy-atom molecules, clusters, and the majority of real suspensions can be considered as rough spheres colliding with the surface incompletely. In such cases the diagram of integration of particles with the body offered in [1] can be applied. In case only one kind particles are considered the conditions of slipping particles, the energy stream of chaotic particle movement, their pressure on the body and other conditions on the body, considered in [1] can be used as boundary conditions for air-dynamic equations of solid particle phase. The expression for inter-phase force of weakly-vaporizing particles comply with the results of Anderson and Jackson [5].

The offered kinetic model takes into account main hydro-dynamic effect comparatively fully. It can also be applied to a wider range of gas suspensions than the model of multy-penetrating media, including the latest one.

References

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