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On the independent dynamics controllability of manipulator systems

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1. Introduction

Industrial robots are very complex (nonlinear and coupled) mechanical systems which are assigned to perform various dynamic tasks. The coupled and nonlinear dynamic robot model presents a formidable problem for control engineers. The continuously increasing demands for enhanced performance and productivity emphasize the need of integrated design of the mechanical configuration, drive and control system.

For the present, each degree of freedom of industrial robots is controlled independently and some of the main arguments for it are the following:

— The complete and exact modelling of manipulator dynamics is difficult if not impossible to be realized. It is not so easy some of the parameters like friction forces, backlashes and elastic stress-strain components to be identified.

— There is computation burden and on-line limitations when coupled dynamics models are used in the control systems.

— Joint position and velocity feedback sensors are inherently implemented in the independent PD, PID, etc. joint controllers.

A natural way to overcome the dynamic complexity is to modify the design of arm linkages and joint actuators so that the links are to be driven more independently. The central role in designing a robot with appropriate dynamic characteristics plays the inertial matrix. Recently, some conceptual frameworks have been developed in order to specify the inertia matrix, leading to a significant reduction of the dynamics complexity [1, 2]. Mass distribution techniques and corresponding arm configurations are presented in these works leading to diagonal dominant inertial matrices.

When the geometrical and inertial parameters of manipulator links are specified, some problems on the optimal design of manipulator drive and control systems can be formulated. On the base of a diagonal dominance condition for the inertia matrix, torques limits have been specified in [3] to achieve independent joint dynamics controllability. Such a condition is referred to system with cartesian, cylindrical and spherical geometrical schemes. The best design of gear ratios and motor impedances for achieving quick and isotropic velocity convergence is formulated in [4, 5] as nonlinear programming problems. The objective function in [5] is proved to be convex and an efficient optimization procedure is developed to search for the global optimal design. Although these works concern the problem of inertial and dynamic coupling reduction, the question for independent joint controllability is still open.

To this end, the subject of our work is the minimization of the inertial coupling coefficients [2] and, after that, the determination of the largest domain of the control inputs, in which an independent joint controllability condition is satisfied.

At the first design step, the gear ratios are used as design parameters and for the second step— the extremal control levels. As a typical example, a dynamic model of system with two rotary joints has been taken into consideration. Many industrial robots, walking machines, and other mechanical devices, contain this basic configuration. The design procedure is performed in analytical form.

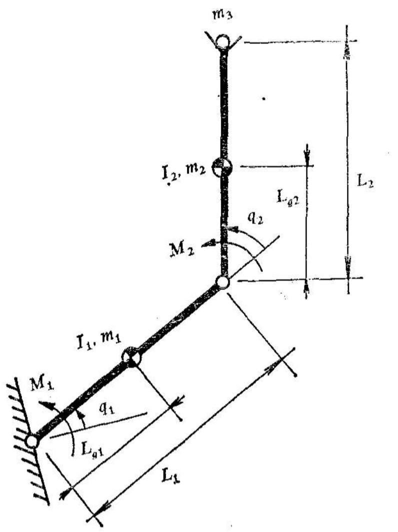

Fig. 1

2. Complete Dynamic Model and Design Objectives

Our consideration involves the complete dynamic model of a two-rotary open kinematic chain, Fig. 1 — an example of high coupled joint dynamics. The parameters and the differential equations of joint motions are as follows [6]:

L_j — length of link j,
m_j — mass of link j,
m_3 — mass of the load,
L_{gj} — length between an end of a link and its center of gravity,
I_j — moment of inertia about the center of gravity,
M_j — torque which acts of link j,
q_j — angular displacement of link j.

(1) $$M_1=(a_1+a_2+2a_3\cos q_2)\ddot{q}_1+(a_2+a_3\cos q_2)\ddot{q}_2 - a_3\dot{q}_2^2\sin q_2-2a_3\dot{q}_1\dot{q}_2\sin q_2+a_4\cos q_1+a_5\cos(q_1+q_2);$$

(2) $$M_2=(a_2+a_3\cos q_2)\ddot{q}_1+a_2\ddot{q}_2+a_3\dot{q}_1^2\sin q_2+a_5\cos(q_1+q_2),$$

where: $a_1=I_1+m_1L_{g1}^2+(m_2+m_3)L_1^2$, $a_2=I_2+m_2L_{g2}^2+m_3L_2^2$, $a_3=(m_2L_{g2}+m_3L_2)L_1$
$a_4=(m_1L_{g1}+(m_2+m_3)L_1)g$, $a_5=(m_2L_{g2}+m_3L_2)g$.

A relation between output torque M_j and input voltage e_j of the servomotor, when the reduction gear train is in use, is the well-known equation:

(3) $$e_j=r_jb_{1j}\dot{q}_j+r_jb_{2j}\ddot{q}_j+(1/r_j)b_{3j}M_j,$$

where: $1/r_j$ — reduction ratio,
b_{ij} — coefficients for servomotor (j).

By substituting M_j from Eqs (3) in Eqs (1) and (2), we have the dynamical equations of the system as follows:

(4) $$\ddot{q}_1=[(b_{22}r_2+(1/r_2)b_{32}a_2)e_1-(1/r_1)b_{31}(a_2+a_3\cos q_2)e_2-f_1]/d,$$
$$\ddot{q}_2=[-(1/r_2)b_{32}(a_2+a_3\cos q_2)e_1+(r_1b_{21}+(1/r_1)b_{31}(a_1+a_2+2a_3\cos q_2))e_2-f_2]/d,$$

(5) where: $d=\det M=\det(m_{ij})$,

(6) $$m_{11}=r_2b_{22}+(1/r_2)b_{32}a_2,\quad m_{12}=(1/r_1)b_{31}(a_2+a_3\cos q_2)$$
$$m_{21}=(1/r_2)b_{32}(a_2+a_3\cos q_2),\quad m_{22}=r_1b_{21}+(1/r_1)b_{31}(a_1+a_2+2a_3\cos q_2);$$

(7) $$f_1 = m_{11}(1/r_1)b_{31}(a_3\dot{q}_2^2 \sin q_2 + 2a_3\dot{q}_1\dot{q}_2 \sin q_2 - a_4 \cos q_1 - a_5 \cos(q_1+q_2))$$
$$-m_{11}r_1b_{11}\dot{q}_1 + m_{12}r_2b_{12}\dot{q}_2 + m_{12}(1/r_2)b_{32}(a_3\dot{q}_1^2 \sin q_2 + a_5 \cos(q_1+q_2))$$

(8) $$f_2 = -m_{21}(1/r_1)b_{31}(a_3\dot{q}_1^2 \sin q_2 + 2a_3\dot{q}_1\dot{q}_2 \sin q_2 - a_4 \cos q_1 - a_5 \cos(q_1+q_2))$$
$$+m_{21}r_1b_{11}\dot{q}_1 - m_{22}r_2b_{12}\dot{q}_2 - m_{22}(1/r_2)b_{32}(a_3\dot{q}_1^2 \sin q_2 + a_5 \cos(q_1+q_2)).$$

From $a_1a_2 > a_3^2$, it follows that the generalized inertial matrix M is positive definite and $m_{11}m_{22} > m_{12}m_{21}$. We define:

(9) $$k^2 = m_{12}m_{21}/(m_{11}m_{22}) < 1$$

as a generalized coefficient of the inertial coupling [2]. This coefficient is configuration — dependent. In our case it depends on q_2 with maximum value at $q_2=0$ and minimum value at $q_2 = \arccos\rho$, where $\rho = \max\{-1; -a_2/a_3\}$.

As small is the coefficient of inertial coupling, as independent [2], and stable [7] controls of the joint motions can be. So the first step in our design procedure is:

A. Minimization of k with respect to gear ratios r_1 and r_2: $r_i \le r_{i\,\max}$.

Further, if the robot is supplied with individual *P*, *PD*, *PID*, etc. joint controllers, then the following condition for independent joint controllability (*IJC*) should be satisfied:

(10) $$\operatorname{sign}\ddot{q}_i = \operatorname{sign} e_i, \quad i=1,\ 2.$$

When such a condition takes place, several advantages in the control of the manipulator dynamics can be achieved:

— Robustness of many feedback control schemes [8, 9, 10, 11];

— Efficient loading of joint actuators (discrepancy in the signs of the control input (voltage) and the corresponding acceleration may result in violating current limitations, energy loss, wear, etc.);

— Controllability and optimal point-to-point control synthesis of robot dynamics [12, 13, 14, 15];

— Success in the path planning of the motion in the presence of torque and control constraints [16], obstacles, etc.

To satisfy IJC conditions (10), it is necessary the following inequalities to be fulfilled:

(11) $$\left|\begin{array}{l} m_{11}|e_1| - |m_{12}|\,|e_2| > 0 \\ -|m_{21}|\,|e_1| - m_{22}|e_2| > 0, \end{array}\right.$$

(12) where: $|e_i| \le e_{i\,\max}$, $i=1,\ 2$.

Inequalities (11) and (12) are equivalent to the following ones:

(13) $$\left|\begin{array}{l} r_e m_{11}u_1 - |m_{12}|\,u_2 > 0 \\ -r_e|m_{21}|\,u_1 + m_{22}u_2 > 0, \end{array}\right.$$

(14) $$0 \le u_i \le 1, \quad i=1,\ 2,$$

where: $u_i = |e_i|/e_{i\,\max}$, $r_e = e_{1\,\max}/e_{2\,\max}$.

Denote $S = Area(D_c)$, where D_c is the set of all (u_1, u_2) satisfying (13) and (14). The quantity S will serve as a measure of the joint actuators' ability to satisfy the *IJC* conditions. So the second part of our design procedure is the following:

B. Maximization of S with respect to the parameter r_e: $|m_{12}|/m_{11} \le r_e \le m_{22}/|m_{21}|$

3. The Design Procedure.

A. From (6) and (9) follow that the minimum value of the inertial coupling coefficient k is obtained for the maximum gear ratios $k_0 = k_{\min} = k(r_{1\,\max};\ r_{2\,\max})$.

Moreover, one can see from (7) and (8) that the maximum gear ratios correspond to the minimum values of the nonlinear (Coriolis and centrifugal) interaction forces as well as to the minimum values of the gravitation torques.

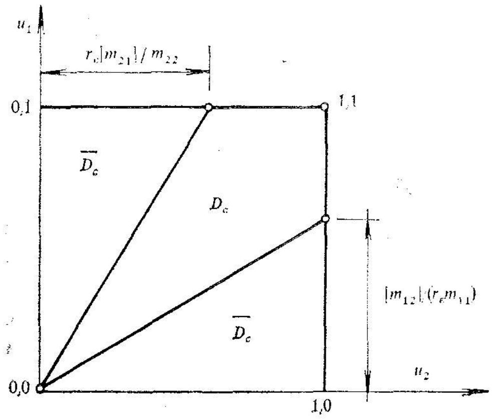

Fig. 2

B. For the next design step, we take the geometrical representation of the set D_c as in Fig. 2. Then the area of this domain can be expressed as follows:

$$S = Area(D_c) = 1 - 1/2 . (r_e |m_{21}|/m_{22} + |m_{12}|/(r_e m_{11})).$$

Thus the maximum value of S

(15) $$S_{\max} = 1 - (m_{12} m_{21}/(m_{11} m_{22}))^{1/2} = 1 - k_0$$

is obtained, when r_e is the geometric mean of its bounds

(16) $$r_e = r_e^0 = ((m_{22}/|m_{21}|) . (|m_{12}|/m_{11}))^{1/2}.$$

With this value of the ratio of $e_{1\max}$ and $e_{2\max}$, the domain D_c is symmetrical about the line passing through points (0; 0) and (1; 1). And what is more, in this case the latter point which corresponds to $e_1 = e_{1\max}$ and $e_2 = e_{2\max}$ is at the maximum distance from the domain $\overline{D}_c$ in which conditions (13) are not satisfied.

4. Application

The so obtained optimal ratio of the control input bounds can be utilized to specify gain coefficients in single-input single-output joint controllers. E. g., if we take the simplest case of P-controller

(17) $$e_i = -k_i \triangle q_i$$

($\triangle q_i$: position error), then the optimal ratio of the gain coefficients is given from the equation

(18) $$(k_1 \triangle_1)/(k_2 \triangle_2) = r_e^0,$$

where: $\triangle_i$ are the extremal tolerances of $|\triangle q_i|$.

5. Conclusion

A closed form solution of a problem for optimal design of manipulator with two rotary joints has been presented. The main guideline in our design procedure is to provide independent joint controllability and the design parameters are the gear ratios and the ratio of the control inputs bounds. The present work, together with the previously obtained one [3], gives a methodology for optimization of the drive system design in the case of any non-redundant manipulator with independent joint controllers. Realizing the proposed concepts, one can improve the robustness of many feedback control schemes and other robot performances in dynamic operations.

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