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On the determination of air friction factor during the spinning of polymer melts

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Introduction

The spinning of polymer melts is a process of extruding a thermoplastic polymer at a constant temperature T_0, constant mass flow W, through a spinneret with "n" orifices and capillary diameter d_0. At a length L from the spinneret is placed the take-up device which provides a constant drawing rate V_L. Along the whole length of the spinway from the spinneret ($x=0$) up to take-up point ($x=L$) the liquid polymer jets are deformed nonisothermally, cooled and are transformed into filaments. The cooling is usually carried out by a heat exchange with the surrounding air medium in special shafts.

According to the basic theoretical formulations of melt spinning (1) the force balance exerting on a single filament at position x is given by the equation

$$F_{ext}=F_{rheo}(x)+F_{aero}(x)+F_{inert}(x)+F_{surf}(x)+F_{grav}(x), \tag{1}$$

where F_{ext} is the take-up tension, F_{rheo}, F_{aero}, F_{inert}, F_{surf} and F_{grav} are the components of the take-up tension, created by the rheological resistance of the material, the air drag, the inertia force, the surface tension of the polymer melt, and the mass of the filament, respectively.

As the rheological force F_{rheo}, is a complex function of the parameters of the polymer and the spinning process usually it is determined by the force balance (Eq. (1)) after an independent calculation of the rest components and the measurement of the take-up tension F_{ext} at the end of the spinway. Following this procedure one of the most complex problems is the determination of F_{aero}. According to aerodynamics the force of the aerodynamic friction of a cylinder with an infinite ratio of the length L to the diameter d, moving with velocity V in a gaseous medium with density ρ is calculated by the equation

$$F_{aero}(x)=\int_0^L \rho . \frac{V^2}{2} . \pi . d . C_f . dx, \tag{2}$$

where C_f is the air friction factor.

In previous investigations on the melt spinning (1) an air friction for an infinite flat surface have been used for the calculation of F_{aero}. Later, on the basis of a great number of experiments with monofilaments (2-4), empirical correlations for the friction factor like $C_f = A.ReB$ have been derived, where Re is the criterion of Reynolds. The coefficients A and B are valid only for the spinning conditions investigated, mainly filament filament velocity and diameter.

In a research work of Kwon and Prevorsek (5) on a laboratory setup have been measured air drag about 40 % higher than the value predicted by the existing theories.

Nevertheless, the spinning of monofilaments is the simplest case of melt spinning there is not an unanimous view of the determination of the air friction factor.

In contrast to the theory and the investigations cited in the actual industrial process several dozens of filaments are extruded simultaneously from the spinneret in the production of filaments, and up to several thousands — in the production of staple fibres. Because of constructional reasons the space between the filaments is about several milimetres. In this case the air carried away by the moving filaments is summarized in a total flow along the filament bundle obtained. Reasonably, it may be assumed that the air friction of the filaments in the core and on the periphery of the bundle will differ considerably.

At such conditions it is not correct to calculate the total air drag of the bundle as a product of the number of filaments and the air drag of a single filament, using the air friction factor derived for a monofilament.

In the present work a possibility for an estimation of the air friction factor and the calculation of F_{aero}, accounting the mutual effect of the filaments, is considered.

Description of the method

Let us examine a filament bundle containing n filaments with a diameter d_L after the point of hardening, moving down with a velocity V_L. Let the filaments be set up in a circle with a diameter D, corresponding to the circle in which the spinneret orifices are disposed. Along the spinway the diameter of the filament bundle is gradually decreased from D at a position $x=0$ to d_L at a position $x=L$, where the filaments are gathered together in a thread. As the thread diameter d_L is much smaller than D the geometry of the filament bundle may be accepted as a cone with a diameter of the base D and height L. This geometry is given schematically in Fig. 1, a.

Let then the filament bundle be gathered by compulsion in a thread with a diameter d_L at a length l_1 from the spinneret, and $l_0 < l_1 < L$, where l_0 is the position of hardening of the filaments and the following conditions are fulfilled: $dV_x/dx = 0$, $V_x = V_L = \text{const}$. So, the geometry of the filament bundle in the region of the spinway from position $x=0$ to position $x=l_1$ is a cone with a diameter of the base D and height l_1, and in the region from position $x=l_1$ to position $x=L$ it can be considered as a moving monofilament with a diameter d_L.

Let us gradually shift the point of gathering the bundle into a thread from $x=l_1$ to $x=L$. So m configurations will be obtained and the height of the turned upside-down cone for every new configuration will be increased. The take-up tension will gradually be increased. In the case when $l_m L$ the initial configuration (0) will be reached. The procedure described is given in Fig. 1, a and b. Each configuration will be characte-

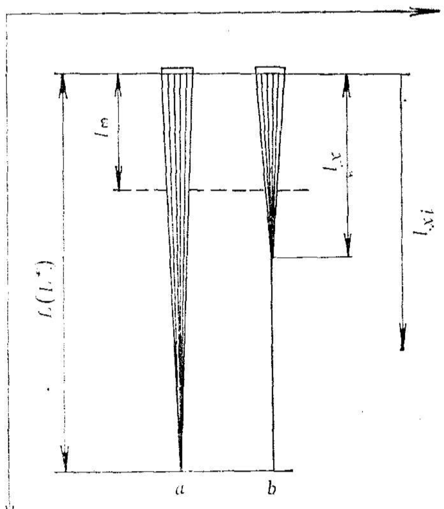

Fig. 1. Scheme of the experimental procedure

rized with the take-up tension F_{ext} at the end of the spinway. The take-up tension will gradually be increased as the length of the cone and, respectively, the surface getting in contact with the air medium will be increased. The zero configuration will be characterized with the highest take-up tension, i. e. $F_{ext_1} < F_{ext_2} < \dots F_{ext_m} < F_{ext_0}$.

Let us then make the force balance for the different configurations of the filament bundle. For simplifying the following assumtions can be made:

— the air drag of the filament bundle in the region from the spinneret to position $x = l_0$ does not depend on the configuration of the filament bundle and it has a constant value;

— along the spinway the velocity and the diameter of the filaments are constant, V_L and d_L, respectively;

— after gathering of the filament bundle the thread obtained is considered as a monofilament with an effective diameter d_L. The diameter d_L is equal to the diameter of the circumference in which n circles with diameter d_L are tightly packed.

It is seen, considering the force balance (Eq. (1)), that the components F_{inert}, F_{surf} and F_{grav} do not depend on the mutual effect of the filaments and the configuration of the filament bundle. It is known (1) also that the components F_{surf} and F_{grav} are many times smaller than the other and usually they are neglected. That is why the force balance can be written in the following way

(3) $$F_{ext} = F_{rheo}(n) + F_{aero}(n) + n \,.\, F_{inert},$$

$F_{rheo}(n)$ and $F_{aero}(n)$ show that those components depend on the number of the filaments and their location in the bundle.

In this case the air drag of a bundle, containing n mutually effecting filaments with diameter d_L and velocity V_L can be given by the expression

(4) $$F_{aero}(n) = \int_x^L \rho \,.\, \frac{V_L^2}{2} \,.\, \pi \,.\, d_L \,.\, \bar{C}_f \,.\, dx,$$

where $\bar{C}_f$ is the mean air friction factor of the filaments in the bundle by which the mutual effect of the filaments is accounted.

The force balance of the zero configuration of the filament bundle (Fig. 1, a) is as follows

(5) $$F_{ext(0)}(n) = F_{rheo(0)}(n) + \int_0^L \rho \,.\, \frac{V_L^2}{2} \,.\, \pi \,.\, d_L \,.\, \bar{C}_f \,.\, dx$$

and after the expansion of the integral limits:

(5a) $$F_{ext_{(0)}}(n) = F_{rheo_{(0)}}(n) + \int_0^L \rho \,.\, \frac{V_L^2}{2} \,.\, \pi \,.\, d_L \,.\, \bar{C}_f \,.\, L.$$

The force balance of i configuration of the filament bundle will be given by the expression

(6) $$F_{ext_{(i)}}(n) = F_{rheo_{(i)}}(n) + \int_0^{l_i} \rho \,.\, \frac{V_L^2}{2} \,.\,.\, d_L \,.\, \bar{C}_f \,.\, dx + \int_{l_i}^L \rho \,.\, \frac{V_L^2}{2} \,.\, \pi \,.\, d_L \,.\, C_f \,.\, dx$$

and after the expansion of the integrals:

(6a) $$F_{ext_{(i)}}(n) = F_{rheo_{(i)}}(n) + \rho \,.\, \frac{V_L^2}{2} \,.\, \pi \,.\, d_L \,.\, n \,.\, \bar{C}_f \,.\, l_i + \rho \,.\, \frac{V_L^2}{2} \,.\, \pi \,.\, d_L \,.\, C_f \,.\, (L - l_i) + n \,.\, F_{inert},$$

where C_f is the air friction factor of a monofilament.

The change of the take-up tension, caused by the change of the geomerty of the filament bundle will be

(7) $$\Delta F_{ext_{(i)}}(n)=F_{ext_{(0)}}(n)-F_{ext_{(i)}}(n).$$

After the substitution of equation (7) by the expressions (5a) and (6a) and making transformations the following expression will be obtaind

(8) $$\Delta F_{ext_{(i)}}(n)+\rho\cdot\frac{V_L^2}{2}\cdot\pi\cdot(L-l_i)\cdot[n\cdot d_L\cdot\bar{C}-d_L^*\cdot C_f^*]+\Delta F_{rheo_{(i)}}(n).$$

The change of the rheological component of the take-up tension $F_{rheo_{(i)}}(n)$ is a result of the change of the cone height and the space between the filaments at the apex of the cone because of the shifting of the point where the bundle is gathered into a thread. When the apex is at the top of the spinway where the temperature is still high the change of the space between the filaments effects on the heat exchange with the surrounding medium. When $l_i \rightarrow L$, $\Delta F_{rheo_{(i)}}(n)\rightarrow 0$ as the location of the filaments along the height of the cone is practically unchanged. $\Delta F_{rheo_{(i)}}(n)$ is a small value as the apex of the cone is always located in a place where the filaments are hardened That is why $F_{rheo_{(i)}}(n)$ may be replaced by $\bar{C}_{f_i}$, which means the error of the determination of C_f, caused by the change of the rheological force which inclines to zero when $(L-l_i)\rightarrow 0$. After the transformation equation (8) became

(9) $$F_{ext_{(i)}}(n)=\rho\cdot\frac{V_L^2}{2}\cdot\pi\cdot(L-l_i)\cdot[n\cdot d_L(\bar{C}_f+\Delta\bar{C}_f)-d_L^*\cdot C_f^*].$$

The unknown quantities in equation (9) are d_L, C_f and $(\bar{C}_f+\Delta\bar{C}_f)$. The equivalent diameter of the thread, d_L, can be calculated by

(10) $$d_L=d_L\sqrt{\frac{n}{k}},$$

where n is the number of the filaments, and k is filling factor. k depends on n and ranges from 0.60 up to 0,85. The function $k=f(n)$ can be derived experimentally.

The friction factor of the thread C_f is taken from literature data on the basis of V_L aud d_L values.

Let us work out a series of equations like equation (9) for the different configurations of the filament bundle. As d_L and C_f are constants for given spinning conditions from each equation can be calculated the corresponding value of $(\bar{C}_f+\Delta\bar{C}_{f(i)})$. The following relation can be worked out

(11) $$(\bar{C}_f+\Delta\bar{C}_{f(i)})=f(L-l_i).$$

The value of $\bar{C}_f$ is determined from this relation when $(L-l_i)$ —— 0, i. e.

(12) $$C_f=\lim_{(L-l_i)}\rightarrow\bar{0}\,(C_f-\overline{\Delta C}_{f(i)}).$$

This can be carried out by a graphical extrapolation or after finding out the analytical mode of the relation $(\bar{C}_f+\Delta\bar{C}_{f(i)})=f(L-l_i)$. This method is fulfilled in practice in the following way.

At the end of the spinway the take-up tension $F_{ext_{(0)}}$, is measured by a sensitive tensiometer. After that by means of a suitable thread guide element placed on a distance 1 metre from the spinneret (position l_l), the filament bundle is gathered into a thread and the take-up tension $F_{ext_{(l)}}$, is registered. Gradually the thread guide element move upside-down to the measuring head of the tensiometer. In this region the take-up tension $F_{ext_{(i)}}$ can be registered continuously by synchronizing the speed of the thread guide element and the speed of the recorder. If there is not a suitable recorder the measurements are carried out at definite positions along the spinway from l_1 to L.

At the utter position when the thread guide element reaches the measuring head of the tensiometer the take-up tension $F_{ext(m)}$ is measured. As the perfect case $F_{ext\,(m)}$ must be equal to $F_{ext(0)}$. In practice, there is a little difference $F_{ext} = F_{ext(0)} - F_{ext(m)}$ which is caused by the friction of the filaments with the thread guide element. If ΔF^*_{ext} is higher than the error of the tensiometer the dependence $F_{ext(i)} = f(L - l_i)$ must be corrected by ΔF^*_{ext}. The procedure described is given in Fig. 2.

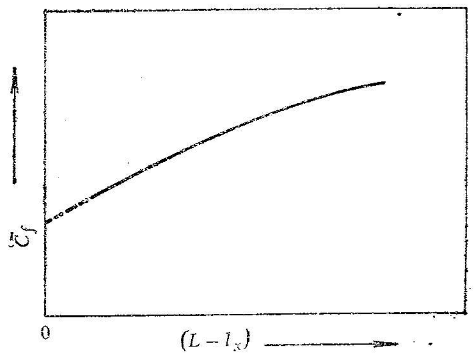

Fig. 2. Representation of the Practical Application of the Method

The region $(L - l_i)$ is divided into at least ten intervals. For every value of $(L - l_i)$ from the hatched area the corresponding value of $F_{ext(i)}$ is calculated. This value is used to be substituted equation (9). The corresponding value $(\bar{C}_f + \Delta_{f(i)})$ is calculated and the relation $(\bar{C}_f + \Delta \bar{C}_{f(i)}) = f(L - l_i)$ is worked out.

The method for the determination pf $\bar{C}_f$ described contains a series of assumptions made for its easy practical application. Nevertheless, this method eliminates most of the initial assumptions (which are included in $\bar{C}_{f(i)}$) and results in determining a numerical value of the mean air friction factor C_f, of the filaments in the bundle. This friction factor includes all mutual effects of the filaments, which theoretical description now is impossible.

Conclusion

A method for the determination of the air drag during the spinning of polymer melts has been described. The method can be applied for multifilament yarns. An average air friction factor dependent on the number of the filaments and their mutual disposition in the bundle has been introduced. Nevertheles, the simplifications done, the calculation of the air friction factor using this method is more correct than the use of the air fraction factor derived for a monofilament.

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