БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ BULGARIAN ACADEMY OF SCIENCES
ТЕОРЕТИЧНА И ПРИЛОЖНА МЕХАНИКА THEORETICAL AND APPLIED MECHANICS
София, 1989, год. XX, № 1 Sofia, 1989, Year XX, No 1

On the problem of determining the stress-strain state of a multi-layer specimen under dynamic loading

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In this work the problem of modelling the stress-strain state of a multi-layer specimen, subjected to a dynamic deformation pulse, is considered. The specimen consists of several cylinders with equal diameters and lengths made of the same material. The modified Hopkinson pressure bar is used for the loading. Our aim is to investigate the stresses and strains appearing in the specimen during the process of wave propagation. We propose a method here, which considers the specimen homogeneous (one-layer) in the process of solving the problem. The presence of several layers is taken into account in the mechanical-mathematical model adopted.

Experimental statement and data

A multi-layer specimen with length l is loaded on its end by a deformation pulse $P(t)$, $0 \leq t \leq t_1$, which is measured. On the other end of the specimen, the transfered pulse $P_1(t)$ is registered.

The experimental data available are the values of the pulses P and P_1 in discrete number of points (moments), i. e. $P_i = P(t_i)$, $t_i = i\Delta t$, $P_i^1 = P_1(t_i')$, $t_i' = t_0 + i\Delta t$, $i = 1, 2, \dots, m$.

Modelling problem

We make it our aim to construct a model of wave propagation process, in the case of a multi-layer specimen. The experiment described is modelled by one-dimension wave propagation problem. Mathematically the problem is determined by the next system of partial differential equations

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t}, \tag{1.1}$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial x}, \tag{1.2}$$

$$\Phi(\sigma, \varepsilon, \dot{\sigma}, \dot{\varepsilon}, \dots) = 0. \tag{1.3}$$

Here, the axis Ox is chosen on the direction of wave propagation. $\sigma(x, t)$ denotes the normal stress on plates with normals paralel to Ox, $\varepsilon(x, t)$ — the strain in the same direction, $v(x, t)$ — the velocity of points in the direction of Ox. Equation (1.1) is the motion equation, (1.2) — the geometric equation, and (1.3) — the physics equation. The third equation models the mechanical properties of the specimen material. Our problem is modelling the properties of a multi-layer specimen, when loaded as described. We have the following premises for the choice of the model:

a) when a homogeneous specimen with the same length l is considered, then the pulse $P_1(t)$ differs from $P(t)$ negligibly. That is true for the differences in the form and in the intensity. So in the case $n=1$ no energy dissipation is observed during the process of wave propagation;

b) on the contrary, in the case of more than one layer $n=1, 2, 3, 4\ldots$ the pulse $P_1(t)$ differs from the pulse $P(t)$ substantially.

On the grounds of these facts, we consider the energy dissipation due to the traverse from one layer to another only, part of the pulse being reflected and another part continues its propagation. Hence, we have to assume a model that reflects the energy absorbtion during the process of wave propagation, the absorbtion depending on the number of layers only.

It is well-known that models of visco-elastic and visco-plastic bodies reflect the pulse fading during its wave propagation. Using the data available about the input and output pulses one may choose such a model of the type mentioned, describing the dynamic behaviour of the specimen well enough [2]. Having in mind the effectiveness of the applications and the simplicity of the model, we made use of Maxwell's model. We assume the viscosity coefficient depending on the number of layers n only, and the Young modulus E being equal to the elasticity modulus of the material the cylinders are made of. The process of one-dimension wave propagation will be defined by the next system of partial differential equations

$$\frac{\partial\sigma}{\partial x}=\rho\frac{\partial v}{\partial t}, \tag{1.1}$$

$$\frac{\partial\varepsilon}{\partial t}=\frac{\partial v}{\partial x};\quad \varepsilon=\varepsilon^e+\varepsilon^a, \tag{1.2}$$

$$\sigma=E\varepsilon^e, \tag{1.3'}$$

$$\dot{\varepsilon}^a=\sigma/\eta, \tag{1.3''}$$

where $\eta=\eta(n)$.

The system is solved under the following initial conditions

$$\sigma(x,\ 0)=0, \tag{1.4}$$

$$\ddot{u}(x,\ 0)=0. \tag{1.5}$$

The boundary condition is determined by the loading on the end $x=0$

$$\varepsilon(0,\ t)=-P(t) \tag{1.6}$$

and $P(0)=P(t_1)=0$.

We have to determine the coefficient $\eta=\eta(n)$ such that the results obtained after solving the above formulated problem for $\varepsilon(l,\ t)$ coincide with the experimental ones for the pulse P_1 on $x=l$, best.

Mathematical formulation of the problem

Mathematically the problem stated is a problem for parametric identification of the parameter η. These problems are reduced to function minimization sums [3]. The function minimized defines the deflection between the theoretical and the experimental results. Most often the next function is used $f(\eta)=\sum_{i=1}^{m}[P_1(t_i)-\varepsilon(l,$

t_i, $\eta)]^2$. It defines the mean square declination between the two functions. We search its minimum with respect to η. First of all, we need a method for solving the formulated problem for wave propagation, for being able to obtain $\varepsilon(l, t_i)$ in the mo-

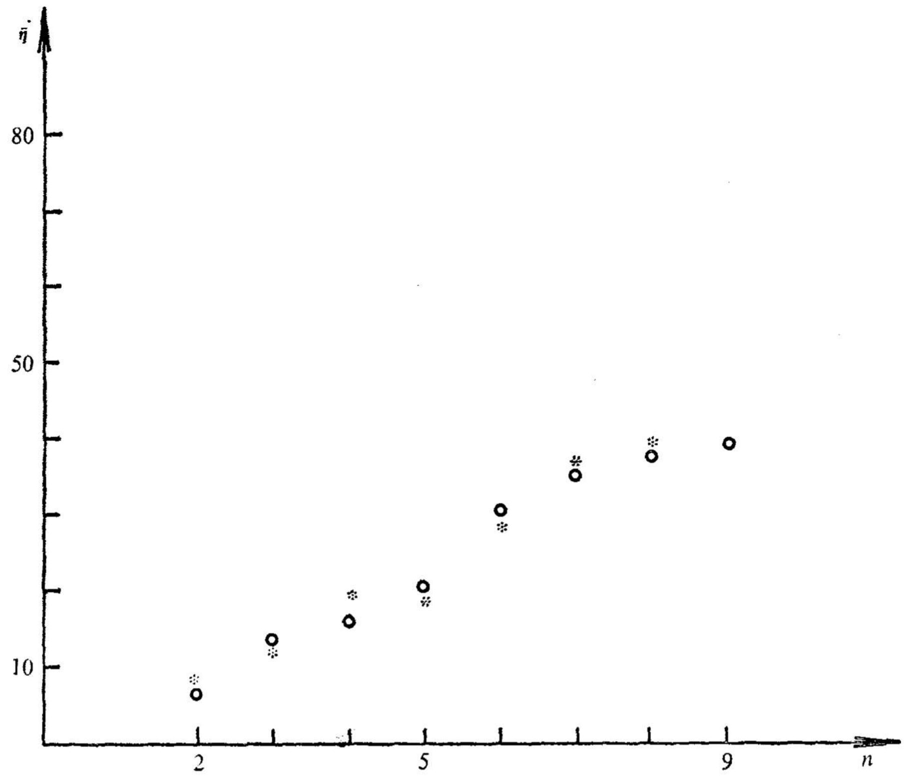

Fig. 1

ments t_i, $i=1, 2, \dots m$. The system (1.1), (1.2), (1.3′), (1.3″) is reduced to one partial differential equation and it looks like that

$$\frac{\partial^2\sigma}{\partial x^2}-\frac{1}{c_2}\frac{\partial^2\sigma}{\partial t^2}-\frac{\rho}{\eta}\frac{\partial\sigma}{\partial t}=0, \tag{2}$$

where $c=\sqrt{E/\rho}$.

We apply the Laplas transform to (2) and we obtain

$$\frac{\partial^2\bar{\sigma}}{\partial x^2}-\frac{1}{c^2}p^2\bar{\sigma}-\frac{\rho}{\eta}p\bar{\sigma}=0, \tag{2'}$$

where $\bar{\sigma}$ is the Laplas transform of σ. We know the solution of equation (2′) is of the form [4]

$$\bar{\sigma}(x, p)=A(p)\exp\left[-x/c\sqrt{p^2+\frac{E}{\eta}p}\right]. \tag{3}$$

Taking into account the boundary condition and making the inverse transform, we obtain for the deformations

$$(3.1)\qquad \varepsilon(l,t,\eta)=\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,(k-1)!}\left(\frac{E}{4\eta}\right)^{2k}\int_{l/c}^{t}P(t-\tau)\,e^{-Er/2\eta}\sqrt{\tau^2-l^2/c^2}\,d\tau$$

and finally for $f(\eta)$

$$(3.2)\qquad f(\eta)=\sum_{i=1}^{m}\left[P_{1i}-\sum_{k=0}^{m}\int_{l/c}^{t}P(t-\tau)e^{-Er/2\eta}\sqrt{r^2-l^2/c^2}\,d\tau\right]^2$$

$$A_k=(-1)^k\left(\frac{E}{4\eta}\right)^{2k}/(k!\,(k-1)!).$$

There are two groups of methods for determining the function minimum [5]:
1) methods using derivatives,
2) direct methods which do not use derivatives.

Because of the complex type of the function $f(\eta)$ (see (3.2)) we prefer using the second group methods, the unknown value searched iteratively, as a limit of a sequence of values $\eta_0, \eta_1, \ldots, \eta_n, \ldots$ We shall use the Fybonatchi method [3], which is a symmetric one. Initially the minimum is searched in the whole interval $[a, b]$, the interval being narrowed at every step and at every step a new value η_i is obtained for the minimum. Here raises the question for cancelling the iterative procedure. It is proved for the Fybonatchi method, that the number of iterations needed for obtaining a point η_k remoted from η_*, η_* being the exact minimum, on a distance less than ε, equals k which satisfies inequality

$$(4)\qquad (b-a)/2F_{k+2}<\varepsilon\leq(b-a)/2F_k,$$

where F_k and F_{k+2} are the Fybonatchi numbers. It turned out that $k=50$ is enough for obtaining an error about $e\approx10^{-2}$.

Conclusions and comments

Now we may calculate $f(\eta)$ and find its minimum for every given $P(t_i)$ and $P_1(t_i')$, hence for each η we may find the corresponding η. This way we may determine the relation $\eta=\eta(n)$ for the given intervals of n and η, which will hold under the loading and circumstances mentioned above. We have obtained that relation by means of regression analysis and it looks like that

$$(5)\qquad \eta=a_1e^{bn}+a_2n+a_3$$

for $n=1, 2, \ldots, 10$ and $10^8\leq\eta\leq79.10^8$ N. cm/s. The values for the coefficients a_1 a_2, a_3 and b for steel 3 are the next

$$a_1\approx12.934,\qquad b\approx2.13$$

$$a_2\approx-3.8194.10^4,\qquad a_3\approx1.2341.10^2.$$

On Fig. 1 the relation (5) is shown for stell 3. The evaluations for the variances of the coefficients are as follows

$$\hat{V}(a_1)=7.1.10^{-1}\qquad \hat{V}(a_2)=2.12.10^2$$

$$\hat{V}(b)=5.10^{-2}\qquad \hat{V}(a_3)=2.15.$$

The relative standard deviation $l_r=\sqrt{\frac{1}{r-2}\sum_{i=1}^{r}\frac{|\bar{\eta}_i-\hat{\eta}_i|}{\bar{\eta}_i}}$ is about $l_r\approx3.5\%$. Here $\hat{\eta}_i$ are the experimental values for η and $\bar{\eta}_i$ are those obtained by following (5), r is the number of measurements.

Of course, one may raise the question is the model chosen adequate? We can make a test comparing the parameters η obtained for equal numbers n, but different loadings (different input and output pulses). We have made such tests for $n=2$ with four different pulses P and for $n=4$ with three different pulses. The difference between the values for η, in both cases appeared to be in the eighth cipher.

In this work we propose a method for modelling the wave propagation, under dynamic loading in multi-layer specimens, the fading of the pulse due to the passing over the layers, which is taken into account with the help of Maxwell's model, whose viscosity coefficient η depends on the number of layers n.

The model constructed gives us the possibility to predict the stresses and strains and its maximum values, which may arise when we have a given number of layers and a tangible loading.

One may use the relation $\eta=\eta(n)$ in other cases of multi-layer structures, subjected to dynamic loading, and thus one may determine their stress-strain states during the process of their deformation.

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Received 17. 03. 1987