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Calculation of transitional two-dimensional boundary layer under the influence of free stream turbulence

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1. Introduction

In the flow past viscous fluid bodies even slight disturbances in the free stream flow could cause turbulization of the boundary layer formed on the surface of the body. It is very important to consider this effect in the design of cooling systems for the turbomachinery blades and compressors, in calculating the characteristics of airfoils, etc.

Publications [1, 2] include a detailed account of the accumulated experience up to now concerning the influence of the outer turbulence on the development of the boundary layer. The basic conclusion drawn in these works is that the success of numerical modelling depends strongly on the chosen turbulence model. According to the classification assumed at present the turbulence models are divided into two groups: models using turbulence viscosity and models for transfer of turbulent Reynolds stresses.

The first group of models can be divided further into three subgroups according to the number of additional equations necessary for the determination of turbulent viscosity. That is why these models are often called models with n equations ($n=0, 1, 2$). In the zero-equation model the turbulent viscosity is determined by the mean flow characteristics and it is on account of this reason that the influence of disturbances in the external flow can be regarded only empirically. It can be clearly seen why these models are considered unsuccessful in this field. For that purpose more convenient are the models in which the turbulence viscosity is expressed by the mean turbulence characteristics In this case the differential equation for the kinetic energy of turbulence pulsation is used. This equation allows the direct consideration of the influence of free stream turbulence using the boundary condition for turbulent energy. In the models of the one equation ($n=1$) the turbulence length scale is given empirically which is their serious disadvantage. This problem is solved in the models with two equations ($n=2$). The most famous among them is the so-called $k-\varepsilon$ model where the length scale is determined by solving the differential equation for the dissipation rate of turbulence energy.

The second group covers models having differential equations for direct transfer of turbulent stresses and these models are thought of the most perspective in turbulence theory. In spite of that they are rarely used in practical calculations for two reasons: first, the process of creation and approbation of Reynolds stress models hasn't come to its end yet; second, using the model a greater number of differential equations must be solved which leads to the increase of the expense of numerical resources. In [3] there is a comparison between the model of two equations and the Reynolds stress model in the calculation of a two-dimensional turbulent boundary layer under the influence of disturbances in the free stream. The comparison shows that the k—ε model works almost as well as the other one.

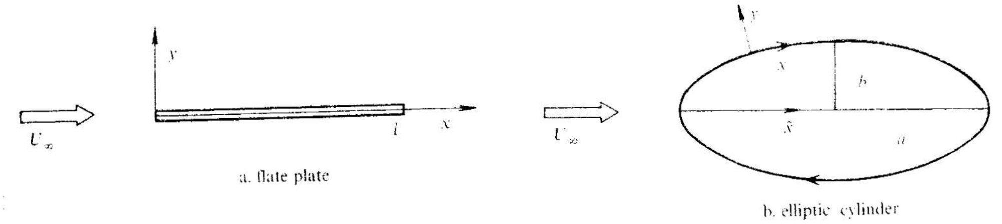

Fig. 1. Sketch of the bodies and coordinate systems

We can conclude from the brief survey of the models in turbulence theory that the two-equation models (the k—ε model) are the most appropriate for practical calculations of two-dimensional transitional boundary layer. Similar conclusions are also made in [1, 2].

According to Rodi and Scheuerer [1, 2] successful calculations can be made using the k—ε model for a disturbance level in the external flow $Tu \geqq 1\%$. It is obvious from the results published by them that a fairly good coincidence with the experiment was obtained for $Tu \geqq 2\%$ while for $Tu < 2\%$ there is a bad agreement with the experiment.

In the present work an effort is made for improving the calculations by the k—ε model for $Tu < 2\%$. A detailed description of the method is given in 2.5. This method is used for two cases of flow past bodies — a flat plate and an elliptic cylinder. In Fig. 1 are shown the drawings of the bodies and respective coordinate system. We assume that there is an uniform viscous flow past the bodies with a velocity U_∞ under zero attacking angle and the disturbances in the free stream flow are uniformly distributed along the whole flow.

2. Mathematical Model

2.1. Mean Flow Equations

The flows used in this work are two-dimensional, steady, viscouse and incompressible. They are described by the following system of equations:

Continuity equation —

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0; \tag{1}$$

Momentum equation —

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}-\overline{u'v'}\right). \tag{2}$$

In the case of a flat plate $dp/dx=0$, and for elliptic cylinder the pressure gradient is determined by the solution for potential flow (4). The turbulent stress remains to be obtained.

2.2. Turbulence Model

The two-equation model chosen here uses the hypothesis of Boussinesq according to which the turbulent stresses are proportional to the gradient of longitudinal velocity:

(3) $$-\overline{u'v'}=\nu_t \frac{\partial u}{\partial y}.$$

The turbulent viscosity is defined by the kinetic energy of turbulent fluctuations and its dissipation rate:

(4) $$\nu_t=c_\mu f_\mu \frac{k_2}{\varepsilon}.$$

The meanings of k and ε are obtained by equations of transfer of these quantities

(5) $$u\frac{\partial k}{\partial x}+v\frac{\partial k}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right]+P_k-\varepsilon;$$

(6) $$u\frac{\partial \varepsilon}{\partial x}+v\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial y}\left[\left(\nu+\frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right]+\frac{\varepsilon}{k}(c_1P_k-c_2f\varepsilon);$$

where

(7) $$P_k=\nu_t\left(\frac{\partial u}{\partial y}\right)^2.$$

The values of turbulent model constants and functions given on Table 1 are determined by Zubkov [5]. In the present work this model is used and not that version of Lam and Bremhorst [6] recommended in [1, 2] because the numerical experiments show that Zubkov's model allows to work with a greater step in the main direction of the flow which makes the calculation time shorter.

Table 1.

Turbulence model parameters.

c_μ	σ_k	σ_ε	c_1	c_2
0.095	1	1.3	1.6	2

f_μ	f
$-\exp(-2.5)+\exp\left(-\frac{125}{50+Re_t}\right)$	$\left[1-0.3\exp(-Re_t^2)\right]\left[-\exp(-10)+\exp\left(-\frac{250}{25+y_+^3}\right)\right.$

2.3. Boundary Conditions and Initial Values

No slip boundary conditions on the surface of the bodies for u, v and k are used:

(8) $$\text{at } y=0 \quad u=v=k=0.$$

It is known that the dissipation has a value different from zero on the solid surface. That is why the boundary condition for dissipation is preferred in calculations which provides the following value:

(9) $$\text{at } y=0 \quad \frac{\partial \varepsilon}{\partial y}=0 \quad \text{or} \quad \varepsilon=2\nu\left(\frac{\partial k^{1/2}}{\partial y}\right)^2.$$

In [5] Zubkov makes a detailed investigation on the influence of different boundary conditions for ε, and he arrives at the conclusion that if $\varepsilon=0$, there is no essential influence on the velocity profiles and turbulent energy, as well as on the other characteristics of the boundary layer. It is for this reason that the zero boundary condition is used in these calculations which provides the greatest stability of the solution when the step in the main direction increases.

At the outer edge of the boundary layer we place the following boundary conditions:

(10) $$\text{at } y=\delta \quad u=U_p \quad k_\delta=1.5(0.01\,Tu)^2U_\infty^2.$$

The boundary value for turbulent kinetic energy is determined from the assumption that the disturbances in external flow are not changed and they have the same magnitude in all directions.

A soft boundary condition for the dissipation is used:

(11) $$\text{at } y=\delta \quad \frac{\partial \varepsilon}{\partial y}=0.$$

The initial values of laminar velocity profiles are obtained from the Blasius' solution for a flat plate and from flat front stagnation point's solution for elliptic cylinder.

The meanings of the turbulent characteristics in the initial section are determined by the velocity and they are in agreement with the chosen boundary conditions:

(12) $$k=k_\delta\frac{u}{U_p} \qquad \varepsilon=0.202\,\frac{k^{3/2}}{l_m},$$

where l_m is the mixing length.

2.4. Numerical Method

The numerical solution of the system of differential equations is made by a package of programmes created on the basis of the implicit marching algorithm of Patankar and Spalding. A detailed description of these programmes as well as the numerical parameters could be found in [7].

2.5. Calculation of Transitional Boundary Layer when $Tu \leq 2\%$

As we have already noticed the studies made in [1 2], show that the $k-\varepsilon$ model does not predict well the transition at lower levels of free stream turbulence. This inference is confirmed also in the present work. In Fig. 2 the numerical results for the local friction coefficient are compared with the experimental data at $Re=3.8\cdot10^6$ and $Tu=0.7\%$ taken from [8]. The results obtained when applying the $k-\varepsilon$ model immediately in the initial section (designated as a traditional method) show that the transition begins at $\tilde{x}_s=0.06$ and ends at $\tilde{x}_e=0.1$ while in the experiment $\tilde{x}_s=0.2$ and $\tilde{x}_e=0.5$. Aiming at the maximum approach of calculations to the experiment in the present work the place of transition at low Tu is offered to be determined by empirical criterion of Abu-Ghannam and Shaw [9]. The calculations begin by evaluation of a laminar boundary layer following when Re_θ will approach the value Re_{θ_s} determined by the criterion. The site where $Re_\theta = Re_{\theta_s}$ is considered to be the beginning of transitional zone and then the calculations continue according to the traditional method. As can be seen from Fig. 2 the point of transition determined in this way (the present method) lies very near to the experimental one.

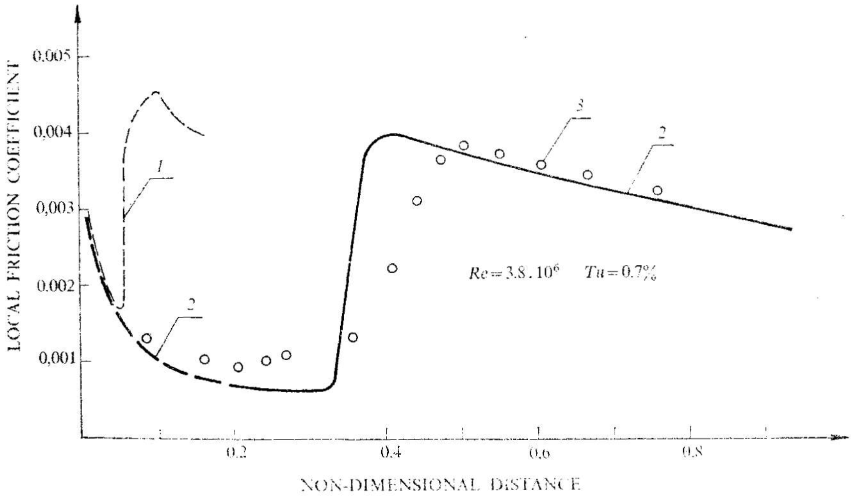

Fig. 2. Application of different methods for transitional flat plate boundary layer with low free stream turbulence: 1. — traditional method; 2. — present method; 3. — experiment

The criterion of Abu-Ghannam and Shaw generalizes great quantities of experimental data and determines Re_{θ_s} in the point of transition depending on Tu:

— zero pressure gradient

(13)
$$Re_{\theta_s} = 163 + \exp(6.91 - Tu),$$

— with a pressure gradient

(14)
$$Re_{\theta_s} = 163 + \exp\left[F(\lambda_\theta) - \frac{F(\lambda_\theta)}{6.91} Tu\right],$$

where

(15)
$$F(\lambda_\theta) = 6.91 + 12.75\,\lambda_\theta + 63.64\lambda_\theta^2 \quad \text{when} \quad \lambda_\theta < 0$$

and

(16)
$$F(\lambda_\theta) = 6.91 + 2.48\,\lambda_\theta - 12.27\lambda_\theta^2 \quad \text{when} \quad \lambda_\theta > 0.$$

The use of the intermittency factor γ in determining the turbulent viscosity plays an important role for improvement the efficiency of calculations in the transitional zone. According to the studies in [8] the most appropriate for the transitional flat plate boundary layers is the intermittency factor suggested by Dhawan and Narasimha:

(17a)
$$\gamma = 1 - \exp(-0.412\,\zeta^2), \quad \zeta = (\tilde{x} - \tilde{x}_s)/\tilde{L}_\gamma,$$

where $\tilde{L}_\gamma$ is obtained by the formula:

(17b)
$$\tilde{L}_\gamma = 5Re^{-0.2}\,\tilde{x}_s^{0.8}.$$

For elliptic cylinder is used intermittency coefficient of Chen and Thyson [8]:

(18a) $$\gamma = 1 - \exp\left[-G(x-x_s)\int_{x_s}^{x} \frac{dx}{U_p}\right],$$

where parameter G is defined from the equation:

(18b) $$G\nu^2 A^2/U_p^3 = 3Re_{x_s}^{-1.34}, \quad A = 60.$$

3. Results

3.1. Flat Plate

An important check for the method described in 2.5 is the comparison of the numerical results for a transitional boundary layer on a flat plate with the experimental data:

a. Feiereisen and Acharya ($Re = 3.8\ 10^6$, $Tu = 0.7\ \%$) [8];
b. Abu-Ghannam and Shaw ($Re = 1.8\ 10^6$, $Tu = 1.25\ \%$) [9];
c. Schubauer and Klebanoff ($Re = 6\ 10^6$, $Tu = 0.03\ \%$) [9];
d. Dhawan and Narasimha ($Re = 1.1\ 10^6$, $Tu = 1.3\ \%$) [9].

For the experiment of Schubauer and Klebanoff the calculations show some delay in the determination of $\tilde{x}_s$ which can be explained by the worse definition of the beginning of the transition according to the criterion of Abu-Ghannam and Shaw at very low Tu. In the other cases $\tilde{x}_s$ is determined with very good precision which shows the advantages of the empirical criterion chosen here.

For the sake of brevity the results of only one of the experiments pointed out are presented in the paper now, as well as graphs only for the local friction coefficient.

The numerical results shown in Fig. 2 suggest a faster transition than in the experiment which leads to some differences in finding the end of the transition zone and also in calculating the flow characteristics. These discrepancies are due to the turbulence model and can be surmounted by a change of the parameters of the model or the intermittency factor but providing the necessary detailed measurements of the flow are made.

Although the flaws already pointed out are quite evident the investigations show and prove the reasonability of this method at low Tu.

3.2. Elliptic Cylinder

The second part of this study shows the obtained results for a transitional boundary layer on an elliptic cylinder in the simultaneous influence of the pressure gradient and free stream turbulence. In this case the calculations begin from the front stagnation point and finish near the separation point. Before using systematically the elaborated technology again a check is being made. The aim of this check is to establish the possibilities for monotonous approaching the numerical results for an elliptic cylinder to these for a flat plate when decreasing the ratio of the axes b/a. The results from this test are shown in Fig. 3 where the change of the local friction coefficient is given depending on the dimensionless distance $\tilde{x}$ measured from the front stagnation point in the direction of the big semi-axis. When the ratio of semi-axes is $b/a = 1/36$, there is a very good correspondence with the numerical results for a flat plate. Some differences are observed in the initial zone of the transition as well as in the rear part of the body which are due the influence of pressure gradient.

In Fig. 4 are presented the results for an elliptic cylinder with a greater ratio of the semi-axes $b/a=1/5$ and with different free stream levels. It is obvious that with the increase of the degree of external disturbances, the boundary layer is turbulized faster which results in increasing the viscous friction. These effects are successfully shown by the numerical procedure.

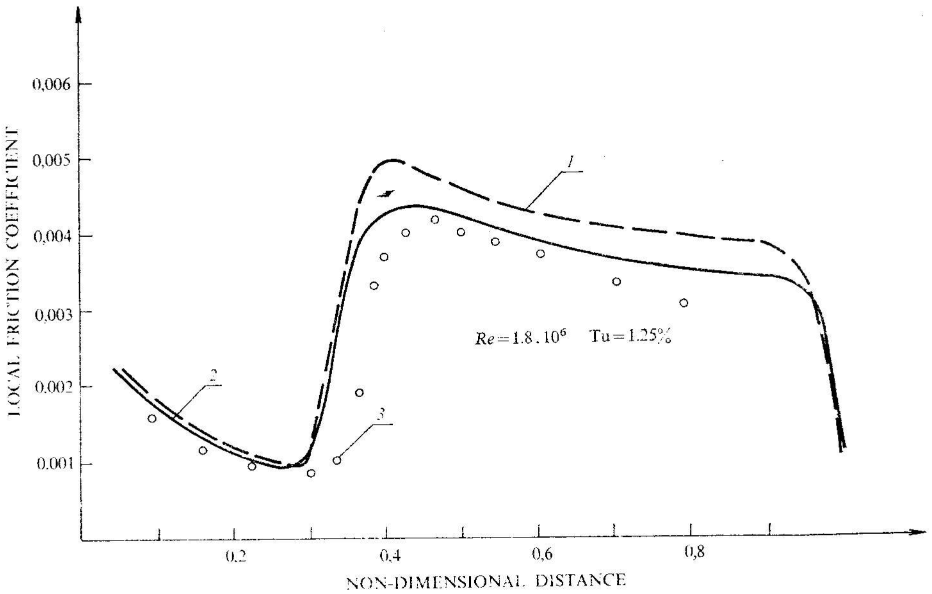

Fig. 3. Test of procedure for transitional boundary layer with non-zero pressure gradient: 1. — elliptic cylinder $b/a=1/12$; 2. — elliptic cylinder $b/a=1/36$; 3. — flat plate calculations

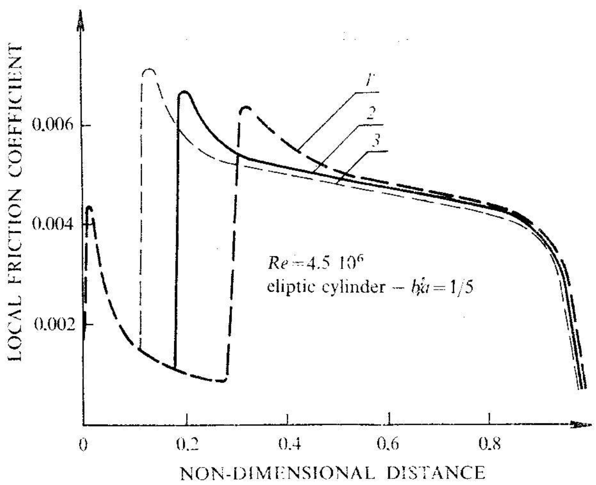

Fig. 4. Predictions of local friction coefficient for laminar-turbulent boundary layer under free stream turbulence: 1. — $Tu=0.75$ %; 2. — $Tu=1$ %; 3. — $Tu=1.5$ %

The results from the investigation of the influence of pressure gradient are demonstrated in Fig. 5. The change of pressure gradient is achieved by the change of the ratio of semi-axes b/a. The strong favorable (negative) gradient acting on the flow in the boundary layer of the first half of the body makes the flow stable and leads to a delay of the transition. The picture is quite different for the flow in the second part of the body. Here under the influence of the adverse (positive) pressure gradient the

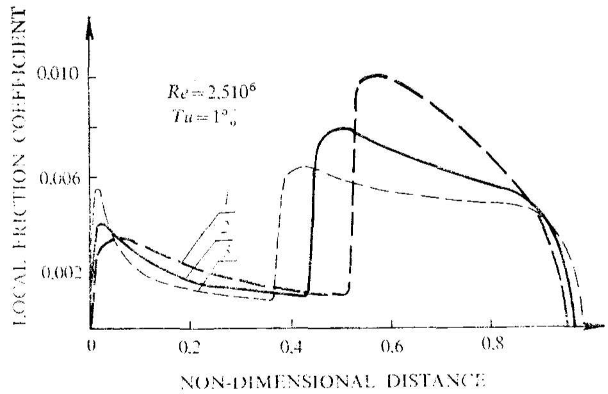

Fig. 5. Calculations of local friction coefficient for transitional boundary layer on different elliptic cylinders: 1. — $b/a=1/2$; 2. — $b/a=1/3$; 3. — $b/a=1/5$

flow is quickly turbulized which results in the rapid increase of viscous friction. When the ratio of semi-axes increases, the boundary layer separates much more earlier.

The numerical experiments carried out show that the elaborated technology for calculation of transitional boundary layer on an elliptic cylinder allows successful work also for a flow under pressure gradient.

4. Conclusions

What we have just reported in this work for the transitional boundary layer lets the following conclusions to be drawn:

1. The empirical criterion of Abu-Ghannam and Shaw defines well the beginning of transitional zone in the case of flow past a flat plate and an elliptic cylinder under the influence of free stream turbulence which guarantees the use of this criterion even in more complex situations;
2. The turbulence model chosen here represents well the picture of the flow in the transitional zone but it does not allow the precise determination of the end of this zone to be done;
3. In spite of the inaccuracy in the determination of the end of transitional zone the technology presented in this work admits of the successful calculation of a transitional boundary layer with and without pressure gradient.

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NOMENCLATURE

n — number of turbulence model equations

x, y — coordinate system (shown in Fig. 1)

$\tilde{x}$ — non-dimensional distance (see Fig. 1)

$y_+ = \frac{y u_\tau}{\nu}$, dimensionless value of y

a, b — length of elliptic cylinder semi-axes

l — flat plate length

l_m — mixing length

U_∞ — free stream reference velocity

u, v — components of mean velocity in x, y directions

u_τ — friction velocity

u', v', w' — components of fluctuating velocity

$k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$, turbulence kinetic energy

ε — dissipation rate

ν — kinetic viscosity

p — pressure

ρ — density

$Tu = 100\frac{\sqrt{\overline{u'^2}}}{U_\infty}$, free stream turbulence

δ — boundary layer thickness

θ — boundary layer momentum thickness

U_p — potential flow velocity

k_δ — turbulence kinetic energy at edge of boundary layer

$\lambda_\theta = \frac{\theta^2}{\nu}\frac{dU_p}{dx}$, pressure gradient parameter

γ — intermittency factor

$Re = \frac{U_\infty}{\nu}L$, Reynolds number: flat plate—$L = l$; elliptic cylinder—$L = 2a$

Re_θ — momentum thickness Reynolds number

$Re_t = \frac{k^2}{\nu\varepsilon}$, turbulent Reynolds number

$Re_x = \frac{U_\infty x}{\nu}$, local Reynolds number

$\tilde{L}_\gamma$ — non-dimensional transition length over which intermittency factor increase from 0.25 to 0.75

Subscripts

s — start of transition

e — end of transition