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Independent joint controllability of manipulator systems

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I. Introduction

The continuously increasing demands for enhanced productivity and improved precision have imposed special requirements for the integrated design of the mechanical part, drive and control systems. One of the main reasons for such a design is the coupled nonlinear dynamic behaviour of manipulator systems.

In most practical cases, each degree of freedom of industrial robots is controlled independently and some of the main arguments are the following:

— the complete and exact modelling of manipulator dynamics is difficult if not impossible to be performed;

— there is computation burden and on-line limitations when coupled dynamics models are used in the control systems;

— joint position and velocity feedback sensors are inherently implemented in the independent *PD*, *PID* and so on controllers.

In case of heavier manipulation loads and higher velocities, the conventional control methods can not ensure the required performance of robots because of large dynamic couplings. A natural way to overcome the dynamic complexity is to modify the design of arm linkages actuators so that the links are to be driven more independently. The central role in designing a robot with appropriate dynamics characteristics plays the inertial matrix. Recently [1,2], some conceptual frameworks have been developed in order to specify the inertia matrices, leading to a significant reduction of the inertial and dynamics coupling coefficients. When the geometrical and inertial parameters of manipulator links are fixed, some problems on the optimal design of the drive and control systems can be formulated. The best design of gear ratios and motor impedances for achieving quick and isotropic velocity convergence is performed in [3,4], solving nonlinear programming problems. The objective function in [4] is proved to be convex and an efficient optimization procedure is developed to search for the global optimal design. Although these works concern the problem of the inertial and dynamic coupling reduction, the question for independent joint controllability (*IJC*) is still open.

We want to solve the problem to find those control bounds (levels) at which the sign of any control input is the same as the sign of the corresponding acceleration, irrespective of the other joint motions. Providing such a condition for *IJC*, several advantages can be achieved:

— robustness of many feedback control schemes with independent joint controllers [7—11];

— admissible loading of joint actuators (the discrepancy in the signs of the control input and the corresponding acceleration may result in violating current limitations, energy loss, wear and other effects);

— controllability and optimal point-to-point control synthesis of robot dynamics [12–15];

— successful path planning of the manipulator motion in the presence of torque and control constraints, obstacles, etc. [16].

This *IJC*-condition is firstly discussed in [5,6]. We find in [5] those control levels which satisfy this condition for any geometrical configuration of the manipulator if a condition for diagonal dominant inertia matrix is fulfilled. The problem for IJC of a planar manipulator with two revolute joints (in which case a diagonal dominant condition may not be fulfilled) is considered in [6]. Some optimal design concepts are developed in this work using gear ratios and the ratio of the control input bounds as design parameters. In this case *IJC*-condition is satisfied for various control input ratios, depending on the geometrical configuration.

Now we are about to deal with the problem of IJC of manipulation robots in the general case of open-loop kinematic chain. A simple weak condition on the inverse inertia matrix is proved to be necessary and sufficient for the *IJC*. This condition is considered for a planar manipulator with three revolute joints.

2. Dynamic model and IJC

The following considerations will concern only those n manipulator joints which are inertially coupled. Via Lagrange formulation, the corresponding dynamic equations of joint motions could be written in the following form:

$$(1) \qquad \sum_{j=1}^{n} d_{ij}(q)\,\ddot{q}_j + c_i(q,\dot{q}) + s_i\dot{q}_i + g_i(q) = T_i(t), \quad i=1,\ldots,n$$

where: q_i are joint coordinates; $d_{ij}(q)$ are inertial coupling coefficients; $c_i(q,\dot{q})$ present centrifugal and Coriolis forces, $c_i(q,\dot{q}) = \dot{q}^T C_i(q)\,\dot{q}$ — quadratic forms; s_i — viscous friction coefficients; $g_i(q)$ stand for gravitation forces and Coulumb friction; T_i — joint torques;

The work space for joint coordinates is a compact set in the joint space

$$(2) \qquad Q = \{q \mid q_i^{min} \le q_i \le q_i^{max}\}.$$

As the inertia matrix (d_{ij}) is positive definite, its inverse (d_{ij}^{-1}) exists and is positive definite, too.

In order to drive the manipulator links, each independently of the others, the following condition should take place:

$$(3) \qquad sign\, T_i = sign\, \ddot{q}_i, \quad i=1,\ldots,n.$$

In what follows, we shall assume that the velocity and gravitation torques in (1) are compensated and the torques T_i are to provide decentralized control. Making use of Eqs (1), the condition (3) is then equivalent to the following one:

$$(4) \qquad sign\, T_i = sign \sum_{j=1}^{n} d_{ij}^{-1}\, T_j, \quad i=1,\ldots,n$$

More definitely, we will look for such bounds (levels) $\bar{T}_i$ of control inputs that:

$$(5) \qquad sign\, \bar{T}_i = sign\left(\sum_{j\neq i} d_{ij}^{-1}\, T_j + d_{ii}^{-1}\,\bar{T}_i\right), \quad i=1,\ldots,n$$

whenever $T_j \in [-\bar{T}_j;\bar{T}_j]$, $j \neq i$

Therefore the IJC-conditions (5) are satisfied if only

$$(6) \qquad d_{ii}^{-1}\,\overline{T}_i > \sum_{j\neq i} |d_{ij}^{-1}|\,\overline{T}_j,\ i=1,\ldots,n.$$

In other words, we have to prove that the following system of inequalities:

$$(7) \qquad \sum_{=1}^{n} D_{ij}\,u_j > 0,\ i=1,\ldots,n$$

has got positive solution, i.e., $u_i > 0$, $i=1,\ldots,n$
where: $D_{ij} = s_{ij}\,|d_{ij}^{-1}|$, $(s_{ii}=1,\ s_{ij}=-1 \text{ if } i\neq j)$.

In order to attain this property, the matrix $D=(D_{ij})$ should be positive definite as the inertia matrix (d_{ij}) is. We come to the following theorem:

Theorem: The system of inequalities (7) has got positive solution if and only if the matrix D is positive definite.

The proof is given in the APPENDIX.

To determine the best solution of the system of inequalities (7) we may follow [6] or in general we take a point equidistant from the sides of the polygon given by the equations of (7). So, we take the positive solution of the following system of equations:

$$(8) \qquad \sum_{j=1}^{n} D_{ij}\,u_j = U\Delta_i,\ i=1,\ldots,n,\ U>0$$

where:

$$\Delta_i = \left(\sum_{j=1}^{n} D_{ij}^2\right)^{\frac{1}{2}}$$

U — the distance of the corresponding point from the above mentioned hyperplanes.

3. Example ($n=3$, Fig. 1)

We take into consideration a planar manipulator with three revolute joints. The inertial matrix coefficients are the following [17]:

$$d_{11} = r_1^2(m_1/4 + m_2 + m_3)$$
$$d_{12} = r_1 r_2(m_2/2 + m_3)\cos(\theta_1 - \theta_2) = d_{21}$$
$$d_{13} = r_1 r_3 m_3 \cos(\theta_1 - \theta_3) = d_{31}$$
$$d_{22} = r_2^2(m_2/4 + m_3)$$
$$d_{23} = r_2 r_3 m_3 \cos(\theta_2 - \theta_3) = d_{32},\ d_{33} = r_3^2 m_3$$

The manipulator links are the most inertially coupled when $\theta_1=\theta_2=\theta_3$. In this case we find a condition that geometrical and mass parameters should satisfy in order to provide *IJC*. In case $n=3$ the matrix D will be positive definite if only $detD>0$. After simple calculations, we come to the following condition:

$$(9) \qquad m_1/m_3 - 64\ m_2/m_1 > 32.$$

It means, actually, that the links' masses should strongly decrease, in progressing from the base to the end-effector. This fact is in accordance with the theory developed in [2].

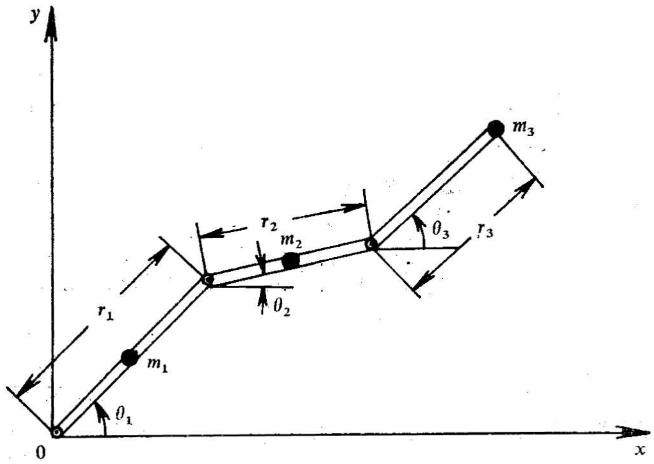

Fig. 1

4. Conclusion

A general condition on the inverse inertia matrix is proved to be a necessary and sufficient condition for the independent joint controllability of the coupled dynamic systems of manipulation robots. The results so obtained can effectively be used in the design of the mechanical structure and the corresponding drive-control system, as well. Besides, optimal gain coefficients ratios in regard to the individual joint controllers can be determined as in [6] to ensure the best mutual compensation of the inertial couplings at any manipulator configuration.

APPENDIX — Proof of the Theorem

a) sufficiency

As the matrix D is positive definite, *det* $D>0$ and the system of inequalities (7) has got infinitely many solutions. Applying mathematical induction with respect to n, we shall prove that any solution u of this system is positive, i. e., $u_i>0$ for $i=1,\ldots,n$:

i—$n=1$ — it is true;

ii — if the assertion is true for $n=\bar{n}$, then we prove it will be true for $n=\bar{n}+1$:

Firstly, we prove that at least one $u_k>0$.

Assume that $\forall u_k \le 0$, $k=1,\ldots,\bar{n}+1$. Then, it follows from (7) that:

$$\sum_{i=1,\, j=1}^{\bar{n}+1} D_{ij} u_i u_j \le 0 \tag{A1}$$

which contradicts the positive definiteness of the matrix D $(u \neq 0)$. Therefore, there exists at least one $u_k>0$, e. g., $u_{\bar{n}+1}>0$. Then the first $\bar{n}$ numbers u_i will satisfy the system of the corresponding inequalities (7) whose matrix is positive definite, too. Hence, $u_k>0$, $\forall k=1,\ldots,\bar{n}+1$.

This part of the Theorem is found to be a version of the S t i e l t i j e s theorem [18].

b) necessity

There is a positive solution $u_i=p_i>0$ of the system (7):

$$\text{(A2)} \qquad \sum_{j=1}^{n} D_{ij}p_j>0,\ i=1,\ldots,n.$$

Then the matrix D is positive definite.

Let $x\neq0$ and $x_i=\sqrt{p_i}y_i$. Making use of (A2), we obtain the following relations:

$$\text{(A3)} \qquad \sum_{i=1,j=1}^{n} D_{ij}x_ix_j=\sum_{i=1,j=1}^{n} D_{ij}\sqrt{p_i}\sqrt{p_j}y_iy_j>\sum_{i=1}^{n}\left(\sum_{j\neq i}-D_{ij}p_j\right)y_i^2$$

$$+\sum_{i\neq j} D_{ij}\sqrt{p_i}\sqrt{p_j}y_iy_j=\sum_{i\neq j}|d_{ij}^{-1}|(p_jy_i^2+p_iy_j^2-2\sqrt{p_i}\sqrt{p_j}y_iy_j)\geq0$$

Thus the proof is completed.

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