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On the optimal control synthesis for a class of nonlinear mechanical systems

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1. Introduction

Many industrial robots, walking machines and other mechanical devices are mechanical systems, each degree of freedom of which being controlled individually. The dynamic performance of those systems is characterized mainly by the positioning accuracy, the movement execution time and the energy loss. These contradictory requirements are even more difficult to satisfy because of the existing couplings between the subsystems. To insure existence of feasible solutions we assume a decentralized controllability condition in the sense that the sign of any control input (at least at extreme value) is the same as the sign of the corresponding acceleration, regardless of the other subsystem's influence. For the individually controlled mechanical systems to behave well such a condition is found to be a reasonable one [1, 2, 3].

An attempt to solve that complex highly nonlinear problem of the optimal control theory is presented in [4]. A suboptimal solution is obtained there, utilizing control laws being optimal for one-degree-of-freedom mechanical systems.

The direct optimization approach is improved in the present work employing spline approximations of the control laws corresponding with the Pontryagin's maximum principle. The final switching times of the trial control functions are used in a natural way to solve the given TPBVP with values of all other describing parameters being fixed. Existence of a sufficiently large set of feasible solutions is guaranteed by the decentralized controllability condition and thus a satisfactory suboptimal solution can be obtained.

2. Problem Statement and Pontryagin's Maximum Principle

Consider mechanical systems with dynamics described by the following coupled nonlinear differential equations:

$$(1)\qquad \ddot{x}(t)=g(x(t),\ \dot{x}(t))+f(x(t))u(t)$$

where: $x(t)\in R^n$, $u(t)\in R^n$, $g(\cdot,\cdot)\in C(R^n\times R^n;\ R^n)$,

$$f(\cdot)\in C(R^n;\ R^{n\times n}).$$

According to [5] such systems with the number of degrees of freedom equal to the number of control functions are regarded as being "systems with full control".

Boundary conditions:

(2) $$x(t^0)=x^0,\quad \dot{x}(t^0)=0 \text{ — initial state}$$

(3) $$x(t^f)=x^f,\quad \dot{x}(t^f)=0 \text{ — final state,}$$

where t^f is not given in advance.

Performance index:

(4) $$J=\int_{t^0}^{t^f}(1+1/2\alpha u^T(t)u(t))dt,\quad \alpha>0.$$

Control constraints:

(5) $$|u_i(t)|\leq M_1,\quad i=1,\ldots,n \Leftrightarrow u\in U.$$

The problem is to find $u\in U$ that will drive the system (1) from the initial state (2) to the final state (3) so that the weighted time-energy loss functional (4) to be minimal and the constraints (5) to be satisfied.

According to [6, 7] the problem so defined is a totally nonsingular control problem. Applying the Pontryagin's maximum principle in the latter work, the following optimal control laws have been derived:

(6) $$u_i(t)=\begin{cases}+M_i & p_i(t)/\alpha>M_i\\ p_i(t)/\alpha & -M_i<p_i(t)/\alpha<M_i\\ -M_i & p_i(t)/\alpha<-M_i\end{cases}$$

where $p_i(t)$ are the costate functions.

Generally speaking it does not make much sense to compute with great effort the exact solution of the mathematical model and to effectuate then this solution in an approximate way and with big efforts for the real system. In such cases the direct computation of such an "approximation" will be of a greater value.

3. Direct Search Approach

Relying on the optimal control laws (6), we suggest the spline approximations as depicted in Fig. 1 to construct a set of trial control functions u_i, where: $k_i=1,\ldots,\bar{k}_i$ is the index of the switching times of function u_i and $L^{k_i}>0$ are the corresponding slopes.

The time interval (t^0, t_i^f) within the control function u_i is actuated, is devided by the switching times into $\bar{k}_i+1$ subintervals $(t^{k_i}; t^{k_i+1})$ where $t^{k_i}=t^0$ if $k_i=0$ and $t^{k_i+1}=t_i^f$ if $k_i=\bar{k}_i$.

On describing the control functions, it is necessary to define some other parameters. As shown in Fig. 1, the time t^* in an interval $(t^{k_i}; t^{k_i+1})$ is calculated through

(7) $$t^*=(L^{k_i}t^{k_i}+L^{k_i+1}t^{k_i+1})/(L^{k_i}+L^{k_i+1})\quad \text{when } k_i=1,\ldots,\bar{k}_i-1.$$

We set $t^*=t^0$ if $k_i=0$ and $t^*=t^{\bar{k}_i}$ if $k_i=\bar{k}_i$.

We define also sign-switching functions $S_i(t)$ corresponding to control functions $u_i(t)$ as follows:

(8) $$S_i(t)=S^{k_i}=\text{const.},\quad t\in(t^{k_i}; t^{k_i+1}),$$

where $S^{k_i+1} = -S^{k_i}$, $k_i = 0, \ldots, \bar{k}_i$ and $S_i(0) = \pm 1$.

The control function $u_i(t)$ on an interval $(t^{k_i}; t^{k_i+1})$ where $k_i = 0, \ldots, \bar{k}_i$ is determined by

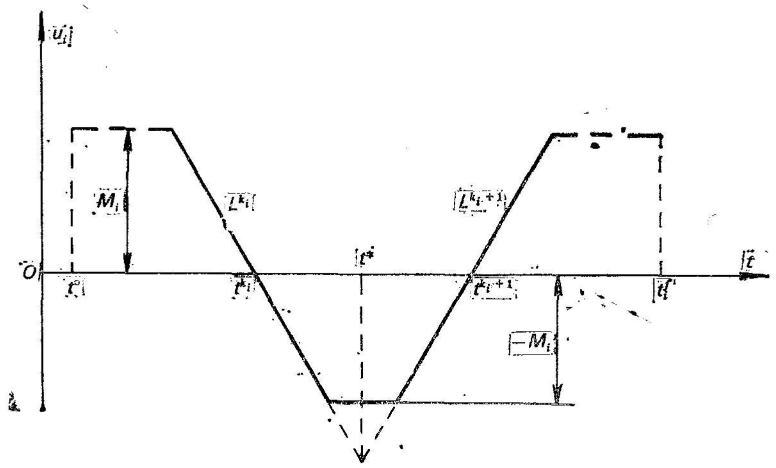

Fig. 1. A characteristic part of a trial control function u_i

(9) $$u_i(t) = \begin{cases} S_i(t)M_i(t), & |t - t^k| \geq M_i/L^k \\ S_i(t)\,|t - t^k|\,L^k, & |t - t^k| < M_i/L^k \end{cases}$$

where $k = k_i$ if $t \leq t^*$ and $k = k_i + 1$ if $t \geq t^*$.

Therefore we need the following parameters to describe a trial control function:

— number $\bar{k}_i$ of the switching times (in practice $\bar{k}_i \leq 2$)

— switching times t^{k_i}, $k_i = 1, \ldots, \bar{k}_i$

— initial value $S_i(0) = +1$ or -1

(10) — slopes L^{k_i}, $k_i = 1, \ldots, \bar{k}_i$.

In this way the optimal control laws can be perameterized by the maximum principle quite naturally. We devide the set of describing control parameters into two subsets. One of them consists of the final switching times $t^{\bar{k}_i}$, $i = 1, \ldots, n$, thus forming a vector $\bar{t}$. With fixed values of all the other control parameters their set denoted by p_{opt}, the n-dimensional vector $\bar{t}$ is assigned to solve the two-point boundary-value problem *TPBVP* (1÷3) and thus obtaining a feasible solution. And varying the values of the parameters p_{opt} we are looking for the optimum (the best suboptimal feasible solution as regards the performance index (4)).

The main point of our direct search approach is the solution of the TPBVP.

Denote by

(11) $$x(t) = x(t, \bar{t}, p_{opt})$$

the dependence of motion on $\bar{t}$ and p_{opt}. We determine the final times of the particular coordinate motions $x_i(t)$ quite naturally:

(12) $$t_i^f:\ t_i^f > t^{\bar{k}_i} \,\&\, \dot{x}_i(t_i^f, \bar{t}, p_{opt}) = 0, \quad i = 1, \ldots, n.$$

Thus the final conditions $(3)_2$ for the velocities are satisfied with $t^f = \max t_i^f$, $i=1 \ldots, n$. The correspondent reached positions at these final times are denoted by

$$F_i^{p_{opt}}(\bar{t}) = x_i(t_i^f, \bar{t}, p_{opt}), \quad i=1, \ldots, n. \tag{13}$$

So, for the other final condition $(3)_1$ to be fulfilled, the missed distance from the required final position must be zero:

$$F_i^{p_{opt}}(\bar{t}) - x_i^f = 0, \quad i=1, \ldots, n. \tag{14}$$

This system of n equalities is regarded as being a system of n shooting equations with respect to the n-dimensional vector $\bar{t}$. So that the main steps of the optimization procedure we propose are the following:

1. Guess $\bar{k}_i$; t^{k_i}; L^{k_i}; $S_i(0)$; for all $k_i = 1, \ldots, \bar{k}_i$, $i=1, \ldots, n$.
2. Perform a test movement. Check Eqs. (14): if "yes" Go to 4, if "no" Go to 3.
3. Update $\bar{t}$. Go to 2.
4. If the value of the performance index (4) is not satisfactory then update p_{opt} and Go to 2. Else Stop.

4. Discussion

To succeed in carrying out the optimisation procedure above proposed, that is to succeed in finding sufficiently large number of feasible solutions, we need the following decentralized controllability (DC) conditions on (1) to be fulfilled:

$$|f_{ii}(x(t)) M_i| > |g_i(x(t), \dot{x}(t))| + \sum_{j=1,\, j\neq i}^{n} f_{ij}(x(t)) u_j|, \quad i=1, \ldots, n \tag{15}$$

$\forall u \in U$, $\forall (x, \dot{x}) \in X \times V$ where $X \ni x^0$, x^f and $V \ni 0$ are parallelipipeds in R^n.

In the presence of the DC-conditions (15) one can easily verify the following statements:

Lemma 1: The final times t_i^f in (12) are finite and the reached positions $x_i(t_i^f)$ thus can be defined by (13) as functions F_i^{opt} of $\bar{t} = (t^{\bar{k}_1}, \ldots, t^{\bar{k}_n})$.

Lemma 2: F_i ("opt" is omitted) are continuous functions of $\bar{t}$ [8, 9].

Lemma 3: There exist bounds t_i^- and t_i^+, $t_i^+ > t_i^-$, $i=1, \ldots, n$ such that the function F considered on the cube $P = \{\bar{t} \mid t_i^- \leq t^{\bar{k}_i} \leq t_i^+\}$ has the following property: for any pair of points on the boundary of P: $\bar{t} \in bdP$, $\bar{t}_{sym} \in bdP$ symmetrical about the centre of the cube P there exists i:

$$(F_i(\bar{t}) - x_i^f)(F_i(\bar{t}_{sym}) - x_i^f) < 0. \tag{16}$$

Now, we are in the position to apply a known theorem asserting existence of solution of a nonlinear vector equation [10]: Theorem 1 (Borsuk-Ulam): Let $G \in C(P; R^n)$

$$G(z) = -G(z_{sym}), \quad \forall z, z_{sym} \in bdP \tag{17}$$

then $\exists z^* \in P$: $G(z^*) = 0$.

Relying on Lemmas 1÷3 and Theorem 1 we can state the following theorem:

Theorem 2: With p_{opt} fixed, there exists a solution of the shooting equations (14). Some details about the proof of this theorem are given in [11].

Therefore the TPBVP (1÷3) is solvable employing the control laws described in section 3 (7÷10) and one can varying p_{opt} synthesize a satisfactory number of feasible solutions. Thus in the presence of the DC-conditions (15) a guarantee in the realization of our direct search optimization procedure can be given.

5. Conclusion

A problem on the optimal control of mechanical systems with individually controllable degrees of freedom is taken into consideration. The performance index is a weighted minimum time-energy loss criterion. A direct search optimization procedure is suggested involving control laws matching the Pontryagin's maximum principle. In other words, the suboptimal solution obtained by means of that procedure is an appropriate spline approximation to the optimal solution itself. To ensure the success of the optimization procedure an inherent decentralized controllability condition is assumed.

Another advantage of the method proposed is that the direct search procedure can be implemented on the mechanical system itself for a final adjustment of the control parameters. This is required in most cases when the dynamic model is not complete and exact.

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